UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MSG 252 – Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all nine** [9] questions.

[Arahan : *Jawab **semua sembilan** [9] soalan].*

1. Consider the following LP.

$$\begin{aligned} \text{Maximize} \quad & z = x_1 - x_2 + 2x_3 \\ \text{Subject to} \quad & 2x_1 - 2x_2 + 3x_3 \le 5 \\ & x_1 + x_2 - x_3 \le 3 \\ & x_1 - x_2 + x_3 \le 2 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

You are given the facts that the basic variables in the optimal solution are (x_2, x_6, x_3) where x_6 is the slack variable of the third constraint. Use the given information to find

(a) the optimal solution to this problem.
(b) the optimal solution to its dual.

[10 marks]

2. Consider the following LP and its dual.

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 7x_2 + 4x_3 \\ \text{Subject to} \quad & x_1 + 2x_2 + x_3 \le 10 \\ & 3x_1 + 3x_2 + 2x_3 \le 1 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Dual:

$$\begin{aligned} \text{Minimize} \quad & z = 10y_1 + y_2 \\ \text{Subject to} \quad & y_1 + 3y_3 \ge 2 \\ & 2y_1 + 3y_2 \ge 7 \\ & y_1 + 2y_2 \ge 4 \\ & y_1, y_2 \ge 0 \end{aligned}$$

Show how the weak duality theorem applies to this problem.

[10 marks]

3. Consider the symmetric primal-dual problem.

Primal:	Max	$z = \mathbf{cx}$	Dual:	Min	$w = \mathbf{yb}$
	Subject to	$\mathbf{Ax} \le \mathbf{b}$		Subject to	$\mathbf{yA} \ge \mathbf{c}$
		$\mathbf{x} \ge 0$			$\mathbf{x} \ge 0$

If x^* and y^* are feasible solutions to the primal and dual respectively and $cx^* = y^*b$, prove that x^* and y^* are the optimal solutions to their respective problems.

[10 marks]

1. *Pertimbangkan masalah PL berikut.*

$$\begin{aligned} \textit{Maksimumkan} \quad & z = x_1 - x_2 + 2x_3 \\ \textit{Terhadap} \quad & 2x_1 - 2x_2 + 3x_3 \le 5 \\ & x_1 + x_2 - x_3 \le 3 \\ & x_1 - x_2 + x_3 \le 2 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Anda diberitahu bahawa pembolehubah asas pada penyelesaian optimum adalah (x_2, x_6, x_3) *yang mana* x_6 *ialah pembolehubah lalai bagi kekangan ketiga.. Gunakan maklumat yang diberkan untuk mendapatkan*

(a) penyelesaian optimum bagi masalah ini.

(b) penyelesaian optimum bagi masalah dualnya.

[10 markah]

2. *Pertimbangkan masalah PL berikut dan dualnya.*

$$\begin{aligned} \textit{Maksimumkan} \quad & z = 2x_1 + 7x_2 + 4x_3 \\ \textit{terhadap} \quad & x_1 + 2x_2 + x_3 \le 10 \\ & 3x_1 + 3x_2 + 2x_3 \le 1 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

$$\begin{aligned} \textit{Dual: Minimumkan} \quad & z = 10y_1 + y_2 \\ \textit{terhadap} \quad & y_1 + 3y_3 \ge 2 \\ & 2y_1 + 3y_2 \ge 7 \\ & y_1 + 2y_2 \ge 4 \\ & y_1, y_2 \ge 0 \end{aligned}$$

Tunjukkan bagaimana teorem kelalaian lemah digunakan pada masalah ini.

[10 markah]

3. *Pertimbangkan masalah primal-dual simmetri berikut.*

Primal:	*Maks* $z = cx$	*Dual:*	*Min* $w = yb$
	terhadap $Ax \le b$		*terhadap* $yA \ge c$
	$x \ge 0$		$x \ge 0$

Jika x^* *and* y^* *masing-masing adalah penyelesaian tersaur bagi masalah primal dan dual dan* $cx^* = y^*b$, *buktikan bahawa* x^* *dan* y^* *adalah penyelesaian optimum bagi masalah masing-masing.*

[10 markah]

4. Consider the following LP problem and its optimal tableau.

$$\begin{aligned} \text{Maximize } z &= -5x_1 + 5x_2 + 13x_3 \\ \text{Subject to } & -x_1 + x_2 + 3x_3 \le 20 \\ & 12x_1 + 4x_2 + 10x_3 \le 90 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Basic	x_1	x_2	x_3	x_4	x_5	solution
z	0	0	2	5	0	100
x_2	−1	1	3	1	0	20
x_5	16	0	−2	−4	1	10

The slack variables of constraint 1 and 2 are represented by x_4 and x_5 respectively. Conduct sensitivity analysis by independently investigating each of the following changes in the original model.

(a) Change the right-hand side of constraint 1 to $b_1 = 20$.

(b) Change the right-hand sides to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 100 \end{bmatrix}$

(c) Change the coefficients of x_1 to $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$

(d) Introduce a new variable x_6 with $\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix}$

[20 marks]

5. Use the dual simplex method to solve the following LP.

$$\begin{aligned} \text{Minimize } z &= 5x_1 + 2x_2 + 4x_3 \\ \text{Subject to } & 3x_1 + x_2 + 2x_3 \ge 4 \\ & 6x_1 + 3x_2 + 5x_3 \ge 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

[10 marks]

6. Solve the following 0-1 problem.

$$\begin{aligned} \text{Maximize } & z = 70x_1 + 40x_2 + 60x_3 + 80x_4 \\ \text{Subject to } & 5x_1 - 2x_2 + 8x_3 - 10x_4 \le 20 \\ & 4x_1 + 2x_2 + 2x_3 + 5x_4 \le 6 \\ & x_1, x_2, x_3, x_4 = 0, 1 \end{aligned}$$

[10 marks]

4. *Pertimbangkan masalah PL berikut dan tablo optimumnya.*

$$\begin{aligned} \text{Maximumkan } z &= -5x_1 + 5x_2 + 13x_3 \\ \text{terhadap } & -x_1 + x_2 + 3x_3 \le 20 \\ & 12x_1 + 4x_2 + 10x_3 \le 90 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Asas	x_1	x_2	x_3	x_4	x_5	*Penyelesaian*
z	0	0	2	5	0	100
x_2	–1	1	3	1	0	20
x_5	16	0	–2	–4	1	10

Pembolehubah lalai bagi kekangan 1 dan 2 masing-masing diwakili oleh x_4 dan x_5. Lakukan analisis kepekaan secara berasingan bagi setiap perubahan berikut dalam model asal.

(a) *Tukar nilai sebelah kanan kekangan 1 kepada $b_1 = 20$.*

(b) *Tukar nilai-nilai sebelah kanan kepada* $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 100 \end{bmatrix}$

(c) *Tukar pekali x_1 kepada* $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$

(d) *Perkenalkan pembolehubah baru x_6 dengan* $\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix}$

[20 markah]

5. *Gunakan kaedah simpleks dual untuk menyelesaikan masalah PL berikut.*

$$\begin{aligned} \text{Minimumkan } z &= 5x_1 + 2x_2 + 4x_3 \\ \text{terhadap } & 3x_1 + x_2 + 2x_3 \ge 4 \\ & 6x_1 + 3x_2 + 5x_3 \ge 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

[10 markah]

6. *Selesaikan masalah 0-1 berikut.*

$$\begin{aligned} \text{Maksimumkan } z &= 70x_1 + 40x_2 + 60x_3 + 80x_4 \\ \text{terhadap } & 5x_1 - 2x_2 + 8x_3 - 10x_4 \le 20 \\ & 4x_1 + 2x_2 + 2x_3 + 5x_4 \le 6 \\ & x_1, x_2, x_3, x_4 = 0, 1 \end{aligned}$$

[10 markah]

7. The manager of a shop that installs commercial carpeting has developed the following mixed IP model.

$$\begin{aligned} \text{Maximize } z &= 10x_1 + x_2 + 7x_3 \quad \text{(profit)} \\ \text{Subject to } & 4x_1 + 8x_2 + 5x_3 \leq 850 \text{ hours} \\ & 2x_1 + 3x_2 + x_3 \leq 400 \text{ square meters} \\ & x_1, x_2, x_3 \geq 0, \; x_3 \text{ integer} \end{aligned}$$

The following simplex tableau shows the optimal solution for the LP relaxation of the problem. It is known that x_4 and x_5 are the slack variables in the first and second constraints of the original problem respectively.

Basic	x_1	x_2	x_3	x_4	x_5	**Solution**
z	0	$31\frac{1}{3}$	0	$\frac{2}{3}$	$11\frac{1}{3}$	$2033\frac{1}{3}$
x_3	0	$\frac{2}{3}$	1	$\frac{1}{3}$	$-\frac{2}{3}$	$16\frac{2}{3}$
x_1	1	$\frac{7}{6}$	0	$-\frac{1}{6}$	$\frac{5}{6}$	$191\frac{2}{3}$

Find the optimal integer solution by using the cutting plane method.

[10 marks]

8. The manager of XYZ company has formulated the following model:

x_1 = amount of product 1
x_2= amount of product 2
x_3= amount of product 3

$$\begin{aligned} \text{Maximize } z &= 4x_1 + 2x_2 + 3x_3 \quad \text{(profit)} \\ \text{Subject to } & 2x_1 + x_2 + 4x_3 \leq 160 \text{ hours} \\ & x_1 + 2x_2 + 3x_3 \leq 90 \text{ square meters} \\ & x_1 \geq 10 \text{ units} \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

After discussions with several colleagues, the manager now believes that a goal programming model would be more appropriate. Accordingly the manager has decided on these priorities.

a. Minimize the underutilization of labour.
b. Achieve a satisfactory profit level of RM300.
c. Try to avoid making less than 10 units of x_1.

Reformulate this problem as a GP problem.

[10 marks]

7. *Pengurus sebuah kedai yang memasang karpet perniagaan telah merumuskan model PI bercampur berikut.*

$$\begin{aligned} &\text{Maximumkan } z = 10x_1 + x_2 + 7x_3 \quad \text{(keuntungan)} \\ &\text{terhadap} \quad 4x_1 + 8x_2 + 5x_3 \leq 850 \text{ jam} \\ &\qquad\quad 2x_1 + 3x_2 + x_3 \leq 400 \text{ meter per segi} \\ &\qquad\quad x_1, x_2, x_3 \geq 0, \; x_3 \text{ integer} \end{aligned}$$

Tablo simpleks berikut menunjukkan penyelesaian optimum bagi masalah PL tak tegang masalah tersebut. Diketahui bahawa x_4 dan x_5 masing-masing adalah pembolehubah lalai bagi kekangan pertama dan kedua masalah asal.

Asas	x_1	x_2	x_3	x_4	x_5	**Penyelesaian**
z	0	$31/3$	0	$2/3$	$11/3$	$2033\ 1/3$
x_3	0	$2/3$	1	$1/3$	$-2/3$	$16\ 2/3$
x_1	1	$7/6$	0	$-1/6$	$5/6$	$191\ 2/3$

Dapatkan penyelesaian optimum integer dengan menggunakan kaedah satah potongan.

[10 markah]

8. *Pengurus syarikat XYZ Co. telah merumuskan model berikut:*

x_1 = *amaun produk 1*
x_2 = *amaun produk 2*
x_3 = *amaun produk 3*

$$\begin{aligned} &\text{Maksimumkan } z = 4x_1 + 2x_2 + 3x_3 \quad \text{(keuntungan)} \\ &\text{terhadap} \quad 2x_1 + x_2 + 4x_3 \leq 160 \text{ jam} \\ &\qquad\quad x_1 + 2x_2 + 3x_3 \leq 90 \text{ meter per segi} \\ &\qquad\quad x_1 \geq 10 \text{ unit} \\ &\qquad\quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

Setelah berbincang dengan beberapa orang rakan kerja, pengurus percaya bahawa model pengaturcaraan gol (PG) adalah lebih sesuai. Sesuai dengan itu pengurus telah menetapkan prioriti- prioriti berikut.

a. *Minimumkan kurang penggunaan buruh.*
b. *Capai keuntungan yang memuaskan sebanyak RM300.*
c. *Elak daripada menghasilkan kurang daripada10 unit produk .*

Rumuskan semula masalah ini sebagai suatu masalah PG.

[10 markah]

9. Given the following problem:

$$\begin{aligned} \text{Maximize } z &= 120x_1 + 50x_2 \\ \text{Subject to } & 14x_1 + 5x_2 \leq 70 \\ & 2x_1 + 3x_2 \leq 18 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

(a) Graph this problem and identify the LP relaxation solution on the graph.
(b) Determine the optimal LP solution graphically.
(c) Solve the remainder of the problem using branch and bound method.

[10 marks]

9. *Diberikan masalah berikut:*

$$\begin{aligned} &\textit{Maksimumkan } z = 120x_1 + 50x_2 \\ &\textit{terhadap} \quad 14x_1 + 5x_2 \leq 70 \\ &\qquad\qquad 2x_1 + 3x_2 \leq 18 \\ &\qquad\qquad x_1,\ x_2 \geq 0 \textit{ dan integer} \end{aligned}$$

(a) *Grafkan masalah ini dan tentukan ruang bagi masalah PL tak tegang.*
(b) *Tentukan penyelesaian optimum bagi masalah PL tak tegang secara graf.*
(c) *Selesaikan masalah ini seterusnya dengan kaedah cabang dan batas.*

[10 markah]

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