UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MSG 253 – Queueing Systems and Simulation
[Sistem Giliran dan Simulasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIFTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

1. (a) A queueing process that is based on the birth and death process has the following birth and death rates:

$$\lambda_n = \frac{\lambda}{(n+1)}, \quad \text{for } n = 0, 1, 2, 3, \cdots$$

$$\mu_n = \mu, \quad \text{for } n = 1, 2, 3, \cdots$$

(i) Draw a rate-diagram to represent the queueing system.

(ii) Show that $P_n = \dfrac{\rho^n}{n!} e^{-\rho}$, where $\rho = \dfrac{\lambda}{\mu}$.

(iii) Next, show that $L_q = \rho - 1 + e^{-\rho}$.

[50 marks]

(b) The *Pulau Pinang Daily Bugle* operates a popular sport wire service, which allows people to call the special number requesting scores of national and local sports events. The newspaper has assigned one clerk to answer the questions of those who call in. On average, calls arrive at the rate of 45 per hour and are assumed to follow the Poisson distribution. It takes (on average) 1 minute to handle each call and the service time is assumed to be exponential. If the clerk is busy when a call arrives, the caller is placed on hold until the clerk is free.

(i) What proportion of the time does the clerk actually spend handling these calls?
(ii) How many callers, on average, are waiting for the clerk?
(iii) How many callers, on average, are being served or waiting for service?
(iv) What is the average time, in minute, each caller waits for the clerk?
(v) What is the average time, in minute, that each caller spends waiting and having his or her question answered?
(vi) What is the probability that exactly four callers are waiting or being served?
(vii) What is the probability that more than six callers are waiting or being served?
(viii) If a caller has to wait, what is the average time spent waiting?
(ix) What is the average number of callers waiting during those periods that a waiting line exists?

1. (a) *Suatu proses giliran yang berasaskan kepada proses lahir dan mati mempunyai kadar kelahiran dan kematian seperti berikut:*

$$\lambda_n = \frac{\lambda}{(n+1)}, \text{ bagi } n = 0, 1, 2, 3, \cdots$$

$$\mu_n = \mu, \text{ bagi } n = 1, 2, 3, \cdots$$

(i) *Lukiskan gambar rajah kadar yang mewakili sistem giliran tersebut.*

(ii) *Tunjukkan bahawa* $P_n = \frac{\rho^n}{n!} e^{-\rho}$, *yang mana* $\rho = \frac{\lambda}{\mu}$.

(iii) *Kemudian, tunjukkan bahawa* $L_q = \rho - 1 + e^{-\rho}$.

[50 markah]

(b) *Syarikat suratkhabar* Pulau Pinang Daily Bugle *menyediakan perkhidmatan talian-sukan bagi membolehkan orang ramai menelefon dan bertanya keputusan acara sukan kebangsaan dan antarabangsa. Seorang kerani ditugaskan untuk menjawab semua panggilan. Pada puratanya, panggilan diterima dengan kadar purata 45 sejam mengikut agihan Poisson. Masa purata untuk melayan setiap panggilan ialah 1 minit mengikut agihan eksponen. Jika kerani sedang sibuk semasa ketibaan sesuatu panggilan, panggilan itu akan diletakkan dalam keadaan menunggu sehinggalah kerani itu bersenang.*

(i) *Apakah peratusan masa daripada masa bekerjanya yang digunakan oleh kerani untuk menjawab panggilan?*

(ii) *Berapakah bilangan pemanggil secara purata yang menunggu kerani?*

(iii) *Berapakah bilangan pemanggil secara purata yang sedang dilayan atau sedang menunggu kerani?*

(iv) *Berapakah masa purata, dalam minit, setiap pemanggil menunggu kerani?*

(v) *Berapakah masa purata, dalam minit, setiap pemanggil menunggu dan dilayan oleh kerani?*

(vi) *Apakah kebarangkalian bahawa empat orang pemanggil sedang menunggu atau sedang dilayan?*

(vii) *Apakah kebarangkalian bahawa lebih daripada enam orang pemanggil sedang menunggu atau sedang dilayan?*

(viii) *Jika seseorang pemanggil itu terpaksa menunggu, berapakah masa purata menunggunya?*

(ix) *Berapakah bilangan purata pemanggil yang menunggu sewaktu barisan menunggu terbentuk?*

Suppose that as calls arrive at the switchboard, only five lines are available for incoming calls to the sport wire clerk. One of these is actually connected to the clerk and the other four, if necessary, are used for calls put on hold. If all five lines are in use, any incoming call will receive a busy signal and will be, in effect, turned away. Assume that such callers will then turn to a rival newspaper with their request.

(x) What proportion of time does the clerk actually spend answering questions?
(xi) What proportion of calls are turned away by the busy signal?
(xii) What is the average number of phone lines tied up by the sport wire?
(xiii) How long on average do the callers spend waiting for the clerk?
(xiv) How long does it take from the time a caller is put on hold until the question has been answered?
(xv) A follow-up study has been done that show that 1 percent of customers who use the sport wire service eventually become subscribers to the newspaper. An annual subscription to the paper is estimated to contribute RM10 to profit. The cost of leasing additional phone lines is known to be RM400 per year. How many additional phone lines, if any, should the newspaper lease for the sport wire? (Assume that the sport wire service is opened 12 hours a day, 365 days a year)

[50 marks]

2. (a) A power plant operating 24 hours per day has four identical turbine generators, each capable of producing 3 megawatts of power. The demand at any time is 6 megawatts, so that when all four turbines are in working condition, one is kept on "warm-standby", one on "cold-standby", and the other two are in full operation. If one turbine is down, then two are operating and one is on warm-standby. If two are down, both working turbines are operating. If only one turbine is working, then the company must purchase 3 megawatts of power from another source. If all turbines are down, the company must purchase 6 megawatts.

If a turbine is in the operating mode, its mean time to failure is 3 weeks. If a turbine is in warm-standby, its mean time to failure is 9 weeks. The company has two technicians that can repair failed turbines, but it only takes one technician to effect a repair. On average, a repair can be done in half a week. If a turbine is in cold-standby, it cannot fail. (We assume that all switchovers from warm-standby to working or from cold-standby to warm-standby are instantaneous.) Assuming that all times are exponentially distributed, determine the expected megawatt hours that must be purchased each year.

[40 marks]

(b) Two alternative designs are being considered for the service facility of a finite queueing system. Alternative A has a single server and alternative B has two servers. The cost of the service facility has two components: a fixed cost of RM50,000 per year, which is independent of the number of servers, and a variable cost of RM30,000 per server per year, which is proportional to the number of servers. Simulation analysis has determined the following steady-state probability distributions for the number of customers in the system.

Katakan hanya lima talian disediakan untuk menerima panggilan. Salah satu daripada talian itu disambungkan kepada kerani manakala yang selebihnya digunakan untuk panggilan menunggu. Jika kesemua talian sedang digunakan, sebarang panggilan yang tiba akan menerima isyarat sibuk. Anggapkan bahawa pemanggil yang menerima isyarat sibuk akan berurusan dengan talian yang disediakan oleh syarikat suratkhabar yang lain.

(x) *Apakah peratusan masa daripada masa bekerjanya yang digunakan oleh kerani untuk menjawab panggilan?*

(xi) *Berapakah kadar panggilan yang ditolak disebabkan isyarat sibuk?*

(xii) *Berapakah bilangan purata talian yang digunakan?*

(xiii) *Berapa lamakah pada puratanya seseorang pemanggil itu menunggu kerani?*

(xiv) *Berapa lamakah masa yang diambil bermula daripada masa sesuatu panggilan itu menunggu sehinggalah soalannya terjawab?*

(xv) *Satu kajian telah dibuat dan didapati bahawa 1 peratus daripada pemanggil yang menggunakan talian sukan akan akhirnya melanggan suratkhabar syarikat itu. Setiap langganan tahunan akan menyumbang sebanyak RM10 kepada keuntungan. Kos penyewaan talian telefon tambahan ialah RM400 setahun. Berapakah talian telefon tambahan, sekiranya perlu, yang patut disewa untuk talian sukan? (Anggapkan bahawa talian sukan dibuka 12 jam sehari, 365 hari setahun)*

[50 markah]

2. (a) *Sebuah kilang tenaga yang beroperasi 24 jam sehari mempunyai empat turbin janakuasa, setiap satunya berupaya menghasilkan 3 megawat tenaga. Permintaan tenaga pada bila-bila masa adalah sebanyak 6 megawat, dengan itu apabila kesemua turbin berada dalam keadaan baik, satu di antaranya akan berada dalam keadaan "warm-standby" dan satu lagi dalam keadaan "cold-standby", manakala yang dua lagi akan beroperasi penuh. Jika satu turbin rosak, dua akan beroperasi dan satu dalam keadaan "warm-standby". Jika dua rosak, dua turbin yang elok akan beroperasi. Jika satu turbin sahaja yang elok, syarikat terpaksa membeli 3 megawat tenaga daripada sumber luar. Jika kesemua turbin rosak, syarikat terpaksa membeli 6 megawat tenaga.*

Jika sesuatu turbin itu berada dalam mod beroperasi, masa purata sehingga ia rosak ialah 3 minggu. Jika turbin dalam keadaan "warm-standby", masa purata sehingga ia rosak ialah 9 minggu. Syarikat itu mempunyai dua juruteknik untuk membaiki turbin dan mereka bekerja berasingan. Pada puratanya, kerja pembaikan memakan masa setengah minggu. Jika turbin dalam keadaan "cold-standby", ia tidak akan rosak. (Kita andaikan bahawa semua pertukaran daripada "warm-standby" ke keadaan beroperasi atau daripada "cold-standby" ke "warm-standby" berlaku secara serta-merta.) Dengan anggapan bahawa semua masa adalah bertaburan eksponen, tentukan jangkaan jam megawatt yang harus dibeli setiap tahun.

[40 markah]

(b) *Dua alternatif sedang dipertimbangkan sebagai rekabentuk kemudahan perkhidmatan bagi suatu sistem giliran terhingga. Alternatif A mempunyai seorang pelayan dan alternatif B mempunyai dua pelayan. Kos kemudahan perkhidmatan itu terdiri daripada dua komponen: kos tetap sebanyak RM50,000 setahun yang tidak bergantung kepada bilangan pelayan dan kos boleh berubah RM30,000 per pelayan setahun. Daripada analisis simulasi yang dijalankan, agihan kebarangkalian bilangan pelanggan di dalam sistem semasa keadaan mantap adalah seperti berikut:*

Design A					
n	0	1	2	3	4
P_n	0.12	0.25	0.38	0.15	0.10

Design B					
n	0	1	2	3	4
P_n	0.38	0.30	0.21	0.06	0.05

In addition, the simulation has determined that 10% of the customers balk with design A whereas 5% balk with design B. The arrival rate (including balks) is 1500 customers per year.

Management estimates that a customer who balks costs the company RM100 in lost profits. The cost of customers waiting in the system is RM20 per hour (per customer). Determine which design alternative minimizes the total expected cost. Assume that there are 2000 work hours in a year.

[30 marks]

(c) A state agency that handles compensation for those who are unemployed is considering two options for processing applications. Option 1: Four clerks process applications in parallel from a single queue. Each clerk fills out the required form in the presence of the applicant based on information that is verbally related to the clerk. Processing time is exponentially distributed with a mean of 45 minutes. Option 2: Each applicant first fills out the form without the help of the clerk. The time required to accomplish this is exponentially distributed, with a mean of 65 minutes. When the applicant finishes the form, he or she joins a single queue to await a review by one of the four clerks. The time required to review a form is exponentially distributed with a mean of 5 minutes.

Given that the arrival of applicants is a Poisson process with a mean rate of 4.8 per hour, compare the two options with respect to the expected number of applicants in the system and the expected time in the system.

[30 marks]

3 (a) Ships arrive at a harbor at the rate of 1 ± ½ hours. There are six berths to accommodate them. They also need the service of a crane for unloading and there are five cranes. After unloading, 10% of the ships stay to refuel before leaving; the others leave immediately. Ships do not need the cranes for refueling. It takes 7½ ± 3 hours to unload and 1 ± ½ hours to refuel a ship. Write a GPSSPC program to simulate the clearance of 100 ships from the harbor. The queueing patterns for berths and cranes are to be observed.

[50 marks]

(b) Referring to problem 3(a), perform a hand simulation for the clearance of 10 ships from the harbor. Assume that there are only three berths and two cranes. The inter-arrival time of ships is either 1, 2 or 3 hours, each with equal chances of happening. The unloading time is either 2, 3, 4 or 5 hours with probabilities of 0.1, 0.3, 0.4 and 0.2, respectively. The refueling time is either 1, 1½, or 2½ hours, each with equal chances of happening. The answers to the following questions are to be provided:

Rekabentuk A					
n	0	1	2	3	4
P_n	0.12	0.25	0.38	0.15	0.10

Rekabentuk B					
n	0	1	2	3	4
P_n	0.38	0.30	0.21	0.06	0.05

Sebagai tambahan, analisis simulasi itu juga telah menentukan bahawa 10% daripada pelanggan akan belah dalam rekabentuk A, manakala 5% akan belah dalam rekabentuk B. Kadar ketibaan (termasuk belah) ialah 1500 pelanggan setahun.

Pihak pengurusan menganggarkan bahawa setiap pelanggan yang belah akan mengakibatkan kerugian sebanyak RM100 dalam bentuk kehilangan keuntungan. Kos berkaitan dengan masa menunggu pelanggan ialah RM20 sejam (per pelanggan). Tentukan alternatif manakah yang akan meminimumkan jumlah kos jangkaan. Andaikan bahawa terdapat 2000 jam masa bekerja setahun.

[30 markah]

(c) *Sebuah perbadanan kerajaan negeri yang mengendalikan urusan subsidi kepada mereka yang menganggur sedang menimbangkan dua opsyen untuk memproses permohonan. Opsyen 1: Empat kerani akan memproses permohonan secara selari daripada satu barisan menunggu. Setiap kerani akan mengisi borang untuk pemohon berasaskan kepada maklumat yang diberikan secara lisan kepadanya. Masa pemprosesan adalah mengikut agihan eksponen dengan min 45 minit. Opsyen 2: Setiap pemohon mengisi sendiri borang yang disediakan. Masa yang diperlukan ialah mengikut agihan eksponen dengan min 65 minit. Apabila borang siap diisi, pemohon akan memasuki satu barisan menunggu untuk menunggu giliran penyemakan borang oleh salah seorang daripada empat kerani yang ada. Masa penyemakan borang ialah mengikut agihan eksponen dengan min 5 minit.*

Jika ketibaan pemohon adalah mengikut proses Poisson dengan kadar min 4.8 sejam, bandingkan dua opsyen itu berasaskan kepada bilangan jangkaan pemohon di dalam sistem dan jangkaan masa di dalam sistem.

[30 markah]

3 (a) *Kapal tiba di pelabuhan dengan kadar 1 ± ½ jam. Enam pelantar disediakan untuk kegunaan kapal. Kapal juga memerlukan kren untuk pemunggahan dan terdapat lima kren di perlabuhan itu. Selepas siap pemunggahan, 10% daripada kapal akan terus berlabuh untuk mengisi minyak; yang lainnya akan terus beredar. Semasa pengisian minyak, kren tidak diperlukan. Masa memunggah ialah 7½ ± 3 jam dan masa mengisi minyak ialah 1 ± ½ jam bagi setiap kapal. Tulis satu aturcara GPSSPC untuk melakukan simulasi pemprosesan 100 buah kapal di pelabuhan. Corak barisan menunggu untuk kegunaan pelantar dan kren haruslah diperhatikan.*

[50 markah]

(b) *Dengan merujuk kepada masalah 3(a), lakukan simulasi dengan tangan pemprosesan 10 buah kapal di pelabuhan. Andaikan bahawa terdapat hanya tiga pelantar dan dua kren. Masa di antara ketibaan kapal ialah sama ada 1, 2, atau 3 jam, dengan setiap satunya mempunyai peluang yang sama untuk berlaku. Masa memunggah ialah sama ada 2, 3, 4 atau 5 jam dengan kebarangkalian masing-masingnya ialah 0.1, 0.3, 0.4 dan 0.2. Masa untuk mengisi minyak pula ialah 1, 1½ atau 2½ jam, dengan setiap satunya sama mungkin akan berlaku. Jawapan kepada soalan-soalan berikut mestilah diberikan:*

(i) What is the average time that a ship spent at the harbor?
(ii) What proportion of time are the cranes idle?
(iii) What is the average time that a ship spend waiting for a berth to be available?

(Use the enclosed 2-digit random number table with the first column for *inter-arrival time,* second column for *unloading time,* the third column for *determining whether to refuel or not* and the fourth column for *refueling time.*)

[50 marks]

(i) *Berapakah masa purata sesebuah kapal berada di pelabuhan?*
(ii) *Berapakah peratusan masa bersenang kren?*
(iii) *Berapakah masa purata sesebuah kapal menunggu untuk menggunakan pelantar?*

(Gunakan sifir nombor rawak 2-digit yang disertakan dengan lajur pertama untuk masa antara ketibaan, *lajur kedua untuk masa memunggah, lajur ketiga untuk* menentukan sama ada perlu mengisi minyak ataupun tidak dan lajur keempat untuk masa mengisi minyak)

[50 markah]

APPENDIX 1 / *LAMPIRAN 1*

Formulas for Queueing Theory:

1. *M/M/1*:

$$\rho = \lambda / \mu$$

$$P_n = (1-\rho)\rho^n \quad \text{for } n = 0,1,2,\ldots$$

$$L = \frac{\lambda}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W = \frac{1}{\mu-\lambda} \quad , \quad W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$P[w > t] = e^{-t/w}$$

$$P[w_q > t] = \rho e^{-t/w}$$

2. *M/M/s*:

$$\rho = \frac{\lambda}{s\mu}$$

$$P_0 = \left[\frac{(\lambda/\mu)^s}{s!} \frac{1}{(1-\rho)} + \sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} \right]^{-1}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & , \ \text{if } 0 \le n \le s \\ \dfrac{(\lambda/\mu)^n}{s!\,s^{n-s}} P_0 & , \ \text{if } n \le s \end{cases}$$

$$L_q = \frac{(\lambda/\mu)^s \rho}{s!(1-\rho)^2} P_0$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = W_q + 1/\mu$$

$$L = L_q + \lambda/\mu$$

$$P[w_q > t] = e^{-\mu t} \left[1 + \frac{P_0 \left(\dfrac{\lambda}{\mu} \right)^s}{s!(1-\rho)} \left(\frac{1 - e^{\mu t(s-1-\lambda/\mu)}}{s-1-\lambda/\mu} \right) \right]$$

$$P[w_q > t] = \left[1 - P\{w_q = 0\} \right] e^{-s\mu(1-\rho)t}$$

$$\text{where} \quad P\{w_q = 0\} = \sum_{n=0}^{s-1} P_n$$

APPENDIX 2 / *LAMPIRAN 2*

3. *M/M/s*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{s-1} \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$P_n = \begin{cases} P_0 \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n & \text{, if } 0 \le n \le s \\ P_0 \binom{M}{n} \left(\frac{n!}{s^{n-s} s!}\right) \left(\frac{\lambda}{\mu}\right)^n & \text{, if } s < n \le M \\ 0 & \text{, if } n > M \end{cases}$$

$$L = P_0 \left[\sum_{n=0}^{s-1} n \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} n \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]$$

$$L_q = L - s + P_0 \sum_{n=0}^{s-1} (s-n) \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n$$

$$W = \frac{L}{\lambda(M-L)} \quad , \quad W_q = \frac{L_q}{\lambda(M-L)}$$

4. *M/G/1*:

$$P_0 = 1 - \rho$$

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

$$L = \rho + L_q$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = w_q + \frac{1}{\mu}$$

5. $M / E_k / 1$:

$$L_q = \frac{1+k}{2k} \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W_q = \frac{1+k}{2k} \frac{\lambda}{\mu(\mu - \lambda)}$$

$$W = W_q + 1/\mu$$

$$L = \lambda W$$

APPENDIX 3 / *LAMPIRAN 3*

6. *M/M/1/k*:

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{k+1}} & (\rho \neq 1) \\ \dfrac{1}{k+1} & (\rho = 1) \end{cases}$$

For $\rho \neq 1$

$$L = \frac{\rho\left[1-(k+1)\rho^k + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)}$$

$$L_q = L - (1-P_0) = L - \frac{\rho\left(1-\rho^k\right)}{1-\rho^{k+1}}$$

$$W = L/\lambda' \quad , \quad \lambda' = \mu\left(L - L_q\right)$$

$$W_q = W - 1/\mu = L_q/\lambda'$$

For $\rho = 1$

$$L = \frac{k}{2}$$

7. *M/M/s/k*:

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (0 \leq n < s) \\ \dfrac{1}{s^{n-s}s!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (s \leq n \leq k) \end{cases}$$

$$P_0 = \begin{cases} \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}\,\frac{1-\left(\dfrac{\lambda}{s\mu}\right)^{k-s+1}}{1-\dfrac{\lambda}{s\mu}}\right]^{-1} & \text{for } \left(\dfrac{\lambda}{s\mu} \neq 1\right) \\ \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}(k-s+1)\right]^{-1} & \text{for } \left(\dfrac{\lambda}{s\mu} = 1\right) \end{cases}$$

$$L_q = \frac{P_0 (s\rho)^S \rho}{s!(1-\rho)^2}\left[1-\rho^{k-s+1} - (1-\rho)(k-s+1)\rho^{k-s}\right]$$

APPENDIX 4 / *LAMPIRAN 4*

$$L = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(\rho s)^n}{n!}$$

$$W = \frac{L}{\lambda'} \quad , \quad \lambda' = \lambda(1 - P_k)$$

$$W_q = W - \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda'}$$

8. *M/M/s/s*:

$$P_n = \frac{(\lambda/\mu)^n / n!}{\sum_{i=0}^{s} \left(\frac{\lambda}{\mu}\right)^i / i!} \quad \text{for } (0 \le n \le s)$$

$$P_s = \frac{(s\rho)^s / s!}{\sum_{i=0}^{s} (s\rho)^i / i!} \quad \text{where } \left(\rho = \frac{\lambda}{s\mu}\right).$$

$$L = \frac{\lambda}{\mu}(1 - P_s) \quad , \quad W = \frac{L}{\lambda'} \text{ where } \lambda' = \lambda(1 - P_s)$$

9. $M/M/\infty$:

$$P_n = \frac{(\lambda/\mu)^n e^{-\lambda/\mu}}{n!} \quad \text{for } n = 0, 1, 2, \ldots$$

$$L = \lambda/\mu$$

$$W = \frac{1}{\mu}$$

APPENDIX 5 / *LAMPIRAN 5*

10. *M/M/1*: state-dependent service

$$\mu_n = \begin{cases} \mu_1 & (1 \le n \le k) \\ \mu & (n \ge k) \end{cases}$$

$$P_0 = \left[\frac{1-\rho_1^k}{1-\rho_1} + \frac{\rho \rho_1^{k-1}}{1-\rho} \right]^{-1} \quad (\rho_1 = \lambda / \mu_1, \rho = \lambda / \mu < 1)$$

$$L = P_0 \left[\frac{\rho_1 \left[1 + (k-1)\rho_1^k - k\rho_1^{k-1} \right]}{(1-\rho_1)^2} + \frac{\rho \rho_1^{k-1} \left[k - (k-1)\rho \right]}{(1-\rho)^2} \right]$$

$$L_q = L - (1 - P_0)$$

$$W = \frac{L} {\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1-P_0}{\lambda}$$

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1} \right)^n P_0 & (0 \le n < k) \\ \dfrac{\lambda^n}{\mu_1^{k-1} \mu^{n-k+1}} P_0 & (n \ge k) \end{cases}$$

11. *M/M/1*: finite population of size *M.*

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}$$

$$P_n = \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n P_0 \quad \text{for } n = 1, 2, \ldots, M$$

$$L = M - \frac{\mu}{\lambda} [1 - P_0]$$

$$L_q = M - \frac{\lambda + \mu}{\lambda} (1 - P_0)$$

$$W = \frac{L}{\lambda'} \quad , \quad W_q = \frac{L_q}{\lambda'} \quad \text{where } \lambda' = \lambda(M - L)$$

APPENDIX 6 / *LAMPIRAN 6*

TWO-DIGIT RANDOM NUMBER TABLE

03	26	48	92	38	96	41	04	35	84
71	44	81	46	44	47	07	20	58	04
33	75	06	41	87	72	63	88	59	54
53	71	27	13	37	45	89	61	30	26
41	15	43	91	46	81	57	39	34	86
16	18	75	11	26	80	93	97	29	33
88	50	00	56	70	19	90	00	93	95
13	10	08	15	29	33	75	70	43	05
15	72	73	69	27	75	72	95	99	56
64	10	99	02	18	26	78	69	19	12
98	66	53	86	34	71	09	88	56	08
43	05	06	19	91	78	03	65	08	16
69	82	02	61	98	50	74	84	60	41
06	40	10	24	68	42	39	97	25	55
34	86	83	41	33	83	85	92	32	29
46	05	92	36	82	04	67	05	18	69
28	73	59	56	43	88	61	17	07	48
35	53	49	39	98	14	16	76	69	10
90	90	18	27	75	08	75	17	55	68
62	32	97	16	33	66	02	34	62	26

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