UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

ZCT 205/3 – Quantum Mechanics
[Mekanik Kuantum]

Duration: 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **FIVE** printed pages before you begin the examination.
*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer all **SIX** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan**: Jawab semua **ENAM** soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

...2/-

1. (a) Discuss how Planck's theory explains the black body phenomenon.
[Bincangkan bagaimana teori Planck menerangkan fenomena jasad hitam.]

(50/100)

(b) Explain de Broglie's hypothesis of duality nature of particles.
[Jelaskan hipotesis keadaan dualiti zarah de Broglie.]

(50/100)

2. (a) Clarify the postulates of quantum mechanics.
[Jelaskan postulat-postulat mekanik kuantum.]

(50/100)

(b) (i) Define Hermitian operator.
[Berikan definasi operator Hermitian.]

(ii) Prove that two eigen functions for a Hermitian operator is orthogonal if the corresponding eigen values are different.
[Buktikan yang dua fungsi eigen untuk operator Hermitian adalah ortogonal jika nilai-nilai eigen bersepadanan adalah berbeza.]

(30/100)

(c) $\hat{A}$, $\hat{B}$ and $\hat{C}$ are Hermitian operators. Determine whether
[$\hat{A}$, $\hat{B}$ dan $\hat{C}$ adalah operator Hermitian. Tentukan sama ada]

(i) $\left(\hat{A}\hat{B}\hat{C}\right)^{+}$

(ii) $\left(\hat{A}\hat{B}+\hat{B}\hat{A}\right)^{+}$

are hermitian operators
[adalah operator Hermitian]

(20/100)

3. (a) Show that two commutative operators have the same set of eigen functions.
[Tunjukkan bahawa dua operator komutatif mempunyai set fungsi eigen yang sama.]

(30/100)

(b) Show that
[Tunjukkan bahawa]

(i) $\hat{P}_x$ and $\hat{x}$ are not commutative.
[$\hat{P}_x$ dan $\hat{x}$ adalah tak komutatif.]

(ii) $\hat{P}_x$ and $\hat{P}_y$ are commutative.
[$\hat{P}_x$ dan $\hat{P}_y$ adalah komutatif.]

(20/100)

(c) Using the relationship of non-commutative commutator, derive the Heisenberg uncertainty relationship.
[Menggunakan perhubungan komutator tak-komutatif, terbitkan perhubungan ketidakpastian Heisenberg.]

(50/100)

4. (a) Starting by assuming the wave function for a "free particle" along the x direction is in the form of $\psi = Be^{-i(wt-kx)}$,
[Dengan menganggap fungsi gelombang untuk "zarah bebas" sepanjang arah x dalam bentuk $\psi = Be^{-i(wt-kx)}$,]

(i) derive the 3-D time-dependent Schrödinger Equation.
[terbitkan Persamaan Schrödinger 3-D bersandar masa.]

(50/100)

(ii) show that the Hamiltonian operator, $H = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$.

[tunjukkan bahawa operator Hamiltonian $H = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$.]

(40/100)

(b) If the $\psi_1(x,t)$ and $\psi_2(x,t)$ are the solutions for Schrödinger Equation with potential energy of V(x), show that the linear combination $\psi(x,t) = a_1\psi_1(x,t) + a_2\psi_2(x,t)$ is also the solution for Schrödinger Equation with a_1 and a_2 are constants.

[Jika $\psi_1(x,t)$ dan $\psi_2(x,t)$ adalah penyelesaian-penyelesaian bagi Persamaan Schrödinger dengan tenaga keupayaan V(x), tunjukkan bahawa gabungan linear $\psi(x,t) = a_1\psi_1(x,t) + a_2\psi_2(x,t)$ adalah juga penyelesaian bagi Persamaan Schrödinger dengan a_1 dan a_2 adalah pemalar.]

(10/100)

5.

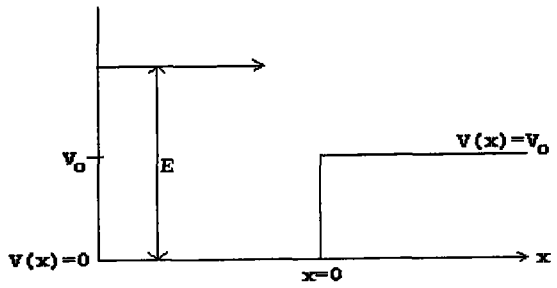

Figure 1 *[Rajah 1]*

(a) Determine the reflection, R and transmission, T factors for the system in Figure 1 when $E > V_0$.

[Tentukan faktor-faktor pantulan, R dan transmisi, T untuk sistem dalam Rajah 1 bila $E > V_o$.]

(50/100)

(b) A particle moves freely in a one-dimensional region of length L and the potential of this system given as

[Suatu zarah bergerak bebas dalam kawasan satu dimensi dengan panjang L dan keupayaan sistem diberikan sebagai]

$V = 0$ for *[untuk]* $0 \le x \le L$

$V = \infty$ otherwise.*[sebaliknya.]*

The time-dependent Schrödinger Equation for this region is

[Persamaan Schrödinger bersandar masa untuk kawasan ini adalah]

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi'(x,t) + V(x)\psi'(x,t) = i\hbar\frac{\partial}{\partial t}\psi'(x,t).$$

Determine the energy and wave function of this system.

[Tentukan tenaga dan fungsi gelombang untuk sistem ini.]

(50/100)

...5/-

6. (a) Discuss three (3) examples of the concept of tunneling effect.
[Bincangkan tiga (3) contoh konsep kesan penerowongan.]

(50/100)

(b) The time-independent Schrödinger Equation for an isotropic 3-D harmonic oscillator is $\left[-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}kr^2\right]\psi(\vec{r})=E\psi(\vec{r})$.

[Persamaan Schrödinger tak bersandar masa untuk suatu ayunan harmonik 3-D isotropik adalah $\left[-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}kr^2\right]\psi(\vec{r})=E\psi(\vec{r})$*.]*

Show that the energy of the system is discrete with the ground state of $3/2\hbar\omega_o$.
[Tunjukkan bahawa tenaga dalam sistem ini adalah diskrit dengan tenaga paras rendah $3/2\hbar\omega_o$*.]*

(50/100)

- ooo O ooo -