A GAME THEORETICAL APPROACH IN THE PARLIMENTARY ELECTORAL SYSTEM

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The aim of this paper is to suggest a methodology for an analysis of the distribution of power in multi-party parliamentary bodies elected on the basis of a proportional electoral system. Concepts of the political spectrum and the power spectrum based on game theoretical concepts of power indices on a single left-right ideological dimension are used.

Keywords: distribution of power, committees, political spectrum, voting rules.

1. Introduction

One of the interesting topics in the theory of public choice is an analysis of power distribution and concentration of power in committee systems including parliamentary bodies and shareholding companies. It is known that the distribution of votes among the groups in a committee is not a sufficient characteristic of their voting power or an influence. The so-called power indices are used to estimate the influence of the members of a committee as a function of a voting rule and of a structure of the committee.

We use the term committee for group of formations called "parties" consisting of members called "deputies", who are making decisions by voting for or against some bills. By a quota we mean a minimal number of votes required to pass a bill in accordance to a voting rule. Speaking about a simple majority rule, we mean that the quota is equal to 50 % of all deputies in the committee plus one. Speaking about a qualified majority, we mean that for passing a bill more than a simple majority is required (usually 66.67 % of votes).

We shall assume that:

(1) Each deputy has one vote; hence the number of votes of each party is equal to the number of deputies.
(2) All deputies of the same party always vote together
(3) If some parties vote together, we say that they form a voting coalition. A voting coalition of several parties' votes as one party.
(4) Any coalition of parties is possible and all coalitions are equally probable.

In multiparty parliaments elected on the basis of a proportional electoral system when voters vote not for a person, but for a party program and list, we can consider assumption 1 through 3 to be an appropriate approximation of real parliamentary behavior; any member of the parliament as a member of a party does not derive his decisive power from a personal confidence of some majority group of voters but from a party that nominated him and that got the confidence of voters, and therefore he is supposed to support an original party program and policy. Assumption 4 may raise some questions about the "propensity" of some party to cooperate with another party.

The distribution of votes among the parties in the committee is not a sufficient characteristic of power or influence distribution. This can be clearly seen from a simple example of the committee with 3 parties and 100 deputies (see Table 1.).

Table 1. The committee with 3 parties and 100 deputies

Parties	Deputies
1	49
2	2
3	49

With respect to a 50 % majority rule all three parties have the same position in the voting process (any two-party coalition is a winning one, no single party can win). In fact, under certain circumstances (if the two large parties 1 and 3 are on the opposite sides of the political spectrum) the role of party 2 could be essential. Quite a different situation can be observed for a qualified majority, say, 66.67 %. In this case party 2 has no influence on the outcome of voting and the cooperation of parties 1 and 3 is needed for approving any bill.

We can see that it makes sense to look for some measures that express the actual distribution of power among the members of a committee better than the data about proportional representation. In this paper we use two of the most well known measures of voting power – the so-called Shapley-Shubik and Banzhalf-Coleman power indices.

2. Shapley-Shubik and Banzhaf-Coleman Power Indices

Let N = {1, … , n} be the set of members (parties) in a certain committee, and $w_i\,(i=1,\dots,n)$ be the (real, non-negative) weight of the i-th member of the committee (e.g. the number of votes of party i). Let

$$t=\sum_{i=1}^{n} w_i \qquad (1)$$

be the total weight of the committee (total number of deputies). Any vector of weights compatible with total weight t we shall call an allocation. Let q be so called majority quota, a real number such that

$$\frac{t}{2}<q<t. \qquad (1a)$$

Any non-empty subset $S \subset N$ we shall call a *coalition*. Given an allocation w and a majority quota q, we shall say that $S \subset N$ is a winning coalition, if

$$\sum_{i\in S} w_i \geq q \qquad (1b)$$

and a losing coalition if

$$\sum_{i\in S} w_i < q \qquad (1c)$$

(i.e. the coalition S is winning, if it has a required majority, otherwise it is losing). Clearly all allocations $w=(w_1,\dots,w_n)$ in a committee belongs to the simplex

$$W=\{w=(w_1,\dots w_n):\sum_{i\in N} w_i = t, \quad w_i \geq 0\} \qquad . \qquad (1d)$$

Let us denote by P the unit simplex

$$Z=\{z:\sum_{i\in N} z_i = 1, \quad z_i \geq 0 (i=1,\dots,n)\}. \qquad (2)$$

A power index is a mapping

$$\pi : W \to Z \qquad (2a)$$

that represents a reasonable expectation of the share of decisional power among the various players, in relation to their strength in the game, given by the ability to create winning coalitions. We denote by $\pi_i(q,w)$ the share of power that the index π grants to the i-th member of a committee with allocation w and quota q. Such a share is called a power index of the i-th member.

To illustrate the reasoning behind Shapley and Shubik's voting power measure (SS-power index) consider a four-member committee characterized by Table 2.

Table 2. Four-member committee

Parties	Deputies
1	20
2	25
3	38
4	17

The committee is faced with a series of motions or "bills", each of which the members will vote "Yes" or "No". Shapley and Shubik consider the process of building coalitional support for a particular bill. Let us suppose that a simple majority is required to pass the bill (51 votes in our case). The bill may be most enthusiastically supported by, say, party 2, second most enthusiastically by 4, next most by 1, and least by 3. Thus 2 would be the first party to join a coalition in support of the bill, followed by 4. At this point the bill would still lose, and in fact it will be able to win only if the coalition can gain the support of the next most enthusiastic member 1. Gaining 1's support may require considerate modifications of the original bill, so that member 1 has considerable say over the form in which the bill will pass, if it passes. Member 1 has crucial power in this situation.

In an abstract setting, we would not have a priori knowledge about possible orders of coalition formation. Shapley and Shubik hence propose that to measure abstract voting power, we should consider all orders equally likely. For each order, one member will be pivotal in the sense as member 1 above: the losing coalition will win precisely when the pivotal member joins it. The pivotal member holds the power. Hence, as our measure of a member's voting power we use the probability that the member will be pivotal, assuming that all orders of coalition formation are equally likely.

Formally the Shapley-Shubik power index is derived from a model of bargaining, which provides the forming of the whole coalition of all members through equiprobable additions of single members to all possible sub coalitions. It assigns to the i-th member of a committee with allocation w and quota q the share of power

$$\pi_i^{ss}(q,w)=\sum_{s\subset N}\frac{(card[S]-1)!(n-card[S])!}{n!} \qquad (3)$$

where card (S) stands for cardinality of a coalition S and the sum is extended to all winning coalitions S for which the i-th member is essential in sense that the coalition is winning with him and losing without him. This definition is consistent with our intuitive interpretation.

For our four-member committee from table 2 with simple majority rule, there are 4! = 24 possible orders of forming the winning coalitions (see Table 3).

Table 3. Possible orders of forming the winning coalitions

123*4	213*4	31*24	412*3
124*3	214*3	31*42	413*2
13*24	23*14	32*14	421*3
13*42	23*41	32*41	423*1
142*3	241*3	34*12	43*12
143*2	243*1	34*21	43*21

We put an asterisk on the pivotal party in each order. Party 3 is pivotal in 12 of the 24 orders, while each of the other parties is pivotal only in 4 of the orders. The Shapley-Shubik power indices of the members are thus 4 out of 24 for party 1, 4 out of the 24 for party 2, 12 out of the 24 for party 3 and 4 out of the 24 for party 4, so we can write

$$\pi=(\pi_1,\pi_2,\pi_3,\pi_4)=\left(\frac{1}{6},\frac{1}{6},\frac{1}{2},\frac{1}{6}\right).$$

The Banzhaf-Coleman power index (BC power index) follows a slightly different logic. To calculate it we have to write down all the winning coalitions and in each of them to note the "swing" voters (if such exist), those who by changing their vote could change the coalition from winning to losing. For our committee from Table 2 we enumerated all possible coalitions in Table 4. Since in each voting situation the committee splits into two parts: those who vote "yes" and those who vote "no" or abstain, we denote the "yes" coalitions by + and "not yes" coalitions by -. There exist exactly 2^n coalitions, 16 in our case. We denoted the "swing" members in winning coalitions by an asterisk.

Formally, let us denote by $C_i(q,w)$ the set of all winning coalitions in a committee with quota q and allocation w the member *i* swing in. The Banzhalf-Coleman power index assigns to each member the share of voting power proportional to the number of swings:

$$\pi_i^{BC}(q,w) = \frac{card[C_i^-(q,w)]}{\sum_{k\in N} card[C_k^-(q,w)]}. \qquad \text{(3a)}$$

We can see, that party 1 is twice in the position of the "swing" member, party 2 also two times, party 3 six times and party 4 two times. There are exactly 12 possible "swing" in the committee. Supposing that in a large number of voting situations all possible coalitions are equally probable, we can evaluate the power of the members as a ratio of the number of swings the member can make to the total number of possible swings. Thus the BC-power indices of the members are

$$\pi^{BC} = \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{1}{6}\right)$$

(in our example the same as Shapley-Shubik indices)

Table 4. All possible coalitions using BC power index

1	2	3	4		
20	25	38	17	51	
+	+	+	+	100	W
+	+	+*	-	83	W
+*	+*	-	+*	62	W
+	+	-	-	45	L
+	-	+*	+	75	W
+*	-	+*	-	58	W
+	-	-	+	37	L
+	-	-	-	20	L
-	+	+*	+	80	W
-	+*	+*	-	63	W
-	+	-	+	42	L
-	+	-	-	25	L
-	-	+*	+*	55	W
-	-	+	-	38	L
-	-	-	+	17	L
-	-	-	-	0	L
2	2	6	2	12	

To see that BC power index can differ from SS power index, let us consider a committee with a quota $q = 51$ and an allocation $w = (50, 25, 25)$. For this committee

$$\pi^{SS} = \left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right)$$

while

$$\pi^{BC} = \left(\frac{3}{5}, \frac{1}{5}, \frac{1}{5}\right).$$

It can be seen from the simple examples given above that the computation of the SS and BC power indices is a combinatorial problem and outlined "naïve" methods can help to understand the indices, but they are not efficient enough for computations.

3. Political Profile and Power Profile

In a democratic society committees are being elected. Let $i = 1,2,\ldots,n$ be parties taking part in the election. Suppose that the parties are ordered in a single left-right ideological dimension in such a way that they are numbered from left to right. Let us denote by v_i the number of votes for the party i in the election and by s_i the number of seats allocated to the party i after the election.

To characterize a political profile of the community and a power profile of the committee and to compare the situation in different countries, we shall use concepts of density distribution and cumulative distribution (of voters preferences in the society and power in the committee).

The political profile of a community may be characterized by the density function

$$f(i,v) = \frac{v_i}{\sum_{k=1}^{n} v_k} \tag{4}$$

(proportion of votes for the parties) and by two cumulative distribution functions: LR-cumulative distribution function

$$F^{LR}(i,v) = \sum_{k=1}^{i} f(k,v) \tag{4a}$$

(total proportion of votes for all the parties at least as left as i including i), and RL-cumulative distribution function

$$F^{RL}(i,v) = \sum_{k=i}^{n} f(k,v) \tag{4b}$$

(total proportion of votes for all the parties at least as right as i including i). Clearly

$$\sum_{i=1}^{n} f(i,v) = 1, \quad f(i,v) \geq 0 \tag{4c}$$

and

$$0 \leq F^{LR}(i,v) \leq 1, \quad 0 \leq F^{RL}(i,v) \leq 1.$$

Since it is difficult to introduce a cardinal measure of a distance on an ideological line, but it is possible to locate a centre, we shall use a diagrammatic representation of density distribution and cumulative distribution. Let us explain an interpretation of diagrams in a single example.

Let us suppose that 3 parties participated in the election with votes obtained $v_1 = 30, v_2 = 30, v_3 = 15$. Party 1 is of left orientation, party 2 is centristic and party 3 is rightist. In this case density distribution is

$$f(1,v) = 0.4, \quad f(2,v) = 0.4, \quad f(3,v) = 0.2$$

LR cumulative distribution is

$$F(1,v)^{LR} = 0.4, \quad F(2,v)^{LR} = 0.8, \quad F(3,v)^{LR} = 1$$

and RL cumulative distribution is

$$F^{RL}(1,v) = 1, \quad F^{RL}(2,v) = 0.6, \quad F^{RL}(3,v) = 0.2 \quad .$$

A political profile of the community is then given by the diagram in Figure 1.

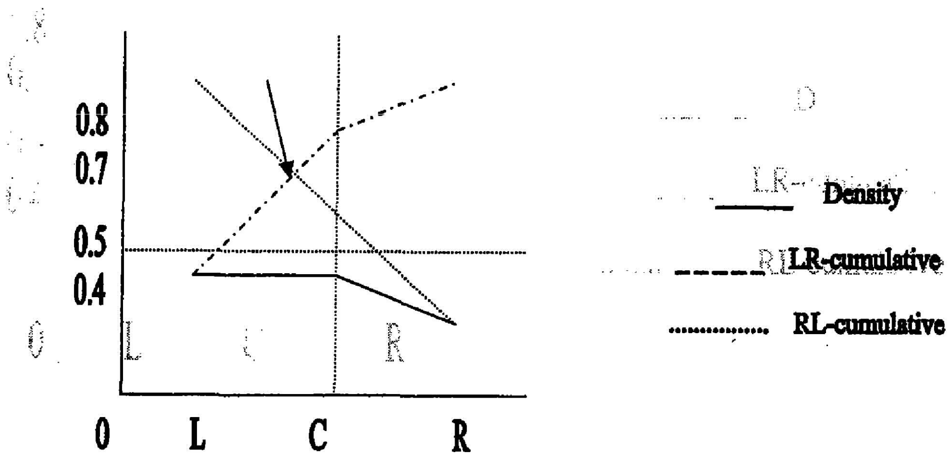

Figure 1. Political profile

The diagram clearly show the left-centristic orientation of the society: the density is declining from left to right and 80 % of voters prefer left and centristic parties, while only 60 % of voters prefer right and centristic parties. The pattern of the political profile is given by the intersection of the LR and RL cumulative distribution curves: if the intersection is left of centre, then we shall speak about the propensity to a left-centristic political profile, if the intersection is right of centre, we shall speak about the propensity to a right-centristic political profile. If the intersection is in the centre, we shall speak about the propensity to a centristic political profile.

After the allocation of seats s_i we can compare a political profile of the community, based on the distribution of votes, with a political profile of parliament, based on the distribution of seats (they may differ due to a "political rounding" and to an eventual threshold).

We have shown before that the distribution of votes may not be identical with the distribution of power. Let us suppose that in our example the representation of the parties in the committee is strictly proportional, e.g. $s_1 = 6, s_2 = 6$ and $s_3 = 3$. The simple majority quota in this case is $q = 8$ and the SS and BC power indices are equal:

$$\pi^{SS} = \pi^{BC} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

Using these indices we can characterize a power profile of the committee by density and the LR, RL cumulative distributions:

$$f(1,\pi) = \frac{1}{3}, \quad f(2,\pi) = \frac{1}{3}, \quad f(3,\pi) = \frac{1}{3}$$

and

$$F^{LR}(1,\pi) = \frac{1}{3}, \quad F^{LR}(2,\pi) = \frac{2}{3}, \quad F^{LR}(3,\pi) = 1$$

$$F^{RL}(1,\pi) = 1, \quad F^{RL}(2,\pi) = \frac{2}{3}, \quad F^{RL}(3,\pi) = \frac{1}{3} \quad .$$

In this case the power is distributed equally on an "ideological interval" among the left, centre and right (see Figure 2).

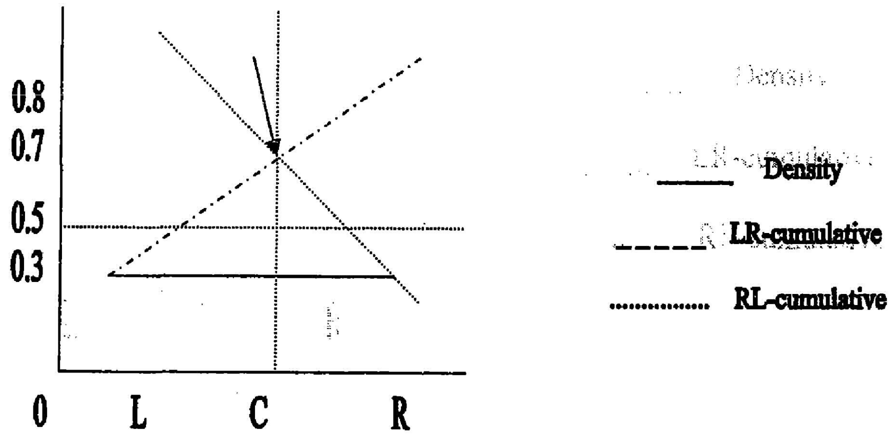

Figure 2. Power Profile for $s_1 = 6, s_2 = 6$ and $s_3 = 3$.

The intersection of LR and RL cumulative distribution curves is in the centre, so the left and centre has the same power as the right and centre and in this sense the centre has a "pivotal" role. Comparing it to the political profile from Figure 1, we can see that the power profile may differ from the political profile, even if the representation of the parties is strictly proportional (the same proportions of seats as votes).

References

Banzhaf, J.F. (1965). Weighted voting doesn't work: a mathematical analysis. *Rutgers Law Review*, **19**: 317-343.
Brams, S. (1975). *Game Theory and Politics*. Free Press, New York.
Coleman, J. (1986). *Individual Interests and Collective Action*. Cambridge University Press, Cambridge.
Gambarelli, G. (1992). Political and Financial Applications of the Power Indices. *Decision Processes in Economics. G. Ricci (ed.)*, 84-106. Springer Verlag, Berlin-Heidelberg.
Holler, M., Packel, E.W. (1983). Power, Luck and the Right Index. *Journal of Economics*, **43**: 21-29.
Shapley, L. S., Shubik, M. (1954). A Method for Evaluating the Distribution of Power in a Committee System. *American Political Science Review*, **48**: 787-792.