
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MAA 101 – Calculus For Science Students I
[Kalkulus Untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

Arahan: Jawab **semua sembilan** [9] soalan.]

1. Evaluate the limit. The L'Hospital rule can be applied whenever it is applicable.

$$(a) \lim_{x \rightarrow 3} \frac{(x^2 + x - 12)^2}{x - 3} \quad (b) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\pi - 2x} \quad (c) \lim_{x \rightarrow \infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}} \quad (d) \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - 1}}{x + 1}$$

[20 marks]

2. (a) Let $f(x) = x^3 + \frac{1}{2}x$. Find $(f^{-1})'(0)$.

(b) Given that

$$f(0) = 1, f'(0) = 2, f(1) = 0, f'(1) = 1, f(2) = 1, f'(2) = 1,$$

$$g(0) = 2, g'(0) = 1, g(1) = 1, g'(1) = 0, g(2) = 2, g'(2) = 1,$$

$$h(0) = 1, h'(0) = 2, h(1) = 2, h'(1) = 1, h(2) = 0, h'(2) = 2,$$

evaluate the following:

(i) $(f \circ h)'(0)$

(ii) $(g \circ f)'(2)$

(iii) $(h \circ g \circ f)'(1)$

[20 marks]

3. Two curves are called *orthogonal* if at each point of intersection, their tangent lines are perpendicular.

(a) Find the point (or points) of intersection of the curves $2x^2 + y^2 = 3$ and $x = y^2$.

(b) Show that the curves $2x^2 + y^2 = 3$ and $x = y^2$ are orthogonal.

[16 marks]

4. Let F be defined by

$$F(x) = \int_0^x \frac{1}{1+t^2} dt, \text{ where } x \text{ is any real number.}$$

(a) Find the critical numbers of F and determine the intervals on which F is increasing and the intervals on which F is decreasing.

(b) Determine the concavity of the graph of F and find the points of inflection.

(c) Using the information in (a) and (b), sketch the graph of F .

[20 marks]

5. Show that the equation

$$a(x-3) + b(x-1) = 0, \quad (a > 0, b > 0)$$

has one real root in the interval $(1, 3)$.

[10 marks]

1. Nilai kan had berikut. Petua L'Hospital boleh digunakan jika perlu.

$$(a) \lim_{x \rightarrow 3} \frac{(x^2 + x - 12)^2}{x - 3} \quad (b) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\pi - 2x} \quad (c) \lim_{x \rightarrow \infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}} \quad (d) \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - 1}}{x + 1}$$

[20 markah]

2. (a) Andaikan $f(x) = x^3 + \frac{1}{2}x$. Cari $(f^{-1})'(0)$.

(b) Diberi

$$f(0) = 1, f'(0) = 2, f(1) = 0, f'(1) = 1, f(2) = 1, f'(2) = 1,$$

$$g(0) = 2, g'(0) = 1, g(1) = 1, g'(1) = 0, g(2) = 2, g'(2) = 1,$$

$$h(0) = 1, h'(0) = 2, h(1) = 2, h'(1) = 1, h(2) = 0, h'(2) = 2,$$

cari nilai berikut:

$$(i) (f \circ h)'(0)$$

$$(ii) (g \circ f)'(2)$$

$$(iii) (h \circ g \circ f)'(1)$$

[20 markah]

3. Dua lengkung dikatakan ortogonal jika pada setiap titik persilangan, tangen mereka adalah berserenjang.

(a) Cari titik (titik-titik) persilangan untuk lengkung-lengkung $2x^2 + y^2 = 3$ dan $x = y^2$.

(b) Tunjukkan bahawa lengkung-lengkung $2x^2 + y^2 = 3$ dan $x = y^2$ adalah ortogonal.

[16 markah]

4. Andaikan F ditakrif sebagai

$$F(x) = \int_0^x \frac{1}{1+t^2} dt, \text{ dengan } x \text{ nombor nyata.}$$

(a) Cari titik genting untuk F dan tentukan selang-selang berlakunya F menaik dan F menyusut.

(b) Tentukan kecembungan graf untuk F dan cari titik lengkok balas.

(c) Dengan menggunakan maklumat di (a) dan (b), lakarkan graf untuk F .

[20 markah]

5. Tunjukkan bahawa persamaan

$$a(x-3) + b(x-1) = 0, \quad (a > 0, b > 0)$$

mempunyai satu punca nyata dalam selang $(1, 3)$.

[10 markah]

6. Set up, but **do not** evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the lines

$$2y = x + 4, y = x \text{ and } x = 0,$$

about

- (a) the x -axis using the washer method.
- (b) the y -axis using the cylindrical shell method.
- (c) the line $x = 4$ using the cylindrical shell method.
- (d) the line $y = 8$ using the washer method.

[20 marks]

7. Evaluate the integral $\int \frac{x^3}{\sqrt{x^2 + 4}} dx$ by using

- (a) trigonometric substitution,
 - (b) substitution $y = x^2 + 4$.
- (Write your final answers in term of variable x .)

[24 marks]

8. The function $f(x) = 2x^2 + 4x - 3$ changes value when x changes from $x_0 = -1.0$ to $x_0 + dx = -0.9$. Find

- (a) the differential Δf
- (b) the value of the estimate df
- (c) the approximation error $|\Delta f - df|$

[10 marks]

9. **True or False.**

- (a) Any continuous function on a closed interval $[a, b]$ has a derivative.
- (b) If f is differentiable on $(1, 4)$, then $\int_2^3 f'(x) dx = f(3) - f(2)$.
- (c) If $f'(c) = 0$ and $f''(c) = 0$, then f has an inflection point at c .
- (d) A continuous function on (a, b) attains an absolute maximum and an absolute minimum at some numbers c and d in (a, b) .
- (e) If f and g are continuous at a , then their product fg is continuous at a .

[10 marks]

6. Bentukkan, tanpa menilai, satu kamiran untuk isipadu bongkah yang diperoleh dengan memutarakan rantau yang dibatasi oleh garis-garis
 $2y = x + 4$, $y = x$ and $x = 0$,

sekitar

- (a) paksi x dengan menggunakan kaedah "washer".
- (b) paksi y dengan menggunakan kaedah kerang silinder.
- (c) garis $x = 4$ dengan menggunakan kaedah kerang silinder.
- (d) garis line $y = 8$ dengan menggunakan kaedah "washer".

[20 markah]

7. Nilai kamiran $\int \frac{x^3}{\sqrt{x^2 + 4}} dx$ dengan menggunakan

- (a) gantian trigonometrik,
 - (b) gantian $y = x^2 + 4$.
- (Tulis jawapan anda dalam pembolehubah x .)

[24 markah]

8. Fungsi $f(x) = 2x^2 + 4x - 3$ bertukar nilai apabila x bertukar dari $x_0 = -1.0$ ke $x_0 + dx = -0.9$. Cari

- (a) pembeza Δf
- (b) nilai anggaran df
- (c) ralat anggaran $|\Delta f - df|$

[10 markah]

9. Benar atau Salah.

- (a) Fungsi selanjar pada selang tertutup $[a, b]$ mempunyai terbitan.
- (b) Jika f terbezakan pada $(1, 4)$, maka $\int_2^3 f'(x) dx = f(3) - f(2)$.
- (c) Jika $f'(c) = 0$ dan $f''(c) = 0$, maka f mempunyai titik lengkok balas di c .
- (d) Suatu fungsi selanjar pada (a, b) memperoleh satu maksimum mutlak dan satu minimum mutlak pada suatu nombor c dan d dalam (a, b) .
- (e) Jika f dan g selanjar pada a , maka hasil darab fg juga selanjar pada a .

[10 markah]