UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MSG 265 – Design and Analysis of Experiments
[Rekabentuk dan Analisis Uji Kaji]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOURTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

[Arahan : *Jawab **semua empat** [4] soalan.]*

1. The yield of a chemical process is being studied. The two most important variables are thought to be the pressure and the temperature. Three levels of each factor are selected, and a factorial experiment with two replicates is performed. The yield data are as follow:

Temperature	Pressure 200	215	230	Total: $y_{i..}$
150	90.4	90.7	90.2	
	90.2	90.6	90.4	542.5
160	90.1	90.5	89.9	
	90.3	90.6	90.1	541.5
170	90.5	90.8	90.4	
	90.7	90.9	90.1	543.4
Total: $y_{.j.}$	542.2	544.1	541.1	1627.4

(i) Complete the following ANOVA table.

Source of variation	Sum of Squares	Degrees of freedom	Mean Square	F_0
Temperature, T				
Pressure, P				
T*P	0.069			
Error			0.018	
Total				

(ii) Write the linear statistical model for this experiment. Obtain the point estimates of the mean yield at each temperature levels.

(iii) Based on the ANOVA table in part (i), analyse the data and write your conclusion. Use $\alpha = 0.05$.

(iv) Partition the sum of squares for temperature into the two polynomial effects and test for its significance. Write your conclusions. Use $\alpha = 0.05$.

(v) Conduct a comparison test to determine which levels of temperature are significantly different. Under what conditions would you operate this process? (Refer to Appendix)

(vi) Suppose that both factors, pressure and temperature are chosen at random. Analyse the data and write your conclusion. Find the point estimates of the variance components.

[100 marks]

1. *Hasil suatu proses kimia telah dikaji. Dua pemboleh ubah yang dirasakan paling penting ialah tekanan dan suhu. Tiga aras bagi setiap faktor dipilih, dan suatu uji kaji faktorial dengan dua replika telah dijalankan. Data bagi hasilnya adalah seperti berikut:*

	Tekanan			
Suhu	200	215	230	*Jumlah:* $y_{i..}$
150	90.4	90.7	90.2	
	90.2	90.6	90.4	542.5
160	90.1	90.5	89.9	
	90.3	90.6	90.1	541.5
170	90.5	90.8	90.4	
	90.7	90.9	90.1	543.4
Jumlah: $y_{.j.}$	542.2	544.1	541.1	1627.4

(i) Lengkapkan jadual ANOVA berikut.

Punca Ubahan	*Hasil Tambah Kuasa Dua*	*Darjah Kebebasan*	*Min Kuasa Dua*	F_0
Suhu, T				
Tekanan, P				
*T*P*	0.069			
Ralat			0.018	
Jumlah				

(ii) Tuliskan model linear statistik bagi uji kaji ini. Dapatkan anggaran titik bagi min hasil pada setiap aras suhu.

(iii) Berdasarkan jadual ANOVA di bahagian (i), jalankan analisis data dan tulis kesimpulan anda. Guna $\alpha = 0.05$.

(iv) Bahagikan hasil tambah kuasa dua suhu kepada dua kesan polinomial dan uji keertiannya. Tulis kesimpulan anda. Guna $\alpha = 0.05$.

(v) Jalankan suatu ujian perbandingan untuk menentukan aras suhu yang berbeza secara bererti. Dalam keadaan manakah akan anda jalankan proses ini? (Rujuk Lampiran)

(vi) Andaikan bahawa kedua-dua faktor, tekanan dan suhu dipilih secara rawak. Jalankan analisis data dan tulis kesimpulan anda. Dapatkan anggaran titik bagi komponen varians.

[100 markah]

2. In a process development study on yield, four factors were studied, each at two levels: time (A: 2.5 hours , 3.0 hours), concentration (B: 14% , 18%), pressure (C: 60 psi , 80 psi), and temperature (D: 225°C , 250°C). A single replicate of a 2^4 design was run, and the resulting data are given as follows:

$(1) = 12$	$c = 17$	$d = 10$	$cd = 19$
$a = 18$	$ac = 15$	$ad = 25$	$acd = 21$
$b = 13$	$bc = 20$	$bd = 13$	$bcd = 17$
$ab = 16$	$abc = 15$	$abd = 24$	$abcd = 23$

The following effect estimates have been calculated:

$A = 4.50$	$AB = -0.75$	$ABC = 1.00$
$B = 0.50$	$AC = 4.25$	$ABD = 0.75$
$C = 2.00$	$AD = 4.00$	$ACD = -0.25$
$D = 3.25$	$BC = 0.25$	$BCD = -0.75$
	$BD = 0.00$	$ABCD = 1.00$
	$CD = 0.00$	

(i) Conduct an analysis of variance using the normal probability plot of the effect estimates for guidance in forming an error term. What are your conclusions? (Refer to Appendix)

(ii) Write down a regression model relating yield to the important process variables.

(iii) Analyse the residuals from this experiment. Are there any indications of model inadequacy? (Refer to Appendix)

(iv) Can this design be collapse into a 2^3 design with two replicates? If so, obtain a table showing the data from the collapsed design.

(v) Suppose that two blocks are required in this experiment. Construct a design with $ABCD$ confounded with blocks. Analyse the data from this design.

[100 marks]

2. *Dalam suatu kajian pembangunan proses terhadap hasil, empat faktor telah dikaji, setiap satu pada dua aras: masa (A: 2.5 jam , 3.0 jam), kepekatan (B: 14% , 18%), tekanan (C: 60 psi , 80 psi), dan suhu (D: 225°C , 250°C). Satu replika tunggal bagi suatu reka bentuk 2^4 telah dijalankan, dan data yang diperoleh diberikan seperti berikut:*

$(1)=12$	$c=17$	$d=10$	$cd=19$
$a=18$	$ac=15$	$ad=25$	$acd=21$
$b=13$	$bc=20$	$bd=13$	$bcd=17$
$ab=16$	$abc=15$	$abd=24$	$abcd=23$

Anggaran kesan berikut telah dihitung:

$A=4.50$	$AB=-0.75$	$ABC=1.00$
$B=0.50$	$AC=4.25$	$ABD=0.75$
$C=2.00$	$AD=4.00$	$ACD=-0.25$
$D=3.25$	$BC=0.25$	$BCD=-0.75$
	$BD=0.00$	$ABCD=1.00$
	$CD=0.00$	

(i) *Jalankan suatu analisis varians dengan menggunakan plot kebarangkalian normal anggaran kesan berikut sebagai panduan untuk menetapkan sebutan ralat. Apakah kesimpulan anda? (Rujuk Lampiran)*

(ii) *Tulis satu model regresi yang menghubungkan hasil dengan pemboleh ubah proses yang penting.*

(iii) *Jalankan analisis bagi reja dari uji kaji ini. Adakah terdapat sebarang tanda ketidakcukupan model? (Rujuk Lampiran)*

(iv) *Bolehkah reka bentuk ini diturunkan kepada suatu reka bentuk 2^3 dengan dua replika? Jika boleh, dapatkan satu jadual yang menunjukkan data dari reka bentuk yang telah diturunkan itu.*

(v) *Andaikan dua blok diperlukan dalam uji kaji ini. Bangunkan satu reka bentuk dengan ABCD dibaur dengan blok. Jalankan analisis varians bagi data dari reka bentuk ini.*

[100 markah]

3. (a) The surface finish of metal parts made on four machines is being studied. An experiment is conducted in which each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine. The data are shown in the following table.

Operator	Machine 1			Machine 2			Machine 3			Machine 4		
	1	2	3	1	2	3	1	2	3	1	2	3
	79 62	94 74	46 57	92 99	85 79	76 68	88 75	53 56	46 57	36 53	40 56	62 47
Total; $y_{ij.}$	141	168	103	191	164	144	163	109	103	89	96	109
Total; $y_{i..}$	412			499			375			294		

Also given,

$$\sum_{i=1}^{4}\sum_{j=1}^{3}\sum_{k=1}^{2} y_{ijk}^2 = 111466 \qquad \sum_{i=1}^{4}\sum_{j=1}^{3} y_{ij.}^2 = 220904$$

$$\sum_{i=1}^{4} y_{i..}^2 = 645806 \qquad \sum_{j=1}^{3} y_{.j.}^2 = 840106$$

(i) Analyze the data and write your conclusions.

(ii) Suppose that the data had been analyzed as a two-factor factorial experiment. Obtain an ANOVA table for this analysis.

[50 marks]

(b) Suppose that you want to investigate the factors that potentially affect the cooking of rice.

(i) What would you use as a response variable in this experiment? How would you measure the response?

(ii) List all the potential sources of variability that could impact the response.

(iii) Complete the first 3 steps of the guidelines for designing experiments, that is, recognition of and statement of the problem, selection of the response variable and choice of factors, levels and ranges.

[50 marks]

3. (a) *Hasil permukaan bahagian logam yang dibuat dari empat mesin telah dikaji. Satu uji kaji telah dijalankan dengan setiap mesin dikendali oleh tiga orang operator dan dua spesimen dari setiap operator telah diambil dan diuji. Oleh kerana lokasi mesin-mesin tersebut, operator yang berlainan diguna pada setiap mesin. Data yang diperoleh ditunjukkan dalam jadual berikut.*

	Mesin 1			*Mesin 2*			*Mesin 3*			*Mesin 4*		
Operator	1	2	3	1	2	3	1	2	3	1	2	3
	79	94	46	92	85	76	88	53	46	36	40	62
	62	74	57	99	79	68	75	56	57	53	56	47
Jumlah; $y_{ij.}$	141	168	103	191	164	144	163	109	103	89	96	109
Jumlah; $y_{i..}$	412			499			375			294		

Juga diberi,

$$\sum_{i=1}^{4}\sum_{j=1}^{3}\sum_{k=1}^{2} y_{ijk}^2 = 111466 \qquad \sum_{i=1}^{4}\sum_{j=1}^{3} y_{ij.}^2 = 220904$$

$$\sum_{i=1}^{4} y_{i..}^2 = 645806 \qquad \sum_{j=1}^{3} y_{.j.}^2 = 840106$$

(i) *Jalankan analisis data dan tulis kesimpulan anda.*

(ii) *Andaikan data tersebut telah dianalisis sebagai satu uji kaji faktorial dua faktor. Dapatkan satu jadual ANOVA bagi analisis ini.*

[50 markah]

(b) *Andaikan bahawa anda ingin mengkaji faktor-faktor yang berpotensi memberi kesan terhadap proses memasak nasi.*

(i) *Apakah pemboleh ubah yang anda akan guna sebagai pemboleh ubah sambutan dalam uji kaji ini? Bagaimana anda akan mengukur sambutan tersebut?*

(ii) *Senaraikan kesemua punca ubahan yang boleh memberi impak pada sambutan.*

(iii) *Lengkapkan 3 langkah pertama bagi garis panduan mereka bentuk uji kaji, iaitu, mengenalpasti dan memberi pernyataan masalah, pemilihan pemboleh ubah sambutan dan pilihan faktor, aras dan julat.*

[50 markah]

4. (a) A researcher uses a 2^{5-2} design to investigate the effect of A (condensation temperature), B (amount of material), C (solvent volume), D (condensation time) and E (amount of material 2) on yield. The results obtained are as follows;

$$e = 23.2 \quad ad = 16.9 \quad cd = 23.8 \quad bde = 16.8$$
$$ab = 15.5 \quad bc = 16.2 \quad ace = 23.4 \quad abcde = 18.1$$

(i) Verify that the design generators used are $I = ACE$ and $I = BDE$.

(ii) Write down the complete defining relation and the aliases for this design.

(iii) Estimate the main effects.

(iv) Prepare an analysis of variance table, using the AB and AD interactions as the error. Given that $SS_T = 89.74$.

[60 marks]

(b) A study was performed on wear of a bearing y and its relationship to x_1 = oil viscosity and x_2 = load. The following output were obtained.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.963[a]	.927	.916	16.359

a. Predictors: (Constant), x2, x1

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	44157.087	2	22078.543	82.505	.000[a]
	Residual	3478.851	13	267.604		
	Total	47635.937	15			

a. Predictors: (Constant), x2, x1

b. Dependent Variable: y

Coefficients[a]

Model		Unstandardized Coefficients	
		B	Std. Error
1	(Constant)	1566.078	61.592
	x1	7.621	.618
	x2	8.585	2.439

a. Dependent Variable: y

(i) Give a multiple linear regression model for this study.

(ii) Based on the given ANOVA table, write the conclusion for this study.

(iii) Compute the t statistics for each model parameter. Write the conclusions that can be made from these quantities.

[40 marks]

4. (a) *Seorang penyelidik mengguna satu reka bentuk 2^{5-2} untuk mengkaji kesan A (suhu pemeluwapan), B (amaun bahan), C (isi padu pelarut), D (masa pemeluwapan) dan E (amaun bahan 2) terhadap hasil. Hasil yang diperoleh adalah seperti berikut:*

$$e = 23.2 \quad ad = 16.9 \quad cd = 23.8 \quad bde = 16.8$$
$$ab = 15.5 \quad bc = 16.2 \quad ace = 23.4 \quad abcde = 18.1$$

(i) *Sahkan bahawa penjana reka bentuk yang diguna adalah $I = ACE$ and $I = BDE$.*

(ii) *Tulis hubungan pentakrif yang lengkap dan alias-alias bagi reka bentuk ini.*

(iii) *Anggarkan kesan-kesan utama.*

(iv) *Bina satu jadual analisis varians, dengan menggunakan saling tindak AB dan AD sebagai ralat. Diberi $SS_T = 89.74$.*

[60 markah]

(b) *Satu kajian telah dijalankan terhadap tahap haus suatu bebola y dan hubungannya dengan x_1 = kepekatan minyak dan x_2 = muatan. Output berikut telah diperoleh.*

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.963[a]	.927	.916	16.359

a. Predictors: (Constant), x2, x1

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	44157.087	2	22078.543	82.505	.000[a]
	Residual	3478.851	13	267.604		
	Total	47635.937	15			

a. Predictors: (Constant), x2, x1

b. Dependent Variable: y

Coefficients[a]

Model		Unstandardized Coefficients	
		B	Std. Error
1	(Constant)	1566.078	61.592
	x1	7.621	.618
	x2	8.585	2.439

a. Dependent Variable: y

(i) *Berikan model regresi linear berganda untuk kajian ini.*

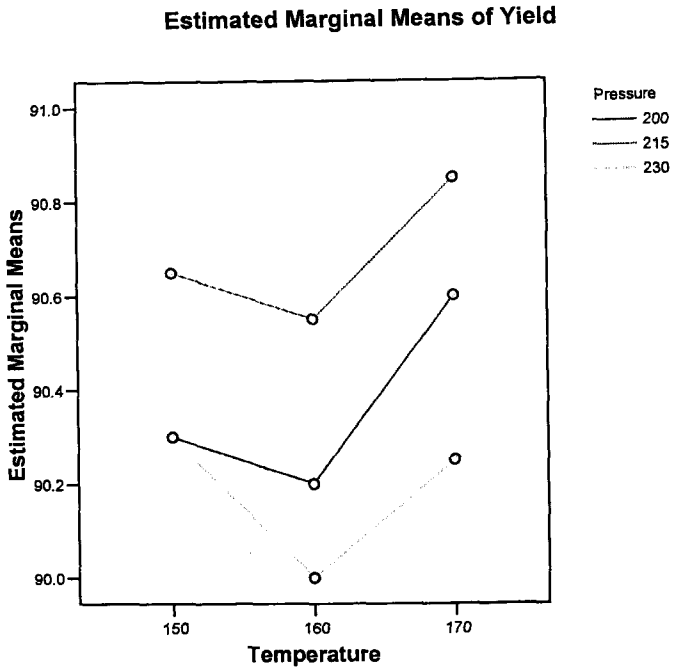
(ii) *Berdasarkan jadual ANOVA yang diberi, tulis kesimpulan kajian ini.*

(iii) *Hitung statistik t bagi setiap parameter model. Tulis kesimpulan yang boleh dibuat dari nilai ini.*

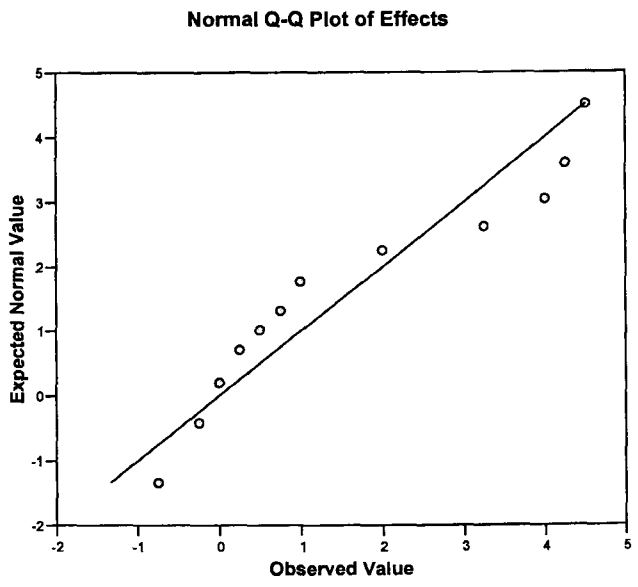
[40 markah]

Appendix / Lampiran

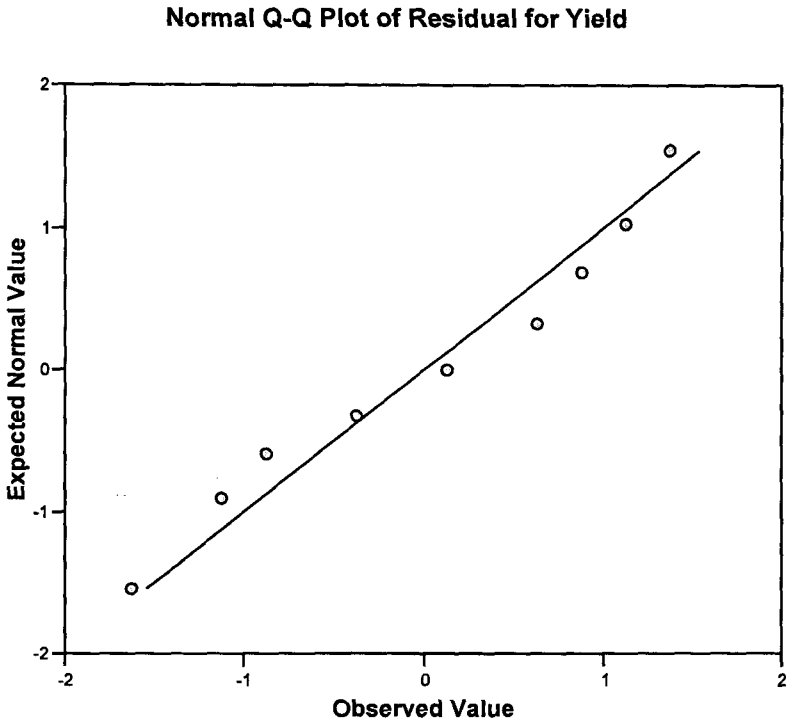
Question 1(v) / Soalan 1(v)

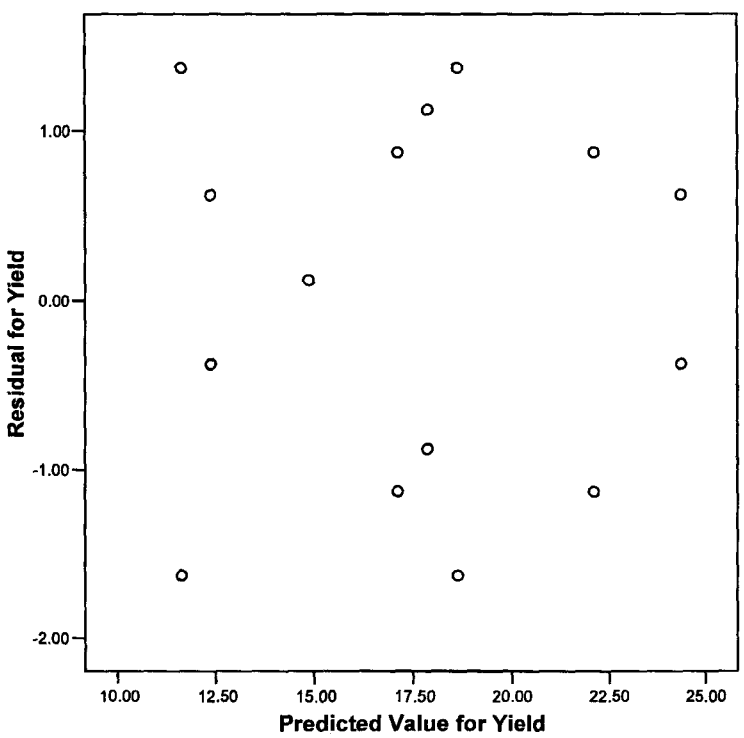


Question 2(i) / Soalan 2(i)



Question 2(iii) / Soalan 2(iii)





MSG265/4 – DESIGN AND ANALYSIS OF EXPERIMENTS

1. Two-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{Subtotal} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = SS_{Subtotal} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{Subtotal} \quad \text{atau} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

Orthogonal Polynomial :

Effect : $\sum_{j=1}^{a} c_j y_{.j.}$

$$SS_{Effect} = \frac{\left(\sum_{j=1}^{a} c_j y_{.j.}\right)^2}{an\sum_{j=1}^{a} c_j^2}$$

Orthogonal Polynomial Coefficients:

	$a = 3$		$a = 4$		
c_j	Linear	Quadratic	Linear	Quadratic	Cubic
1	-1	1	-3	1	-1
2	0	-2	-1	-1	3
3	1	1	1	-1	-3
4			3	1	1
5					

Expected Mean Square
Random Effects Model:

$$E[MS_A] = \sigma^2 + n\sigma^2_{\tau\beta} + bn\sigma^2_{\tau}$$
$$E[MS_B] = \sigma^2 + n\sigma^2_{\tau\beta} + an\sigma^2_{\beta}$$
$$E[MS_{AB}] = \sigma^2 + n\sigma^2_{\tau\beta}$$
$$E[MS_E] = \sigma^2$$

Mixed Model :
(*A*: fixed, *B*: random)

$$E[MS_A] = \sigma^2 + n\sigma^2_{\tau\beta} + \frac{bn\sum_{i=1}^{a}\tau_i^2}{a-1}$$
$$E[MS_B] = \sigma^2 + an\sigma^2_{\beta}$$
$$E[MS_{AB}] = \sigma^2 + n\sigma^2_{\tau\beta}$$
$$E[MS_E] = \sigma^2$$

2. Three-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y^2_{ijkl} - \frac{y^2_{....}}{abcn}$$
$$SS_A = \sum_{i=1}^{a}\frac{y^2_{i...}}{bcn} - \frac{y^2_{....}}{abcn}$$
$$SS_B = \sum_{j=1}^{b}\frac{y^2_{.j..}}{acn} - \frac{y^2_{....}}{abcn}$$
$$SS_C = \sum_{k=1}^{c}\frac{y^2_{..k.}}{abn} - \frac{y^2_{....}}{abcn}$$
$$SS_{Subtotal(AB)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y^2_{ij..}}{cn} - \frac{y^2_{....}}{abcn}$$
$$SS_{AB} = SS_{Subtotal(AB)} - SS_A - SS_B$$
$$SS_{Subtotal(AC)} = \sum_{i=1}^{a}\sum_{k=1}^{c}\frac{y^2_{i.k.}}{bn} - \frac{y^2_{....}}{abcn}$$

$$SS_{AC} = SS_{Subtotal(AC)} - SS_A - SS_C$$

$$SS_{Subtotal(BC)} = \sum_{j=1}^{b}\sum_{k=1}^{c} \frac{y_{.jk.}^2}{an} - \frac{y_{....}^2}{abcn}$$

$$SS_{BC} = SS_{Subtotal(BC)} - SS_B - SS_C$$

$$SS_{Subtotal(ABC)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} \frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$SS_E = SS_T - SS_{Subtotal}$ atau $SS_E = SS_T - SS_A - SS_B - SS_{AB}$

3. Two-stage Nested Design

$$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \sum_{i=1}^{a} \frac{y_{i..}^2}{bn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$