UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2012/2013 Academic Session

January 2013

# EKC 314 – Transport Phenomena
## *[Fenomena Pengangkutan]*

Duration : 3 hours
*[Masa : 3 jam]*

Please ensure that this examination paper contains TEN printed pages and SEVEN printed pages of Appendix before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak dan TUJUH muka surat Lampiran sebelum anda memulakan peperiksaan ini.]*

**Instruction:** Answer **FOUR (4)** questions. Section A is **COMPULSORY**. Answer any **TWO (2)** questions from Section B. All questions carry the same marks.

*[**Arahan:** Jawab **EMPAT (4)** soalan. Bahagian A **WAJIB**. Jawab mana-mana **DUA (2)** soalan dari Bahagian B. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].*

…2/-

Section A : Answer **ALL** questions.
*Bahagian A: Jawab **SEMUA** soalan.*

1. [a] Describe in detail the molecular theory of the viscosity of gases at **LOW** density in terms of the average velocity, $\bar{u}$, the frequency of the bombarded molecules, *Z*, and the distance, *a*, experienced during the last collision between molecules. Define all the terms used in the discussion.

*Terangkan secara terperinci teori kepekatan molekul gas pada ketumpatan rendah dari segi halaju purata* $\bar{u}$*, frekuensi hentaman molekul, Z dan jarak,* a*, yang berlaku semasa pelanggaran sebelumnya di antara molekul-molekul gas. Nyatakan setiap terma yang digunakan di dalam penerangan anda.*

*[8 marks/markah]*

[b] Using the defined equations in Q.1.[a]., compute the average molecular velocity (cm/s) and the mean free path, $\lambda$ (cm), for oxygen at 1 atm and 273.2 K. A reasonable value for *d* is 3 Å. What is the ratio of the mean free path to the molecular diameter under these conditions? (Useful constants are available in Appendix A).

*Dengan menggunakan persamaan-persamaan yang telah dinyatakan dalam S.1.[a]., kira halaju purata molekul (sm/s) dan min jarak bebas, $\lambda$ (sm), untuk oksigen pada 1 atm dan 273.2 K. Suatu nilai yang berpatutan bagi* d *adalah 3 Å. Apakah nisbah min jarak bebas kepada diantara molekul pada keadaan-keadaan ini? (Pemalar-pemalar berguna boleh dirujuk di Lampiran A).*

What would be the order of magnitude of the corresponding ratio in the liquid state?
*Apakah nilai magnitud bagi nisbah tersebut pada keadaan cecair?*

*[8 marks/markah]*

[c] With an aid of a diagram, derive the continuity equation for a fluid flowing within a laminar regime and show that for an incompressible type of fluid, the equation reduces into;

*Dengan menggunakan suatu gambarajah, terbitkan persamaan keselanjaran bagi pergerakan bendalir pada kawasan laminar dan tunjukkan bagi bendalir tidak-boleh-mampat, persamaan tersebut diringkaskan kepada;*

$$(\nabla \cdot \mathbf{v}) = 0$$

where **v** is the velocity vector.
*di mana v adalah vektor halaju.*

*[9 marks/markah]*

2. [a] Predict $D_{AB}$ for chlorine-air mixture at 23.89°C and 1 atm. Treat air as a single substance with Lennard-Jones parameters as given in the appendix (Tables D.1 and D.2). For the estimation, use the Chapman-Enskog theory with the equations given by;

*Ramalkan $D_{AB}$ bagi campuran klorin-udara pada 23.89°C dan 1 atm. Biarkan udara sebagai suatu bahan tunggal dengan parameter Lennard-Jones diberi pada lampiran (Jadual-jadual D.1 dan D.2). Bagi ramalan ini, gunakan teori Chapman-Enskog dan persamaan yang diberi sebagai;*

$$D_{AB} = 0.0018583\sqrt{T^3\left(\frac{1}{M_A}+\frac{1}{M_B}\right)}\frac{1}{p\sigma^2{}_{AB}\Omega_{D_{AB}}}$$

where,
*di mana,*

$$\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B) \text{ and } \varepsilon_{AB} = \sqrt{\varepsilon_A \varepsilon_B}$$

Then, repeat the estimation using the kinetic theory and the corresponding state argument equation given by;

*Kemudian, ulang pengiraan ini dengan menggunakan teori kinetik dan persamaan keadaan berlawanan yang di beri sebagai;*

$$\frac{pD_{AB}}{(p_{cA}p_{cB})^{1/3}(T_{cA}T_{cB})^{5/12}(\frac{1}{M_A}+\frac{1}{M_B})^{1/2}} = a\left(\frac{T}{\sqrt{T_{cA}T_{cB}}}\right)^b$$

where, $p$ represents the pressure in atm, $D_{AB}$ is the diffusivity between two components, $T$ represents the temperature in Kelvin and $M$ is the components' relative molecular mass. The dimensionless constants $a$ and $b$ are values obtained from experimental observation where $a = 2.745 \times 10^{-4}$ and $b = 1.823$. Discuss on the differences between the two correlations used.

*di mana, p mewakili tekanan dalam atm, $D_{AB}$ adalah kemeresapan di antara dua komponen,* T *mewakili suhu dalam Kelvin dan* M *adalah jisim molekul relatif bagi komponen-komponen. Pekali-pekali tanpa-dimensi* a *dan* b *adalah nilai yang diperolehi daripada pemerhatian ujikaji di mana* $a = 2.745 \times 10^{-4}$ *dan* $b = 1.823$. *Bincangkan perbezaan-perbezaan diantara dua korelasi yang telah digunakan.*

*[12 marks/markah]*

[b] [i] State three mechanisms of heat transfer.
*Nyatakan tiga mekanisma pemindahan haba.*

[ii] Can thermal energy be transferred without a medium (solid, liquid or gas)? Give the reason for your answer.
*Bolehkah tenaga haba dipindahkan tanpa medium (pepejal, cecair atau gas)? Berikan sebab kepada jawapan anda.*

*[3 marks/markah]*

[c] [i] A schematic representation in Figure Q.2.[c]. shows a phenomenon where heat is being constantly dissipated through a rectangular fin. The effectiveness of the surface fin for heat transfer is defined as parameter $\eta$, with a condition that the wall is maintained at constant temperature (isothermal). Given:

*Gambarajah skematik dalam Rajah S.2.[c]. menunjukkan suatu fenomena di mana haba dipindahkan secara malar melalui sirip segiempat. Keberkesanan permukaan sirip untuk pemindahan haba ditakrifkan sebagai parameter $\eta$ dengan syarat suhu pada dinding dikekalkan secara malar (isoterma). Diberi:*

$$\eta = \frac{\text{Rate of heat loss from the fin to the air}}{\text{Rate of heat loss from the wall to the air}}$$

$$\eta = \frac{\textit{Kadar kehilangan haba dari sirip ke udara}}{\textit{Kadar kehilangan haba dari dinding ke udara}}$$

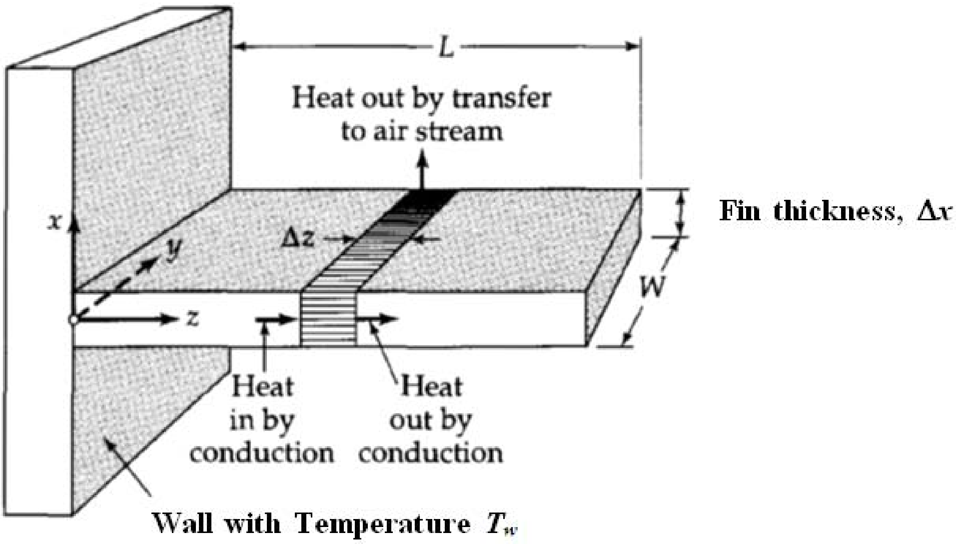


Figure Q.2.[c].
*Rajah S.2.[c].*

The numerical value of the effectiveness of the fin surface can be estimated through the following correlation:

*Nilai berangka keberkesanan permukaan sirip boleh dianggarkan melalui korelasi berikut:*

$$\eta = \frac{\tanh N}{N}$$

where,
*di mana,*

$$N = \sqrt{\frac{2hL^2}{k(\Delta x)}}$$

$h$ = heat transfer coefficient
$k$ = thermal conductivity
$L$ = length protruding from the wall
$\Delta x$ = thickness of the fin

*$h$ = pekali pemindahan haba*
*$k$ = kekonduksian haba*
*$L$ = panjang terkeluar dari dinding*
*$\Delta x$ = tebal sirip*

With the following assumptions:

- as $\Delta x << L$, heat loss through the edge of the fin is negligible
- the shaded area maintained at uniform temperature $T_w$

*Dengan andaian berikut:*

- *apabila* $\Delta x << L$, *kehilangan haba melalui pinggir sirip diabaikan*
- *kawasan berlorek dikekalkan pada suhu seragam* $T_w$

Obtain an expression for the rate of heat loss $Q$ from a rectangular fin through the definition of effectiveness of the fin surface. Explain any term(s) you might introduce.

*Dapatkan ungkapan bagi kadar kehilangan haba* Q *dari sirip segiempat melalui definisi keberkesanan permukaan sirip. Terangkan mana-mana istilah yang mungkin anda perkenalkan.*

*[5 marks/markah]*

[ii] If temperature of the surrounding air is found to be 30$^oC$ and the temperature of the wall maintained at 80$^oC$, calculate the heat loss from a rectangular fin having width 30 cm, length 6 cm and thickness 4 mm.

*Jika suhu udara sekeliling didapati 30$^oC$ dan suhu dinding dikekalkan pada 80$^oC$, kira kehilangan haba dari sirip segiempat yang mempunyai lebar 30 sm, panjang 6 sm dan tebal 4 mm.*

$k$ = 105 W/(m K)
$h$ = 680 W/(m$^2$K)

*[5 marks/markah]*

Section B : Answer any TWO questions.
*Bahagian B: Jawab mana-mana DUA soalan.*

3. A porous catalyst pellet having constant thermal conductivity, $k$, is geometrically modelled as a sphere with radius, $R$. Due to exothermic chemical reaction occurs within the porous pellet, heat is generated at a rate of $S_c$ (J/cm$^3$. s). The generated heat is lost at the outer surface of the pellet to a gas stream at temperature $T_g$ by convective heat transfer with heat transfer coefficient, $h$.

*Suatu pelet mangkin berliang yang mempunyai kekonduksian haba,* k, *yang malar dimodelkan secara geometri sebagai sfera berjejari,* R. *Oleh kerana tindak balas kimia eksotermik berlaku dalam pelet berliang, haba dijana pada kadar* $S_c$ *(J/sm$^3$·s). Haba yang dihasilkan hilang di permukaan luar pelet kepada aliran gas pada suhu* $T_g$ *menerusi pemindahan haba perolakan dengan pekali pemindahan haba* h.

[a] State the general statement of energy balance over a unit element of volume.
*Nyatakan kenyataan umum imbangan tenaga bagi satu elemen unit isipadu.*

*[2 marks/markah]*

[b] Assuming $S_c$ is constant throughout the pellet, show that:
*Andaikan* $S_c$ *adalah malar pada keseluruhan pelet, tunjukkan:*

$$\frac{d}{dr}(r^2 q_r) - r^2 S_c = 0$$

*[6 marks/markah]*

[c] Derive the steady-state temperature profile along the radius of the catalyst pellet.
*Terbitkan profil suhu keadaan mantap sepanjang jejari pelet mangkin.*

*[12 marks/markah]*

[d] What is the theoretical maximum temperature in the system and where does that occur?
*Apakah suhu maksimum teori di dalam sistem tersebut dan di mana ia berlaku?*

*[2 marks/markah]*

[e] Sketch the temperature profile in the solid pellet. Assume both *h* and *k* are constants.

*Lakarkan profil suhu di dalam pelet pepejal. Andaikan kedua-dua* h *dan* k *adalah pemalar.*

*[3 marks/markah]*

4. [a] In the near wall region, and where pressure gradients and gravitational forces are negligible, the Reynolds averaged form of the Navier-Stokes Equation reduces, for a 1-dimensional flow to;

*Pada kawasan berdekatan dengan dinding, di mana kecerunan tekanan dan daya graviti diabaikan, purata-Reynolds daripada Persamaan Navier-Stokes diringkaskan kepada aliran 1-dimensi;*

$$(\mu + \mu_t)\frac{\partial^2 v_x}{\partial y^2} \approx 0 \qquad (1)$$

where, $\mu$ and $\mu_t$ are the molecular and turbulent viscosities, $\overline{v_x}$ the mean velocity in the *x*-direction (parallel to the wall) and y is the distance from the wall. The turbulent velocity is defined as:

*di mana, $\mu$ dan $\mu_t$ adalah kelikatan molekul dan kelikatan gelora, $\overline{v_x}$ ialah halaju min pada arah-x (selari dengan dinding) dan y adalah jarak daripada dinding. Halaju gelora ditakrifkan sebagai:*

$$\mu_t = -\frac{\rho\overline{v_x' v_y'}}{\partial \bar{v}_x / \partial y} \qquad (2)$$

where, $v_x'$ and $v_y'$ are the instantaneous fluctuations in the velocity components in the *x* and *y*-direction respectively. Show that for the region immediately adjacent to the wall (where $\mu >> \mu_t$), equation (2) leads to expression:

*di mana, $v_x'$ dan $v_y'$ adalah ketidakstabilan pada komponen-komponen halaju pada arah x dan y. Tunjukkan bahawa kawasan yang paling berdekatan dengan dinding ($\mu >> \mu_t$), persamaan (2) membawa kepada persamaan:*

$$u^+ = y^+$$

where $u^+$ and $y^+$ are given by;

*di mana $u^+$ dan $y^+$ diberi sebagai;*

$$u^+ = \frac{\overline{v_x}}{u^*} \quad \text{and} \quad y^+ = \frac{u^* \rho y}{\mu}$$

and $u^*$ is the friction velocity defined as
*dan* u$^*$*adalah halaju geseran yang dimaksudkan sebagai;*

$$u^* = \sqrt{\tau_0 / \rho}$$

where, $\tau_0$ is the wall shear stress.
*di mana,* $\tau_0$ *adalah tegasan ricih dinding.*

*[10 marks/markah]*

[b] For the case of fluid further away from the wall, where $\mu_t >> \mu$, the Prandtl Mixing Length hypothesis may be invoked;

*Untuk kes bendalir yang berjauhan dari dinding, di mana* $\mu_t >> \mu$, *hipotesis panjang campuran Prandtl boleh digunakan;*

$$-\overline{v_x' v_y'} = l^2 \left| \frac{\partial \overline{v_x}}{\partial y} \right| \frac{\partial \overline{v_x}}{\partial y} \qquad (3)$$

where the mixing length $l$ is given by;
*di mana panjang campuran,* l *diberi sebagai;*

$$l = \kappa y \qquad (4)$$

where $\kappa$, is the von Karman constant. Show that it follows from equation (2) and from the above Mixing Length hypothesis that;
*di mana* $\kappa$, *adalah pemalar von Karman. Tunjukkan bahawa ia menuruti persamaan (2) di atas. Tunjukkan juga hipotesis panjang campuran di mana;*

$$u^+ = \frac{1}{\kappa} \ln y^+ + C \qquad (5)$$

where $C$ is a constant.
*di mana C adalah suatu pemalar.*

*[10 marks/markah]*

[c] Assuming that $\kappa = 0.41$ and that equations (4) and (5) give ideal values for $u^+$ at $y^+ = 11$, estimate the value of the constant $C$ in equation (5).

*Mengandaikan bahawa* $\kappa$ *= 0.41 dan persamaan (4) dan (5) di atas diberi sebagai nilai-nilai unggul untuk* $u^+$ *pada* $y^+$ *=11, anggarkan nilai pemalar C pada persamaan (5).*

*[5 marks/markah]*

5. Figure Q.5 shows a droplet of methanol, (A) of radius $r_1$, is suspended in a stream of air, (B). We postulate that there is spherical stagnant gas film of radius $r_2$ surrounding the droplet. The concentration of methanol in the gas phase is $x_{A1}$ at $r = r_1$ and $x_{A2}$ at the outer edge of the film, $r = r_2$.

*Gambarajah S.5. menunjukkan suatu titisan metanol, (A) dengan jejari* $r_1$, *yang terampai di dalam aliran udara, (B). Kita menaakulkan bahawa terdapat genangan gas filem sfera statik dengan jejari* $r_2$ *menyelaputi titisan tersebut. Kepekatan bahan metanol dalam fasa gas adalah* $x_{A1}$ *pada* $r = r_1$ *dan* $x_{A2}$ *pada sisi luaran filem,* $r = r_2$.

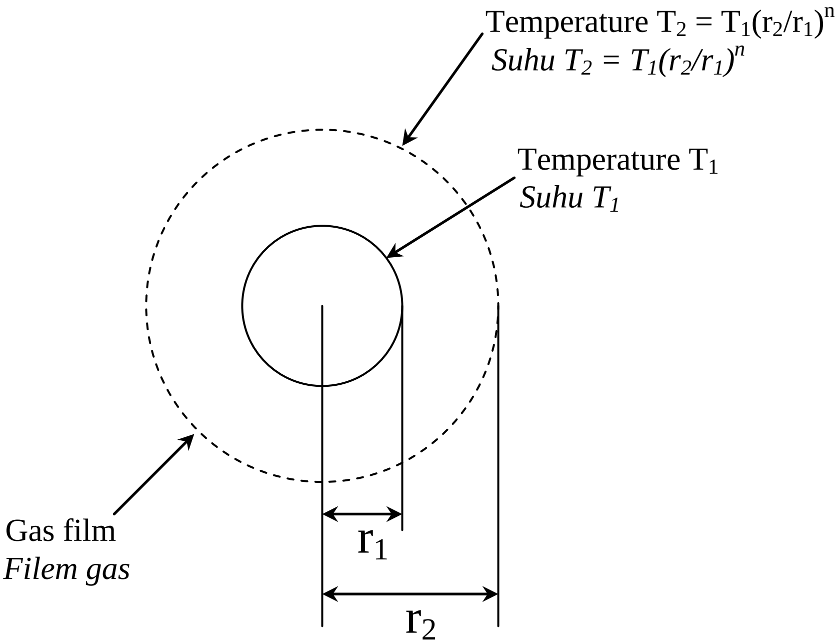


Figure Q.5.
*Rajah S.5.*

[a] By a shell balance, show that for steady-state diffusion $r^2 N_{Ar}$ is a constant within the gas film, and set the constant equal to $r_1^2 N_{Ar1}$ the value at the droplet surface.

*Dengan menggunakan imbangan kerangka, tunjukkan kemeresapan,* $r^2 N_{Ar}$ *pada keadaan mantap adalah pemalar dalam filem gas dan tetapkan pemalar tersebut sama dengan* $r^2{}_1 N_{Ar1}$ *iaitu nilai pada permukaan titisan.*

*[8 marks/markah]*

[b] Show that by using the flux balance equation and the result obtained in [a], the following equation can be obtained;
*Tunjukkan dengan menggunakan persamaan imbangan fluks, dan hasil yang diperolehi daripada [a] membawa kepada persamaan berikut;*

$$r_1^2 N_{Ar1} = -\frac{cD_{AB}}{1 - x_A} r^2 \frac{dx_A}{dr}$$

*[8 marks/markah]*

[c] Integrate this equation between the limits $r_1$ and $r_2$ to get;
*Kamirkan persamaan di atas di antara had-had* $r_1$ *dan* $r_2$ *bagi menghasilkan;*

$$N_{Ar1} = \frac{cD_{AB}}{r_2 - r_1}\left(\frac{r_2}{r_1}\right)\ln\frac{x_{B2}}{x_{B1}}$$

What is the limit of this expression when;
*Apakah had bagi persamaan yang terhasil apabila;*

$$r_2 \rightarrow \infty$$

*[9 marks/markah]*

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**Appendices**

**Appendix A: Conversion Factors and Useful Constants**

| Given a quantity in these units: | Multiply by: | To get quantity in these units: |
|---|---|---|
| Pounds | 453.59 | Grams |
| Kilograms | 2.2046 | Pounds |
| Inches | 2.5400 | Centimeters |
| Meters | 39.370 | Inches |
| Gallons (U.S.) | 3.7853 | Liters |
| Gallons (U.S.) | 231.00 | Cubic inches |
| Gallons (U.S.) | 0.13368 | Cubic feet |
| Cubic feet | 28.316 | Liters |
| Kelvins | 1.800000 | Degrees Rankine |
| Degrees Rankine | 0.555556 | Kelvins |

Table A.1

| Given a quantity in this units | Multiply by table value to convert to these units | $N = kg \cdot m/s^2$ (Newtons) | $g \cdot cm/s^2$ | $lb_m \cdot ft/s^2$ | $lb_f$ |
|---|---|---|---|---|---|
| $N = kg \cdot m/s^2$ | (Newtons) | 1 | $10^5$ | 7.2330 | $2.24881 \times 10^{-1}$ |
| $g \cdot cm/s^2$ | (dynes) | $10^{-5}$ | 1 | $7.2330 \times 10^{-5}$ | $2.24881 \times 10^{-6}$ |
| $lb_m \cdot ft/s^2$ | (poundals) | $1.3826 \times 10^{-1}$ | $1.3826 \times 10^4$ | 1 | $3.1081 \times 10^{-2}$ |
| $lb_f$ | | 4.4482 | $4.4482 \times 10^5$ | 32.1740 | 1 |

Table A.2

| Constant | Values |
|---|---|
| Ideal gas constant, R | 8.3145 J/mol.K<br>82.057 $cm^3$ atm/mol.K |
| Boltzmann constant, k | $1.3806503 \times 10^{-23}$ $m^2$kg/s.K |
| Avogradro's number, $N_A$ | $6.02214 \times 10^{23}$ $mol^{-1}$ |

Table A.3

**<u>Appendix B</u>: Equation of Motion in Terms of $\tau$**

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p - [\nabla \cdot \tau] + \rho \mathbf{g}$$

*Cartesian coordinates $(x, y, z)$:*[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.1

*Cylindrical coordinates $(r, \theta, z)$:*[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.2

*Spherical coordinates $(r, \theta, \phi)$:*[c]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$- \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi$$

Table B.3

**Appendix C: Equation of Motion for a Newtoniam Fluid with Constant $\rho$ and $\mu$**

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p + \boldsymbol{\mu}\nabla^2\mathbf{v} + \rho \boldsymbol{g}$$

*Cartesian coordinates $(x, y, z)$:*

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.1

*Cylindrical coordinates $(r, \theta, z)$:*

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.2

*Spherical coordinates $(r, \theta, \phi)$:*

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$

Table C.3

## Appendix D: Lennard-Jones Potential Parameters and Critical Properties

| | | Lennard-Jones parameters | | | Critical properties[g,h] | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Substance | Molecular Weight $M$ | $\sigma$ (Å) | $\varepsilon/\kappa$ (K) | Ref. | $T_c$ (K) | $p_c$ (atm) | $\tilde{V}_c$ ($cm^3$/g-mole) | $\mu_c \times 10^6$ (g/cm · s) | $k_c \times 10^6$ (cal/cm · s · K) |
| **Light elements:** | | | | | | | | | |
| $H_2$ | 2.016 | 2.915 | 38.0 | *a* | 33.3 | 12.80 | 65.0 | 34.7 | — |
| He | 4.003 | 2.576 | 10.2 | *a* | 5.26 | 2.26 | 57.8 | 25.4 | — |
| **Noble gases:** | | | | | | | | | |
| Ne | 20.180 | 2.789 | 35.7 | *a* | 44.5 | 26.9 | 41.7 | 156. | 79.2 |
| Ar | 39.948 | 3.432 | 122.4 | *b* | 150.7 | 48.0 | 75.2 | 264. | 71.0 |
| Kr | 83.80 | 3.675 | 170.0 | *b* | 209.4 | 54.3 | 92.2 | 396. | 49.4 |
| Xe | 131.29 | 4.009 | 234.7 | *b* | 289.8 | 58.0 | 118.8 | 490. | 40.2 |
| **Simple polyatomic gases:** | | | | | | | | | |
| Air | 28.964[i] | 3.617 | 97.0 | *a* | 132.4[i] | 37.0[i] | 86.7[i] | 193. | 90.8 |
| $N_2$ | 28.013 | 3.667 | 99.8 | *b* | 126.2 | 33.5 | 90.1 | 180. | 86.8 |
| $O_2$ | 31.999 | 3.433 | 113. | *a* | 154.4 | 49.7 | 74.4 | 250. | 105.3 |
| CO | 28.010 | 3.590 | 110. | *a* | 132.9 | 34.5 | 93.1 | 190. | 86.5 |
| $CO_2$ | 44.010 | 3.996 | 190. | *a* | 304.2 | 72.8 | 94.1 | 343. | 122. |
| NO | 30.006 | 3.470 | 119. | *a* | 180. | 64. | 57. | 258. | 118.2 |
| $N_2O$ | 44.012 | 3.879 | 220. | *a* | 309.7 | 71.7 | 96.3 | 332. | 131. |
| $SO_2$ | 64.065 | 4.026 | 363. | *c* | 430.7 | 77.8 | 122. | 411. | 98.6 |
| $F_2$ | 37.997 | 3.653 | 112. | *a* | — | — | — | — | — |
| $Cl_2$ | 70.905 | 4.115 | 357. | *a* | 417. | 76.1 | 124. | 420. | 97.0 |
| $Br_2$ | 159.808 | 4.268 | 520. | *a* | 584. | 102. | 144. | — | — |
| $I_2$ | 253.809 | 4.982 | 550. | *a* | 800. | — | — | — | — |
| **Hydrocarbons:** | | | | | | | | | |
| $CH_4$ | 16.04 | 3.780 | 154. | *b* | 191.1 | 45.8 | 98.7 | 159. | 158. |
| $CH \equiv CH$ | 26.04 | 4.114 | 212. | *d* | 308.7 | 61.6 | 112.9 | 237. | — |
| $CH_2 = CH_2$ | 28.05 | 4.228 | 216. | *b* | 282.4 | 50.0 | 124. | 215. | — |
| $C_2H_6$ | 30.07 | 4.388 | 232. | *b* | 305.4 | 48.2 | 148. | 210. | 203. |
| $CH_3C \equiv CH$ | 40.06 | 4.742 | 261. | *d* | 394.8 | — | — | — | — |
| $CH_3CH = CH_2$ | 42.08 | 4.766 | 275. | *b* | 365.0 | 45.5 | 181. | 233. | — |
| $C_3H_8$ | 44.10 | 4.934 | 273. | *b* | 369.8 | 41.9 | 200. | 228. | — |
| $n$—$C_4H_{10}$ | 58.12 | 5.604 | 304. | *b* | 425.2 | 37.5 | 255. | 239. | — |
| $i$—$C_4H_{10}$ | 58.12 | 5.393 | 295. | *b* | 408.1 | 36.0 | 263. | 239. | — |
| $n$—$C_5H_{12}$ | 72.15 | 5.850 | 326. | *b* | 469.5 | 33.2 | 311. | 238. | — |
| $i$—$C_5H_{12}$ | 72.15 | 5.812 | 327. | *b* | 460.4 | 33.7 | 306. | — | — |
| $C(CH_3)_4$ | 72.15 | 5.759 | 312. | *b* | 433.8 | 31.6 | 303. | — | — |
| $n$—$C_6H_{14}$ | 86.18 | 6.264 | 342. | *b* | 507.3 | 29.7 | 370. | 248. | — |
| $n$—$C_7H_{16}$ | 100.20 | 6.663 | 352. | *b* | 540.1 | 27.0 | 432. | 254. | — |
| $n$—$C_8H_{18}$ | 114.23 | 7.035 | 361. | *b* | 568.7 | 24.5 | 492 | 259. | — |
| $n$—$C_9H_{20}$ | 128.26 | 7.463 | 351. | *b* | 594.6 | 22.6 | 548. | 265. | — |
| Cyclohexane | 84.16 | 6.143 | 313. | *d* | 553. | 40.0 | 308. | 284. | — |
| Benzene | 78.11 | 5.443 | 387. | *b* | 562.6 | 48.6 | 260. | 312. | — |
| **Other organic compounds:** | | | | | | | | | |
| $CH_4$ | 16.04 | 3.780 | 154. | *b* | 191.1 | 45.8 | 98.7 | 159. | 158. |
| $CH_3Cl$ | 50.49 | 4.151 | 355. | *c* | 416.3 | 65.9 | 143. | 338. | — |
| $CH_2Cl_2$ | 84.93 | 4.748 | 398. | *c* | 510. | 60. | — | — | — |
| $CHCl_3$ | 119.38 | 5.389 | 340. | *e* | 536.6 | 54. | 240. | 410. | — |
| $CCl_4$ | 153.82 | 5.947 | 323. | *e* | 556.4 | 45.0 | 276. | 413. | — |
| $C_2N_2$ | 52.034 | 4.361 | 349. | *e* | 400. | 59. | — | — | — |
| COS | 60.076 | 4.130 | 336. | *e* | 378. | 61. | — | — | — |
| $CS_2$ | 76.143 | 4.483 | 467. | *e* | 552. | 78. | 170. | 404. | — |
| $CCl_2F_2$ | 120.91 | 5.116 | 280. | *b* | 384.7 | 39.6 | 218. | — | — |

Table D.1

Collision Integrals for use with the Lennard-Jones Potential for the Prediction of Transport Properties of Gases at Low Densities

| $\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$ | $\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity) | $\Omega_{\mathfrak{D},AB}$ (for diffusivity) | $\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$ | $\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity) | $\Omega_{\mathfrak{D},AB}$ (for diffusivity) |
|---|---|---|---|---|---|
| 0.30 | 2.840 | 2.649 | 2.7 | 1.0691 | 0.9782 |
| 0.35 | 2.676 | 2.468 | 2.8 | 1.0583 | 0.9682 |
| 0.40 | 2.531 | 2.314 | 2.9 | 1.0482 | 0.9588 |
| 0.45 | 2.401 | 2.182 | 3.0 | 1.0388 | 0.9500 |
| 0.50 | 2.284 | 2.066 | 3.1 | 1.0300 | 0.9418 |
| 0.55 | 2.178 | 1.965 | 3.2 | 1.0217 | 0.9340 |
| 0.60 | 2.084 | 1.877 | 3.3 | 1.0139 | 0.9267 |
| 0.65 | 1.999 | 1.799 | 3.4 | 1.0066 | 0.9197 |
| 0.70 | 1.922 | 1.729 | 3.5 | 0.9996 | 0.9131 |
| 0.75 | 1.853 | 1.667 | 3.6 | 0.9931 | 0.9068 |
| 0.80 | 1.790 | 1.612 | 3.7 | 0.9868 | 0.9008 |
| 0.85 | 1.734 | 1.562 | 3.8 | 0.9809 | 0.8952 |
| 0.90 | 1.682 | 1.517 | 3.9 | 0.9753 | 0.8897 |
| 0.95 | 1.636 | 1.477 | 4.0 | 0.9699 | 0.8845 |
| 1.00 | 1.593 | 1.440 | 4.1 | 0.9647 | 0.8796 |
| 1.05 | 1.554 | 1.406 | 4.2 | 0.9598 | 0.8748 |
| 1.10 | 1.518 | 1.375 | 4.3 | 0.9551 | 0.8703 |
| 1.15 | 1.485 | 1.347 | 4.4 | 0.9506 | 0.8659 |
| 1.20 | 1.455 | 1.320 | 4.5 | 0.9462 | 0.8617 |
| 1.25 | 1.427 | 1.296 | 4.6 | 0.9420 | 0.8576 |
| 1.30 | 1.401 | 1.274 | 4.7 | 0.9380 | 0.8537 |
| 1.35 | 1.377 | 1.253 | 4.8 | 0.9341 | 0.8499 |
| 1.40 | 1.355 | 1.234 | 4.9 | 0.9304 | 0.8463 |
| 1.45 | 1.334 | 1.216 | 5.0 | 0.9268 | 0.8428 |
| 1.50 | 1.315 | 1.199 | 6.0 | 0.8962 | 0.8129 |
| 1.55 | 1.297 | 1.183 | 7.0 | 0.8727 | 0.7898 |
| 1.60 | 1.280 | 1.168 | 8.0 | 0.8538 | 0.7711 |
| 1.65 | 1.264 | 1.154 | 9.0 | 0.8380 | 0.7555 |
| 1.70 | 1.249 | 1.141 | 10.0 | 0.8244 | 0.7422 |
| 1.75 | 1.235 | 1.128 | 12.0 | 0.8018 | 0.7202 |
| 1.80 | 1.222 | 1.117 | 14.0 | 0.7836 | 0.7025 |
| 1.85 | 1.209 | 1.105 | 16.0 | 0.7683 | 0.6878 |
| 1.90 | 1.198 | 1.095 | 18.0 | 0.7552 | 0.6751 |
| 1.95 | 1.186 | 1.085 | 20.0 | 0.7436 | 0.6640 |
| 2.00 | 1.176 | 1.075 | 25.0 | 0.7198 | 0.6414 |
| 2.10 | 1.156 | 1.058 | 30.0 | 0.7010 | 0.6235 |
| 2.20 | 1.138 | 1.042 | 35.0 | 0.6854 | 0.6088 |
| 2.30 | 1.122 | 1.027 | 40.0 | 0.6723 | 0.5964 |
| 2.40 | 1.107 | 1.013 | 50.0 | 0.6510 | 0.5763 |
| 2.50 | 1.0933 | 1.0006 | 75.0 | 0.6140 | 0.5415 |
| 2.60 | 1.0807 | 0.9890 | 100.0 | 0.5887 | 0.5180 |

Table D.2

**Appendix E:** Some Ordinary Differential Equations and Their Solutions

| Equation | Solution |
|---|---|
| $\dfrac{dy}{dx} = \dfrac{f(x)}{g(y)}$ | $\int g\,dy = \int f\,dx + C_1$ |
| $\dfrac{dy}{dx} + f(x)y = g(x)$ | $y = e^{-\int f\,dx}(\int e^{\int f\,dx} g\,dx + C_1)$ |
| $\dfrac{d^2y}{dx^2} + a^2 y = 0$ | $y = C_1 \cos ax + C_2 \sin ax$ |
| $\dfrac{d^2y}{dx^2} - a^2 y = 0$ | $y = C_1 \cosh ax + C_2 \sinh ax$ or <br> $y = C_3 e^{+ax} + C_4 e^{-ax}$ |
| $\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2 \dfrac{dy}{dx}\right) + a^2 y = 0$ | $y = \dfrac{C_1}{x} \cos ax + \dfrac{C_2}{x} \sin ax$ |
| $\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2 \dfrac{dy}{dx}\right) - a^2 y = 0$ | $y = \dfrac{C_1}{x} \cosh ax + \dfrac{C_2}{x} \sinh ax$ or <br> $y = \dfrac{C_3}{x} e^{+ax} + \dfrac{C_4}{x} e^{-ax}$ |
| $\dfrac{d^2y}{dx^2} + a\dfrac{dy}{dx} + by = 0$ | Solve the equation $n^2 + an + b = 0$, and get the roots $n = n_+$ and $n = n_-$. Then (a) if $n_+$ and $n_-$ are real and unequal, $y = C_1 \exp(n_+ x) + C_2 \exp(n_- x)$ <br> (b) if $n_+$ and $n_-$ are real and equal to $n$, $y = e^{nx}(C_1 x + C_2)$ <br> (c) if $n_+$ and $n_-$ are complex: $n_\pm = p \pm iq$, $y = e^{px}(C_1 \cos qx + C_2 \sin qx)$ |
| $\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 0$ | $y = C_1 \int_0^x \exp(-\bar{x}^2)\,d\bar{x} + C_2$ |
| $\dfrac{d^2y}{dx^2} + 3x^2\dfrac{dy}{dx} = 0$ | $y = C_1 \int_0^x \exp(-\bar{x}^3)\,d\bar{x} + C_2$ |
| $\dfrac{d^2y}{dx^2} = f(x)$ | $y = \int_0^x \int_0^{\bar{x}} f(\bar{\bar{x}})\,d\bar{\bar{x}}\,d\bar{x} + C_1 x + C_2$ |
| $\dfrac{1}{x}\dfrac{d}{dx}\left(x \dfrac{dy}{dx}\right) = f(x)$ | $y = \int_0^x \dfrac{1}{\bar{x}} \int_0^{\bar{x}} \bar{\bar{x}} f(\bar{\bar{x}})\,d\bar{\bar{x}}\,d\bar{x} + C_1 \ln x + C_2$ |
| $\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2 \dfrac{dy}{dx}\right) = f(x)$ | $y = \int_0^x \dfrac{1}{\bar{x}^2} \int_0^{\bar{x}} \bar{\bar{x}}^2 f(\bar{\bar{x}})\,d\bar{\bar{x}}\,d\bar{x} - \dfrac{C_1}{x} + C_2$ |
| $\dfrac{d^2y}{dx^2} = h(y)$ | $x = \int_0^y \dfrac{d\bar{y}}{\sqrt{2\int_0^{\bar{y}} h(\bar{\bar{y}})\,d\bar{\bar{y}} + C_1}} + C_2$ |
| $x^3 \dfrac{d^3y}{dx^3} + ax^2 \dfrac{d^2y}{dx^2} + bx\dfrac{dy}{dx} + cy = 0$ | $y = C_1 x^{n_1} + C_2 x^{n_2} + C_3 x^{n_3}$, where the $n_k$ are the roots of the equation $n(n-1)(n-2) + an(n-1) + bn + c = 0$, provided that all roots are distinct. |

Table E

**<u>Appendix E</u>:** Some Ordinary Differential Equations and Their Solutions (cont'd)

Error Function:

The error function is defined as

$$\operatorname{erf} x = \frac{\int_0^x \exp(-\bar{x}^2)\, d\bar{x}}{\int_0^\infty \exp(-\bar{x}^2)\, d\bar{x}} = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\bar{x}^2)\, d\bar{x}$$

$$\frac{d}{dx} \operatorname{erf} u = \frac{2}{\sqrt{\pi}} \exp(-u^2) \frac{du}{dx}$$