UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MSG 162 – Applied Statistical Methods
[Kaedah Statistik Gunaan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVENTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

[Arahan *:* *Jawab* ***semua empat*** *[4] soalan.]*

1. (a) A production foreman was concerned about the quality of the outgoing product from his department. He strongly suspected that the percentage of defective items passing through his assembly line during a 30-minute period increased throughout the day. At nine 30-minute periods throughout the day, the assembly line was closely examined to determine the number of defectives being produced. For each of these 30-minute periods, the number of hours that workers had been working (from 8.00 a.m.) was also recorded.

Number of defectives	Number of hours on the job
13	1.0
14	1.5
16	2.5
14	2.0
15	3.5
20	4.5
18	4.0
18	5.5
20	6.0

$\sum$(defectives) = 148 $\sum$(defectives)^2 = 2,490

$\sum$(hours) = 30.5 $\sum$(hours)^2 = 128.25

$\sum$(defectives)(hours) = 535.5

(i) Perform appropriate analysis to obtain the model and its assumptions. Use α = 0.01.

(ii) Suppose the number of hours on the job is 3. Obtain an estimation interval and a prediction interval.

(b) Higher wholesale beef prices over the past few years have resulted in the sale of ground beef with higher fat content in an attempt to keep retail prices down. Four different supermarket chains were chosen, and four 1-kilogram packages of ground beef were randomly selected from each. The fat content (in grams) was measured for each package.

	Package				
Supermarket	1	2	3	4	Total
A	4.2	4.0	4.3	4.5	17.0
B	4.5	4.7	4.4	4.4	18.0
C	5.0	4.0	4.3	4.7	18.0
D	3.8	4.0	3.7	3.7	15.2
Total	17.5	16.7	16.7	17.3	68.2

$\sum\sum Y_{ij}^2$ = 292.84

(i) State a model and its assumptions.

(ii) Perform an appropriate analysis at α = 0.01.

[28 marks]

1. (a) *Seorang penyelia pengeluaran menitikberatkan kualiti produk yang dihasilkan daripada jabatannya. Dia mengesyakki bahawa peratus item yang rosak melalui barisan pemasangannya selama suatu tempoh 30-minit meningkat sepanjang sehari. Pada sembilan tempoh 30-minit sepanjang hari, barisan pemasangan diperiksa secara teliti untuk menentukan bilangan item yang rosak dihasilkan. Bagi setiap tempoh 30-minit, bilangan jam pekerja telah bekerja (daripada pukul 8.00 a.m.) dicatat.*

Bilangan rosak	*Bilangan jam bekerja*
13	*1.0*
14	*1.5*
16	*2.5*
14	*2.0*
15	*3.5*
20	*4.5*
18	*4.0*
18	*5.5*
20	*6.0*

$\sum(rosak) = 148$ $\quad \sum(rosak)^2 = 2,490$

$\sum(jam) = 30.5$ $\quad \sum(jam)^2 = 128.25$

$\sum(rosak)(jam) = 535.5$

(i) *Lakukan analisis yang sesuai untuk mendapatkan suatu model dan anggapannya. Guna* $\alpha = 0.01$.

(ii) *Andaikan bilangan jam bekerja adalah 3. Dapatkan suatu selang anggaran dan suatu selang ramalan.*

(b) *Harga borong daging lembu yang meningkat dalam beberapa tahun yang lepas telah menyebabkan daging lembu cincang dijual dengan kandungan lemak yang tinggi di dalam usaha untuk mengekalkan harga pasaran rendah. Empat pasaraya yang berlainan dipilih, dan empat 1-kilogram bungkusan daging cincang dipilih secara rawak daripada setiap satunya. Kandungan lemak (di dalam gram) disukat bagi setiap bungkusan.*

	Bungkusan				
Pasaraya	*1*	*2*	*3*	*4*	*Jumlah*
A	*4.2*	*4.0*	*4.3*	*4.5*	*17.0*
B	*4.5*	*4.7*	*4.4*	*4.4*	*18.0*
C	*5.0*	*4.0*	*4.3*	*4.7*	*18.0*
D	*3.8*	*4.0*	*3.7*	*3.7*	*15.2*
Jumlah	*17.5*	*16.7*	*16.7*	*17.3*	*68.2*

$\sum\sum Y_{ij}^2 = 292.84$

(i) *Nyatakan suatu model dan anggapannya.*

(ii) *Lakukan suatu analisis yang sesuai pada* $\alpha = 0.01$.

[28 markah]

2. (a) An experiment was conducted to determine the nutritional value of diets for cows that are supplemented by whey. Five diaries were involved in the study. Each cow from a diary was randomly assigned to one of the following diets:

Diet I: water only
Diet II: whey plus 30.2 L of water per day
Diet III: whey plus 15.1 L of water per day
Diet IV: whey only

In addition to the diets, each cow was fed 7.5 kilograms of grain per day. The amount of hay consumed per day (in kilograms per cow) was recorded.

	Diet				
Diary	I	II	III	IV	Total
1	15.4	9.6	9.5	8.4	42.9
2	14.8	9.3	9.4	9.3	42.8
3	15.9	9.8	9.7	8.1	43.5
4	15.5	9.4	9.2	7.9	42.0
Total	61.6	38.1	37.8	33.7	171.2

$\sum\sum Y_{ij}^2 = 1{,}954.72$

(i) State an appropriate model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.01$.
(iii) Perform a follow-up analysis using Duncan multiple range test.
(iv) Determine the following effects on hay consumed:
  a. using whey and water only
  b. using combination of whey with water and whey only

(b) Manufacturers of perishable foods often use preservatives to retard spoilage. One concern is that using too much preservative will change the flavor of the food. Suppose an experiment is conducted using samples of a food product with varying amounts of preservatives added. The length of time until the food shows signs of spoiling and taste rating are recorded for each sample. The taste rating is the average rating for three tasters, each of whom rates each sample on a scale from 1 (good) to 5 (bad).

Sample	Time until spoilage (in days)	Taste rating
1	47	3.6
2	26	4.5
3	94	2.8
4	67	3.3
5	83	2.7
6	36	4.2
7	77	3.9
8	43	3.6
9	109	2.2
10	56	3.1
Total	638	33.9

Perform an appropriate analysis using $\alpha = 0.01$.

[29 marks]

2. (a) *Suatu ujikaji dilakukan untuk menentukan nilai khasiat diet untuk lembu yang ditambah dengan whey. Empat pusat tenusu terlibat di dalam kajian ini. Setiap lembu daripada satu pusat tenusu diumpukkan secara rawak kepada satu daripada diet berikut:*

Diet I: hanya air
Diet II: whey bercampur 30.2 L air sehari
Diet III: whey bercampur 15.1 L air sehari
Diet IV: hanya whey

Tambahan kepada diet tersebut, setiap lembu diberi makan 7.5 kilogram bijiran sehari. Amaun jerami yang dimakan sehari (di dalam kilogram bagi setiap lembu) dicatat.

	Diet				
Pusat Tenusu	*I*	*II*	*III*	*IV*	*Jumlah*
1	*15.4*	*9.6*	*9.5*	*8.4*	*42.9*
2	*14.8*	*9.3*	*9.4*	*9.3*	*42.8*
3	*15.9*	*9.8*	*9.7*	*8.1*	*43.5*
4	*15.5*	*9.4*	*9.2*	*7.9*	*42.0*
Jumlah	*61.6*	*38.1*	*37.8*	*33.7*	*171.2*

$\sum\sum Y_{ij}^2 = 1{,}954.72$

(i) Nyatakan suatu model dan anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.01$.
(iii) Lakukan suatu tatacara lanjutan menggunakan ujian julat berganda Duncan.
(iv) Tentukan kesan berikut ke atas jerami yang dimakan:
a. menggunakan whey dan hanya air
b. menggunakan gabungan whey dengan air dan hanya whey

(b) *Pengusaha makanan mudah rosak kerap menggunakan pengawet untuk melambatkan kerosakan. Suatu yang dibimbangkan adalah menggunakan terlalu banyak pengawet akan menukar rasa makanan tersebut. Andaikan suatu ujikaji dilakukan menggunakan beberapa sampel suatu produk makanan yang ditambah pengawet dengan amaun yang berlainan. Panjang masa sehingga makanan tersebut menunjukkan tanda-tanda rosak dan nilai rasa dicatat bagi setiap sampel. Nilai rasa adalah purata nilai bagi tiga perasa, setiap seorang menilai setiap sampel menggunkan skala daripada 1 (baik) hingga 5 (tidak baik).*

Sampel	*Masa sehingga rosak (di dalam hari)*	*Nilai rasa*
1	*47*	*3.6*
2	*26*	*4.5*
3	*94*	*2.8*
4	*67*	*3.3*
5	*83*	*2.7*
6	*36*	*4.2*
7	*77*	*3.9*
8	*43*	*3.6*
9	*109*	*2.2*
10	*56*	*3.1*
Jumlah	*638*	*33.9*

Lakukan suatu analisis yang sesuai menggunakan $\alpha = 0.01$.

[29 markah]

3. (a) A study was designed to evaluate the effectiveness of different chemicals developed to control fire ants. The type of environmental conditions in which the chemical is placed might have an effect on the effectiveness of the treatment to kill fire ants. To reduce the effect of different colonies of fire ants and the type of mounds they inhibit, the researcher created artificial fire ant mounds and populated them with 50,000 ants having similar ancestry. The researcher randomly assigned two mounds to three different chemicals and placed at three locations with different environmental conditions. The number of fire ants killed (in thousands) during a 1-week period was recorded.

	Chemical						
Location	A		B		C		Total
1	7.2		4.2		5.4		
	9.6	16.8	3.5	7.7	3.9	9.3	33.8
2	8.5		2.9		6.3		
	9.6	18.1	3.3	6.2	6.0	12.3	36.6
3	9.1		1.8		6.1		
	8.6	17.7	2.4	4.2	5.6	11.7	33.6
Total	52.6		18.1		33.3		104

$\sum\sum\sum Y_{ijk}^2 = 712$

(i) State a model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.05$.
(iii) Perform a follow-up analysis.

3. (a) *Suatu kajian dirangka untuk menilai keberkesanan beberapa bahan kimia berlainan yang dirumus untuk mengawal semut api. Jenis keadaan persekitaran untuk diletakkan bahan kimia tersebut mungkin mempunyai kesan keatas keberkesanan rawatan untuk menghapus semut api. Untuk mengurangkan kesan koloni semut api yang berlainan dan jenis busut yang mereka kerumuni, penyelidik mencipta busut semut api tiruan dan diisi dengan 50,000 ekor semut yang berketurunan serupa. Penyelidik tersebut mengumpukkan secara rawak dua busut kepada tiga bahan kimia berlainan dan diletakkan pada tiga lokasi yang mempunyai suasana persekitaran yang berlainan.*

	Bahan Kimia						
Lokasi	*A*		*B*		*C*		*Jumlah*
1	*7.2*		*4.2*		*5.4*		
	9.6	*16.8*	*3.5*	*7.7*	*3.9*	*9.3*	*33.8*
2	*8.5*		*2.9*		*6.3*		
	9.6	*18.1*	*3.3*	*6.2*	*6.0*	*12.3*	*36.6*
3	*9.1*		*1.8*		*6.1*		
	8.6	*17.7*	*2.4*	*4.2*	*5.6*	*11.7*	*33.6*
Jumlah	*52.6*		*18.1*		*33.3*		*104*

$\sum\sum\sum Y_{ijk}^2 = 712$

(i) Nyatakan suatu model dan anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.05$.
(iii) Lakukan suatu tatacara lanjutan.

(b) Researchers from the Department of Fruit Crops at a university used three different preservatives in freezing strawberries. The researchers prepared the yield from a strawberry patch for freezing and randomly divided it into four equal groups. Within each group they treated the strawberries with the appropriate preservative and packaged them into five small plastics for freezing at $0^{\circ}C$. The bags in group I served as a control group, while those in groups II, III, and IV were assigned one of three newly developed preservatives.

After all the bags of strawberries were prepared, they were stored at $0^{\circ}C$ for a period of six months. At the end of this time, the contents of each bag were allowed to thaw and then rated on a scale of 1 to 20 points for discoloration. A low score indicates little discoloration.

	Group				
Plastic Bag	I	II	III	IV	Total
1	20	16	3	2	41
2	7	17	4	1	29
3	19.5	3	16.5	1.5	40.5
4	19	16.5	4.5	14	54
5	18	17.5	4	2	41.5
Total	83.5	70	32	20.5	206

$\sum\sum Y_{ij}^2 = 3{,}187.5$

(i) State a model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.05$.

[29 marks]

(b) *Penyelidik daripada Jabatan Tanaman Buah-buahan di sebuah universiti menggunakan tiga bahan pengawet berlainan dalam membekukan buah strawberi. Penyelidik-penyelidik tersebut menyediakan hasil daripada suatu kebun strawberi untuk dibekukan dan membahagikannya kepada empat kumpulan yang sama. Bagi setiap kumpulan mereka merawat buah strawberi tersebut dengan pengawet berkenaan dan membungkusnya ke dalam lima plastik beg kecil untuk dibekukan pada $0^{o}C$. Beg-beg di dalam kumpulan I merupakan kumulan kawalan, manakala yang di dalam kumpulan II, III, dan IV diumpukkan secara rawak satu daripada tiga bahan pengawet yang baru dirumus.*

Selepas kesemua beg strawberi disediakan, ianya dibekukan pada $0^{o}C$ untuk suatu tempoh selama enam bulan. Pada akhir masa ini, kandungan setiap beg dibiarkan nyahbeku dan dinilaikan pada suatu skala 1 hingga 20 markah untuk kelunturan warna. Skor yang rendah menandakan sedikit kelunturannya.

Kumpulan

Beg Plastik	*I*	*II*	*III*	*IV*	*Jumlah*
1	*20*	*16*	*3*	*2*	*41*
2	*7*	*17*	*4*	*1*	*29*
3	*19.5*	*3*	*16.5*	*1.5*	*40.5*
4	*19*	*16.5*	*4.5*	*14*	*54*
5	*18*	*17.5*	*4*	*2*	*41.5*
Jumlah	*83.5*	*70*	*32*	*20.5*	*206*

$\sum\sum Y_{ij}^2 = 3{,}187.5$

(i) *Nyatakan suatu model dan anggapannya.*

(ii) *Lakukan suatu analisis yang sesuai pada* $\alpha = 0.05$.

[29 markah]

4. Literacy rate is a reflection of the educational facilities and quality of education available in a country, and mass communication plays a large part in the educational process. In an effort to investigate the effect of various mass communication outlets to the literacy rate of a country to, a researcher has obtained the following data for eight countries.

Newspaper copies per 1,000 population	Television sets per 1,000 population	Literacy rate
275	189	.98
259	181	.98
291	185	.85
274	181	.91
265	171	.83
257	165	.75
282	183	.99
264	179	.99

Refer to the appendix output for question 4.

(i) Perform an appropriate analysis to obtain the model and its assumptions. Use $\alpha = 0.10$.

(ii) Obtain an estimation interval and prediction interval for a country with 270 newspaper copies and 180 television sets per 1,000 population.

[14 marks]

4. *Kadar literasi menggambarkan kemudahan pendidikan dan kualiti pendidikan yang wujud di dalam sesebuah negara, dan komunikasi massa memainkan peranan yang besar di dalam proses pendidikan. Di dalam usaha untuk mengkaji kesan pelbagai saluran komunikasi massa kepada kadar literasi sesebuah negara, seorang penyelidik telah mendapatkan data berikut bagi lapan buah negara.*

Bilangan Naskah Akhbar bagi setiap 1,000 populasi	*Bilangan Televisyen bagi setiap 1,000 populasi*	*Kadar Literasi*
275	*189*	*.98*
259	*181*	*.98*
291	*185*	*.85*
274	*181*	*.91*
265	*171*	*.83*
257	*165*	*.75*
282	*183*	*.99*
264	*179*	*.99*

Rujuk kepada output lampiran untuk soalan 4.

(i) Lakukan suatu analisis yang sesuai untuk mendapatkan model serta anggapannya. Guna $\alpha = 0.10$.

(ii) Dapatkan suatu selang anggaran dan selang ramalan bagi sesebuah negara dengan 270 naskah akhbar dan 180 bilangan televisyen bagi setiap 1,000 populasi.

[14 markah]

APPENDIX: FORMULAS

1. **Completely Randomized Design**

$$SST = \sum_i \sum_j Y_{ij}^2 - \frac{Y_{..}^2}{N}$$

$$SSA = \sum_i \frac{Y_{i.}^2}{n_i} - \frac{Y_{..}^2}{N}$$

For any contrast : $L = \sum_i c_i \bar{Y}_{i.}$

$$SSL = \frac{\left(\sum_i c_i \bar{Y}_{i.}\right)^2}{\sum_i \frac{c_{i.}^2}{n_i}}$$

2. **Completely Randomized Block Design**

$$SST = \sum_i \sum_j Y_{ij}^2 - \frac{Y_{..}^2}{N}$$

$$SSA = \sum_i \frac{Y_{i.}^2}{b} - \frac{Y_{..}^2}{N}$$

$$SSB = \sum_j \frac{Y_{.j}^2}{a} - \frac{Y_{..}^2}{N}$$

3. **Latin Square Design**

$$SST = \sum_i \sum_j \sum_k Y_{ijk}^2 - \frac{Y_{...}^2}{N}$$

$$SSR = \sum_i \frac{Y_{i..}^2}{a} - \frac{Y_{...}^2}{N}$$

$$SSC = \sum_j \frac{Y_{.j.}^2}{a} - \frac{Y_{...}^2}{N}$$

$$SSA = \sum_k \frac{Y_{..k}^2}{a} - \frac{Y_{...}^2}{N}$$

...13/-

4. **Two-way Factorial Design**

$$SST = \sum_i \sum_j \sum_k Y_{ijk}^2 - \frac{Y_{...}^2}{N}$$

$$SS\,Cell = \sum_i \sum_j \frac{Y_{ij.}^2}{n} - \frac{Y_{...}^2}{N}$$

$$SSA = \sum_i \frac{Y_{i..}^2}{bn} - \frac{Y_{...}^2}{N}$$

$$SSB = \sum_j \frac{Y_{.j.}^2}{an} - \frac{Y_{...}^2}{N}$$

$$SSE = \sum_i \sum_j \sum_k Y_{ijk}^2 - \frac{Y_{ij.}^2}{n}$$

5. **Regression**

$$b_1 = \frac{SS_{xy}}{SS_x} \quad , \quad b_o = \overline{Y} - b_1\overline{X}$$

$$SSE = SS_y - \frac{\left[SS_{xy}\right]^2}{SSx}$$

$$SS_{xy} = \frac{\Sigma X_i Y_i - \frac{(\Sigma X_i)(\Sigma Y_i)}{n}}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}}$$

$$SS_x = \Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}$$

$$SS_y = \Sigma Y_i^2 - \frac{(\Sigma Y_i)^2}{n}$$

$$Var(b_1) = \frac{\sigma^2}{SS_x}$$

$$Var(\hat{Y}_h) = \sigma^2\left[\frac{1}{n} + \frac{(X_h - \overline{X})^2}{SS_x}\right] \quad , \quad Var(\hat{Y}_h) = MSE\left(\mathbf{X}_h'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_h\right)$$

...14/-

7. **Correlation**

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

$$t = r\sqrt{\frac{n-2}{1-r^2}}$$

8. **Kruskal-Wallis Test**

$$T = \frac{12}{N(N+1)} \sum_i \frac{R_i^2}{n_i} - 3(N+1)$$

9. **Friedman Test**

$$T = \frac{12}{ba(a+1)} \sum_i R_i^2 - 3b(a+1)$$

10. **Cochran Test**

$$T = \frac{a(a-1)\sum_i A_i^2 - (a-1)N^2}{aN - \sum_j B_j^2}$$

11. **Spearman Test**

$$r_s = 1 - \frac{6\sum_i d_i^2}{n(n^2-1)}$$

APPENDIX: OUTPUT

Question 4

Predictor	Coef	SE Coef	T
Constant	-0.2525	0.5079	-0.50
Newspaper	-0.004710	0.002313	-2.04
Television	0.013603	0.003519	3.87

S = 0.05324 R-Sq = 75.7% R-Sq(adj) = 65.9%

Analysis of Variance

Source	DF	SS	MS	F
Regression	2	0.044030	0.022015	7.77
Residual Error	5	0.014170	0.002834	
Total	7	0.058200		

$$(X'X)^{-1} = \begin{bmatrix} 91.0198 & -0.1643 & -0.2587 \\ -0.1643 & 0.0019 & -0.0019 \\ -0.2587 & -0.0019 & 0.0044 \end{bmatrix}$$

APPENDIX: TABLES

Duncan Multiple Range Table

VII. Significant Ranges for Duncan's Multiple Range Test[a]

$r_{.01}(p, f)$

	p											
f	2	3	4	5	6	7	8	9	10	20	50	100
1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
2	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
3	8.26	8.5	8.6	8.7	8.8	8.9	8.9	9.0	9.0	9.3	9.3	9.3
4	6.51	6.8	6.9	7.0	7.1	7.1	7.2	7.2	7.3	7.5	7.5	7.5
5	5.70	5.96	6.11	6.18	6.26	6.33	6.40	6.44	6.5	6.8	6.8	6.8
6	5.24	5.51	5.65	5.73	5.81	5.88	5.95	6.00	6.0	6.3	6.3	6.3
7	4.95	5.22	5.37	5.45	5.53	5.61	5.69	5.73	5.8	6.0	6.0	6.0
8	4.74	5.00	5.14	5.23	5.32	5.40	5.47	5.51	5.5	5.8	5.8	5.8
9	4.60	4.86	4.99	5.08	5.17	5.25	5.32	5.36	5.4	5.7	5.7	5.7
10	4.48	4.73	4.88	4.96	5.06	5.13	5.20	5.24	5.28	5.55	5.55	5.55
11	4.39	4.63	4.77	4.86	4.94	5.01	5.06	5.12	5.15	5.39	5.39	5.39
12	4.32	4.55	4.68	4.76	4.84	4.92	4.96	5.02	5.07	5.26	5.26	5.26
13	4.26	4.48	4.62	4.69	4.74	4.84	4.88	4.94	4.98	5.15	5.15	5.15
14	4.21	4.42	4.55	4.63	4.70	4.78	4.83	4.87	4.91	5.07	5.07	5.07
15	4.17	4.37	4.50	4.58	4.64	4.72	4.77	4.81	4.84	5.00	5.00	5.00
16	4.13	4.34	4.45	4.54	4.60	4.67	4.72	4.76	4.79	4.94	4.94	4.94
17	4.10	4.30	4.41	4.50	4.56	4.63	4.68	4.73	4.75	4.89	4.89	4.89
18	4.07	4.27	4.38	4.46	4.53	4.59	4.64	4.68	4.71	4.85	4.85	4.85
19	4.05	4.24	4.35	4.43	4.50	4.56	4.61	4.64	4.67	4.82	4.82	4.82
20	4.02	4.22	4.33	4.40	4.47	4.53	4.58	4.61	4.65	4.79	4.79	4.79
30	3.89	4.06	4.16	4.22	4.32	4.36	4.41	4.45	4.48	4.65	4.71	4.71
40	3.82	3.99	4.10	4.17	4.24	4.30	4.34	4.37	4.41	4.59	4.69	4.69
60	3.76	3.92	4.03	4.12	4.17	4.23	4.27	4.31	4.34	4.53	4.66	4.66
100	3.71	3.86	3.98	4.06	4.11	4.17	4.21	4.25	4.29	4.48	4.64	4.65
∞	3.64	3.80	3.90	3.98	4.04	4.09	4.14	4.17	4.20	4.41	4.60	4.68

f = degrees of freedom.

[a]Reproduced with permission from "Multiple Range and Multiple F Tests," by D. B. Duncan, *Biometrics*, Vol. 1, No. 1, pp. 1–42, 1955.

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Spearman Table

Table A.19 Critical values of the Spearman test statistic; approximate upper-tail critical values r_s^*, where $P(r_s > r_s^*) \leq \alpha$, $n = 4(1)30$; significance level, α

n	.001	.005	.010	.025	.050	.100
4	—	—	—	—	.8000	.8000
5	—	—	.9000	.9000	.8000	.7000
6	—	.9429	.8857	.8286	.7714	.6000
7	.9643	.8929	.8571	.7450	.6786	.5357
8	.9286	.8571	.8095	.7143	.6190	.5000
9	.9000	.8167	.7667	.6833	.5833	.4667
10	.8667	.7818	.7333	.6364	.5515	.4424
11	.8364	.7545	.7000	.6091	.5273	.4182
12	.8182	.7273	.6713	.5804	.4965	.3986
13	.7912	.6978	.6429	.5549	.4780	.3791
14	.7670	.6747	.6220	.5341	.4593	.3626
15	.7464	.6536	.6000	.5179	.4429	.3500
16	.7265	.6324	.5824	.5000	.4265	.3382
17	.7083	.6152	.5637	.4853	.4118	.3260
18	.6904	.5975	.5480	.4716	.3994	.3148
19	.6737	.5825	.5333	.4579	.3895	.3070
20	.6586	.5684	.5203	.4451	.3789	.2977
21	.6455	.5545	.5078	.4351	.3688	.2909
22	.6318	.5426	.4963	.4241	.3597	.2829
23	.6186	.5306	.4852	.4150	.3518	.2767
24	.6070	.5200	.4748	.4061	.3435	.2704
25	.5962	.5100	.4654	.3977	.3362	.2646
26	.5856	.5002	.4564	.3894	.3299	.2588
27	.5757	.4915	.4481	.3822	.3236	.2540
28	.5660	.4828	.4401	.3749	.3175	.2490
29	.5567	.4744	.4320	.3685	.3113	.2443
30	.5479	.4665	.4251	.3620	.3059	.2400