UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

ZCT 304/3 – Electricity and Magnetism
[Keelektrikan dan Kemagnetan]

Duration: 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **NINE** printed pages before you begin the examination.
[Sila pastikan bahawa kertas peperiksaan ini mengandungi ***SEMBILAN*** *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer all **FIVE** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

[Arahan: Jawab semua ***LIMA*** *soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

1. (a) Three point charges are situated at the corners of a square (side *a*), as shown in Fig. 1 below. Find the electric field $\vec{\mathbf{E}}$ and the electric potential *V* at the fourth corner.
[Tiga cas titik terletak di pepenjuru-pepenjuru satu segi empat sama (sisi a) seperti yang ditunjukkan dalam Rajah 1 di bawah. Cari medan elektrik $\vec{\mathbf{E}}$ *dan keupayaan elektrik V di pepenjuru yang keempat.]*

(b) How much work does it take to bring in another charge, +q, from far away and place in the fourth corner?
[Berapakah kerja yang diperlukan untuk membawa satu lagi cas, +q, dari jarak yang jauh dan meletakkannya di pepenjuru keempat?]

(c) Calculate the amount of work needed to assemble the whole configuration of four charges.
[Hitung amaun kerja yang diperlukan untuk membina keseluruhan konfigurasi empat cas ini.]

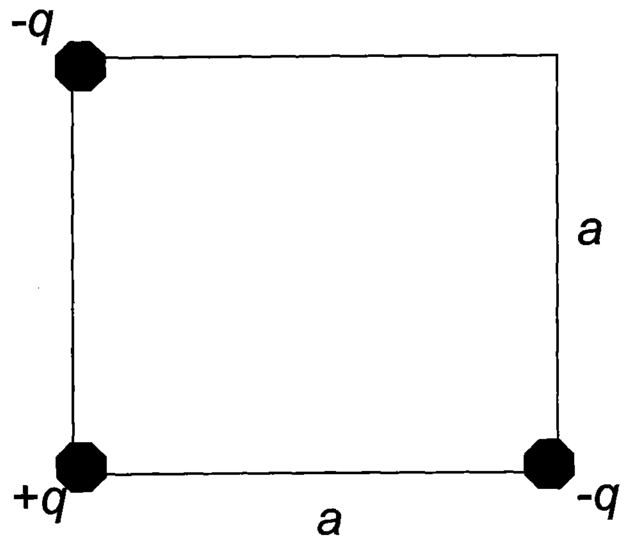

Figure 1 *[Rajah 1]*

(20 marks/*markah*)

2. A parallel plate conductor has been charged such that the lower plate has electric potential $V(x=0)=0$ and the upper plate $V(x=d)=V_0$. The space between the conductor is filled with charges of density $\rho(x)=\alpha x+\rho_0$ (α and ρ_0 are constants).
[Satu plat konduktor selari telah dicas sehingga plat bawah mempunyai keupayaan elektrik $V(x=0)=0$ *dan bagi plat bahagian atas* $V(x=d)=V_0$. *Ruang di antara plat telah diisi dengan cas dengan ketumpatan cas* $\rho(x)=\alpha x+\rho_0$ *(α dan ρ_0 adalah pemalar).]*

(a) Find the electric potential $V(x)$ in the region $0 < x < d$.
[Cari keupayaan elektrik $V(x)$ di kawasan $0 < x < d$.]

(b) Calculate the electric field $\vec{E}$ in this region. Sketch a graph of $|\vec{E}|$ versus x ($0 \le x \le d$).
[Hitung medan elektrik $\vec{E}$ di kawasan ini. Lakarkan graf $|\vec{E}|$ melawan x ($0 \le x \le d$).]

(c) What is the surface charge density on each plate?
[Apakah ketumpatan permukaan cas pada setiap plat?]

Figure 2 *[Rajah 2]*

(20 marks/*markah*)

3. A sphere of radius a carries a uniformly distributed charge Q. It is surrounded, out to radius b, by a linear dielectric material of relative electric permittivity $\varepsilon_r = \kappa$. Please refer to Fig. 3 below.
[Satu sfera berjejari a membawa cas Q yang bertabur seragam. Ia disaluti sehingga ke jejari b, oleh satu bahan dielektrik linear dengan pemalar ketelusan elektrik $\varepsilon_r = \kappa$. Sila rujuk Rajah 3 di bawah.]

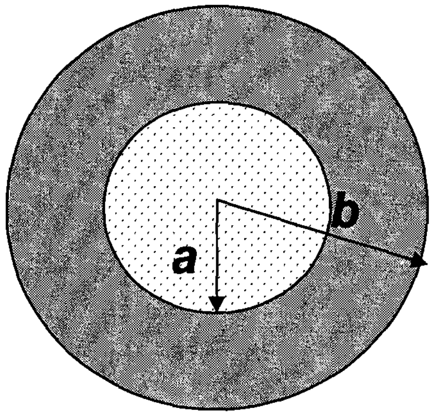

Figure 3 *[Rajah 3]*

(a) Using the Gauss' law find the electric field in the following regions:
[Dengan menggunakan hukum Gauss cari medan elektrik pada kawasan-kawasan yang berikut:]

(i) $r < a$, (ii) $a < r < b$, and *[dan]* (iii) $r > b$

(b) Find the polarization $\vec{\mathbf{P}}$ in the region $a \leq r \leq b$ and then calculate the volume bound charge density ρ_b, dan surface bound charge density σ_b on each surface $r = a$ and $r = b$.
[Cari pengkutuban $\vec{\mathbf{P}}$ pada kawasan $a \leq r \leq b$ dan kemudian hitung ketumpatan isipadu cas terikat ρ_b, ketumpatan permukaan cas terikat σ_b di setiap permukaan $r = a$ and $r = b$.]

(c) Calculate the amount of energy W stored in this configuration of system.
[Hitung amaun tenaga W yang tersimpan di dalam konfigurasi sistem ini.]

(20 marks/*markah*)

4. (a) Two long coaxial solenoids each carries current I, but in opposite directions, as shown in Fig. 4. The inner solenoid (radius a) has n_1 turns per unit length, and the outer one (radius b) has n_2. Find $\vec{\mathbf{B}}$ in each of the three regions:
[Dua solenoid yang panjang dan sepaksi setiap satu membawa arus I, tetapi di arah yang berlawanan, seperti yang ditunjukkan dalam Rajah 4. Solenoid bahagian dalam (jejari a) mempunyai n_1 bilangan lilitan per unit panjang, dan solenoid bahagian luar (jejari b) mempunyai n_2 bilangan lilitan per unit panjang. Dapatkan $\vec{\mathbf{B}}$ di setiap tiga kawasan yang berikut:]

(i) inside the inner solenoid,
[di dalam solenoid bahagian dalam,]

(ii) between them, and
[di antara kedua solenoid, and]

(iii) outside both.
[di bahagian luar keduanya.]

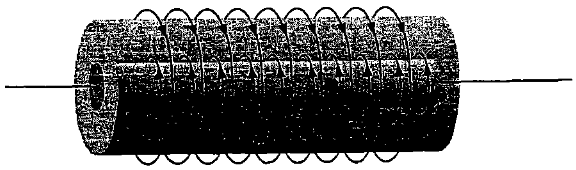

Figure 4 *[Rajah 4]*

(b) If the inner solenoid is filled with a linear insulating magnetic material of magnetic susceptibility χ_m, find magnetization $\vec{\mathbf{M}}$, and the density of bound currents $\vec{\mathbf{K}}_b$ and $\vec{\mathbf{J}}_b$.
[Jika solenoid dalam diisi oleh satu bahan magnet penebat dan linear dengan ketelusan magnet χ_m, dapatkan pemagnetan $\vec{\mathbf{M}}$, dan ketumpatan-ketumpatan arus terikat $\vec{\mathbf{K}}_b$ dan $\vec{\mathbf{J}}_b$.]

(20 marks/*markah*)

5. (a) If an induced elektromotance for a certain circuit is $\varepsilon = -\frac{d}{dt}\Phi_B$ where Φ_B is the magnetic flux over the surface area of the circuit, show that ε obeys the relation $\varepsilon = -\frac{d}{dt}\oint_C \vec{\mathbf{A}} \cdot d\vec{\ell}$. $\vec{\mathbf{A}}$ is the magnetic vector potential.

[Jika aruhan elektromotans bagi suatu litar adalah $\varepsilon = -\frac{d}{dt}\Phi_B$ di mana Φ_B adalah fluks magnet yang melalui/menembusi permukaan litar, tunjukkan bahawa ε juga mematuhi persamaan berikut $\varepsilon = -\frac{d}{dt}\oint_C \vec{\mathbf{A}} \cdot d\vec{\ell}$. $\vec{\mathbf{A}}$ adalah vektor keupayaan magnet.]

(b) Consider two circuits (refer to Fig. 5) consisting of an infinitely long wire carrying a current I_1 and a rectangular coil lying in the same plane as the wire.
[Pertimbangkan dua litar (lihat Rajah 5) yang terdiri dari satu dawai pembawa arus I_1 yang sangat panjang dan satu gelung berbentuk segi empat yang berada pada satah yang sama dengan dawai tadi.]

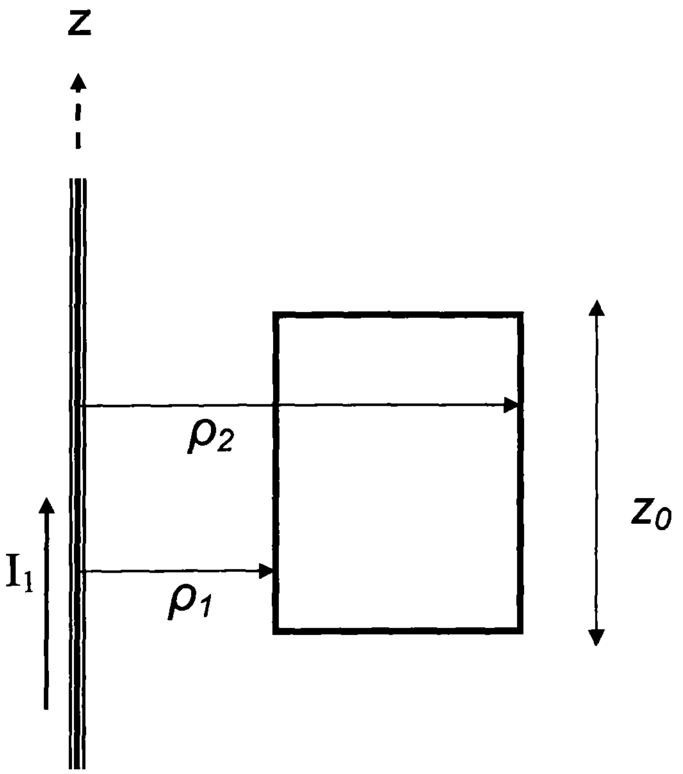

Figure 5 *[Rajah 5]*

(i) Calculate $\vec{\mathbf{B}}$ produced by the wire at a distance ρ from it.
[Hitung $\vec{\mathbf{B}}$ yang dihasilkan oleh dawai pada jarak ρ dari dawai.]

(ii) Calculate $\vec{\mathbf{A}}$ in the region between ρ_1 and ρ_2.
[Hitung $\vec{\mathbf{A}}$ di kawasan antara ρ_1 dan ρ_2.]

(iii) If I_1 varies with time according to $I_1(t) = I_0 \cos(\omega t + \pi)$, calculate the induced electromotance, ϵ, in the rectangular coil. Using Lenz' law, what is the direction of ϵ?
[Jika I_1 berfungsikan masa mengikut $I_1(t) = I_0 \cos(\omega t + \pi)$, hitung elektromotans, ε, yang teraruh pada gelung segi empat tersebut. Dengan menggunakan hukum Lenz, apakah arah ϵ?]

(20 marks/*markah*)

Vector Derivatives

Cartesian Coordinates

$$d\ell = \hat{\mathbf{i}}\,dx + \hat{\mathbf{j}}\,dy + \hat{\mathbf{k}}dz\,, \qquad dV = dx\,dy\,dz$$

$$\nabla f = \hat{\mathbf{i}}\frac{\partial f}{\partial x} + \hat{\mathbf{j}}\frac{\partial f}{\partial y} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \hat{\mathbf{i}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{j}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{k}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates

$$d\ell = \hat{\mathbf{r}}dr + \hat{\phi}r\,d\phi + \hat{\mathbf{k}}dz\,, \qquad dV = r\,dr\,d\phi\,dz$$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial f}{\partial \phi} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{k}}\left[\frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$$d\ell = \hat{\mathbf{r}}dr + \hat{\theta}r\,d\theta + \hat{\phi}r\sin\theta\,d\phi\,, \qquad dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\,A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla\times\mathbf{A} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\,A_\phi) - \frac{\partial A_\theta}{\partial \phi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\phi}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Vector Formulas

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = (\mathbf{C}\times\mathbf{A})\cdot\mathbf{B} = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\,(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}\,(\mathbf{A}\cdot\mathbf{B})$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$$

Derivatives of Sums

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$$\nabla(f+g) = \nabla f + \nabla g$$

$$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$$

$$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$$

Derivatives of Products

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

$$\nabla\cdot(f\mathbf{A}) = f\,(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\times(f\mathbf{A}) = f\,(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,(\nabla\cdot\mathbf{B}) - \mathbf{B}\,(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$$

Second Derivatives

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\nabla f) = 0$$

Integral Theorems

$$\int_V (\nabla\cdot\mathbf{A})\,dV = \oint_S \mathbf{A}\cdot\hat{\mathbf{n}}\,dS$$ Gauss's (divergence) Theorem

$$\int_S (\nabla\times\mathbf{A})\cdot\hat{\mathbf{n}}\,dS = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$$ Stokes's (curl) Theorem

$$\int_{\mathrm{a}}^{\mathrm{b}} (\nabla f)\cdot d\boldsymbol{\ell} = f(\mathrm{b}) - f(\mathrm{a})$$

$$\int_V \left(f\nabla^2 g - g\nabla^2 f\right) dV = \oint_S (f\nabla g - g\nabla f)\cdot\hat{\mathbf{n}}\,dS$$ Green's Theorem

Physical Constants

$c = 2.998 \times 10^8$ m/s	Speed of light
$\mu_0 = 4\pi \times 10^{-7}$ N/A^2 (or H/m)	Permeability constant in vacuum
$\epsilon_0 = \dfrac{1}{\mu_0 c^2} = 8.854 \times 10^{-12}$ C^2/Nm^2 (or F/m)	Permittivity constant in vacuum
$\dfrac{1}{4\pi\epsilon_0} = 10^{-7}c^2 = 8.988 \times 10^9$ Nm^2/C^2	
$e = 1.602 \times 10^{-19}$ C	Magnitude of electron charge
$m_e = 0.9109 \times 10^{-30}$ kg	Electron mass

Useful Integrals

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right)$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Binomial Expansion

$$(1+\epsilon)^p = 1 + p\epsilon + \frac{p(p-1)}{2!}\epsilon^2 + \frac{p(p-1)(p-2)}{3!}\epsilon^3 + \ldots$$

Notation for Position Vector

$$\mathbf{x} = \hat{\mathbf{i}}\,x + \hat{\mathbf{j}}\,y + \hat{\mathbf{k}}\,z$$

$$r = |\mathbf{x}| = \sqrt{x^2+y^2+z^2} \qquad \text{and} \qquad \hat{\mathbf{r}} = \frac{\mathbf{x}}{r}$$

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