FAST STEADY-STATE AND TRANSIENT ANALYSES OF MEMS DEVICES

by

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TABLE OF CONTENTS

CHAPTER ONE : INTRODUCTION

CHAPTER TWO : LITERATURE SURVEY

CHAPTER THREE : ANSYS® SIMULATION OF ELECTRO-THERMAL MICRO-ACTUATOR AND FORMULATION OF FINITE ELEMENT METHOD

CHAPTER FOUR : FORMULATION OF ASYMPTOTIC WAVEFORM EVALUATION (AWE) METHOD

CHAPTER FIVE : RESULTS AND DISCUSSION

CHAPTER SIX : CONCLUSIONS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

SYMBOL	DESCRIPTION	UNIT
$\{ f \}$	Load vector	-
g	Gap between arms of U-shaped micro-actuator	μm
h	Convective heat coefficient	W/m^2K
k	Thermal conductivity	W/mK
k_n	Residues	-
p_n	Poles	-
q	Heat flow	W
t	Time	s
w	Width	μm
wc	Width of cold arm	μm
x, y, z	Cartesian coordinates	-
$\{ y \}$	Displacement solution vector	μm
$[C]$	Capacitance/Damping matrix	-
E	Young ‘s modulus	N/m^2
G	Volumetric heat generation	W/m^3
J	Current density	A/m^2
$[K]$	Conductivity/Stiffness matrix	-
L	Length	μm
Lc	Length of cold arm	μm
Lf	Length of flexure	μm
$Lh1$	Length of first hot arm for double hot-arms micro-actuator	μm
$Lh2$	Length of second hot arm for double hot-arms micro-actuator	μm
$[M]$	Mass matrix	-
M_n	Moments	-
N_i	Shape function	-

R	Resistance	Ω
ROTZ	Rotational displacement about Z-axis	Radian
T	Temperature	^{o}C
ΔT	Temperature increase from ambient temperature	^{o}C
$\{ T \}$	Temperature solution vector	^{o}C
UX	Translational displacement in X-direction	µm
UY	Translational displacement in Y-direction	µm
UZ	Translational displacement in Z-direction	µm
V	Voltage	volt
ZIR	Zero Input Response	-
ZSR	Zero State Response	-
β	Secondary titled angle in cascaded V-shaped micro-actuator	radian
θ	Tilted angle of single V-shaped micro-actuator	radian
ε	Strain	-
σ	Stress	N/m^2
ρ	Density	kg/m^3
ν	Poisson ratio	-
τ	Shear stress	N/m^2
γ	Shear strain	-

LIST OF APPENDICES

LIST OF PUBLICATIONS

ANALISA MANTAP DAN FANA YANG PANTAS BAGI ALATAN MEMS

ABSTRAK

Simulasi unsur terhingga memainkan peranan yang penting dalam pembangunan alatan MEMS (MicroElectroMechanical Systems) dengan membolehkan penilaian sifat pelbagai fizik yang tepat di peringkat awal, tetapi masa pengiraan yang panjang juga diperlukan. Untuk mengatasi kekurangan ini, konsep menggunakan model kerangka diperkenalkan secara khasnya dalam tesis ini untuk membolehkan analisa unsur terhingga mantap yang pantas and tepat bagi ”micro-actuator” elektrik-haba. Model kerangka dapat memberikan keputusan dalam masa pengiraan yang singkat kerana model kerangka mengurangkan darjah kebebasan yang berkenaan ke tahap yang diperlukan sahaja, malah juga mengambil kira sifat bahan yang tidak linear untuk memastikan keputusan yang tepat. Korelasi yang bagus diperolehi apabila dibandingkan dengan penyelesaian dari model unsur terhingga tiga dimensi, di mana perbezaannya adalah kurang daripada 4%, tetapi pengurangan masa pengiraan yang diperolehi adalah lebih dari satu pangkat bagi kelima-lima kes kajian. Berdasarkan model kerangka ini, kajian tentang kesan dimensi geometri terhadap kecekapan output bagi pelbagai “micro-actuator” elektrik-haba juga dijalankan dengan pantas. Selain itu, kaedah Asymptotic Waveform Evaluation (AWE) diperkenalkan secara am untuk menyelesaikan masalah simulasi unsur terhingga fana bagi alatan MEMS dengan pantas and tepat. Berdasarkan konsep penganggaran model unsur terhingga tiga dimensi yang asli dengan model yang melalui pengurangan pangkat, kaedah AWE boleh memberi ketepatan penyelesaian yang sama dalam masa pengiraan yang jauh lebih singkat jika dibandingkan dengan kaedah pengamilan masa konvensional. Dalam tesis ini, AWE berjaya digunakan untuk membina model dengan pengurangan pangkat bagi ”micro-actuator”, ”micro-hotplate” dan juga ”micro-accelerometer”, di mana perbezaan keputusan apabila dibandingkan dengan keputusan ANSYS® adalah kurang daripada 4%, tetapi pengurangan masa pengiraan adalah lebih dari satu pangkat.

Selain itu, juga dipertunjukkan bahawa kaedah AWE berkebolehan untuk mengambil kira pelbagai keadaan sempadan yang rumit, dan dengan itu kaedah AWE boleh digunakan untuk menyelesaikan pelbagai masalah kejuruteraan yang praktikal.

FAST STEADY-STATE AND DYNAMIC ANALYSES OF MEMS DEVICES

ABSTRACT

Finite element simulation plays a crucial role in development of MEMS (MicroElectroMechanical Systems) devices by providing accurate upfront characterization of its multi-physics behaviour, but it also requires substantial amount of computational time. To address this deficit, the concept of using beam model is specifically introduced in this thesis to achieve efficient and accurate steady-state finite element simulation of electro-thermal micro-actuators. Beam model can achieves high computational efficiency by reducing the total degrees of freedoms involved to only those necessary for sufficiently accurate estimate of the bulk mechanical behaviour, while accounting for material non-linearity to ensure solution accuracy. Good correlation is obtained when compared to using three-dimensional finite element model, where the deviation is less than 4% but with more than one order of computational time reduction in all five case studies. Based on this beam model, parametric studies are also efficiently conducted to investigate the effects of geometrical dimensions on the output efficiency of various electro-thermal micro-actuators. In addition, Asymptotic Waveform Evaluation (AWE) method is also introduced in this thesis to efficiently and accurately solve linear dynamics finite element simulation of any MEMS devices in general. Based on the concept of approximating the original three-dimensional finite element model with a reduced order model, AWE method can provide equivalent accuracy as conventional numerical time integration method, but at significantly less amount of computational time. In this thesis, AWE method has been successfully applied to build reduced-order models for a micro-actuator, micro-hotplate and also micro-accelerometer, and it is shown that the achieved computational time reduction is at least one order with less than 4% deviation when compared to ANSYS® solution. Besides that, it is demonstrated that AWE is capable of handling various complex boundary conditions, enabling it to solve various practical engineering problems.

CHAPTER 1
INTRODUCTION

1.0 Overview

In this section, an introduction to the electro-thermal micro-actuators with its steady-state simulation methodology, and also dynamic modeling of MEMS devices, along with the objectives and the outline of the thesis, will be presented. The main contents in this chapter are:

- Introduction to electro-thermal micro-actuators
- Steady-state modeling of electro-thermal micro-actuators
- Dynamic analyses of MEMS devices
- Problem statement
- Project objectives
- Thesis outline

1.1 Introduction to electro-thermal micro-actuators

There are various types of micro-actuators, categorized based on the actuation mechanisms such as piezoelectric, electrostatic, electromagnetic, electro-thermal, etc. Among them, electro-thermal micro-actuator receives substantial attention due to its capability to produce large displacement with high actuation force. Electro-thermal micro-actuator operates by taking advantage of the thermal expansion property of heated material with specially designed structure to produce controlled motion or displacement at specific direction. With such characteristic, it functions to provide actuation mechanism for various micro-devices such as microrelay, microswitch, microgripper, etc.

Figure 1.1 shows a microrelay, which consists of V-shaped electro-thermal micro-actuator, a polysilicon contact head and signal lines with their sidewalls coated with gold. When current is passed through the V-shaped micro-actuator, the thermal expansion caused by resistive heating actuates the in-plane motion of the structure that allows the contact head to move forward and connect the signal lines via sidewall contact. When the current is removed, the microrelay will return to its original position, where the signal lines are disconnected. Another application of electro-thermal micro-actuator is as mechanical microswitch for vibro-tactile display as shown in Figure 1.2. Vibro-tactile displays generate a sensation of motion on the skin by a limited number of mechanical actuators applying light oscillatory pressure via an array of discrete vibrating points. In a hybrid solution, a vibrating plate driven by meso-scale piezo-actuators is used and then the vibrations from this plate are redirected by an array of microswitches to the individual protruding pins (pixels). Each microswitch is made up of a pair of U-shaped electro-thermal micro-actuators to provide locking mechanism on the pixel, and hence in control of turning the pixel on and off as illustrated in Figure 1.2.

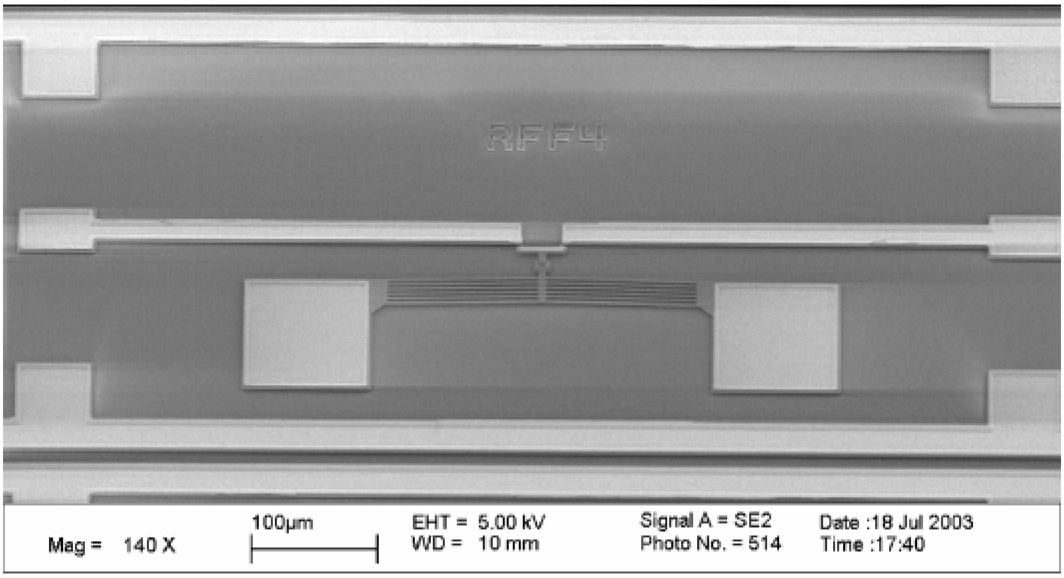

Figure 1.1: SEM image of microrelay which consists of V-shaped electro-thermal micro-actuator (Wang et al., 2004).

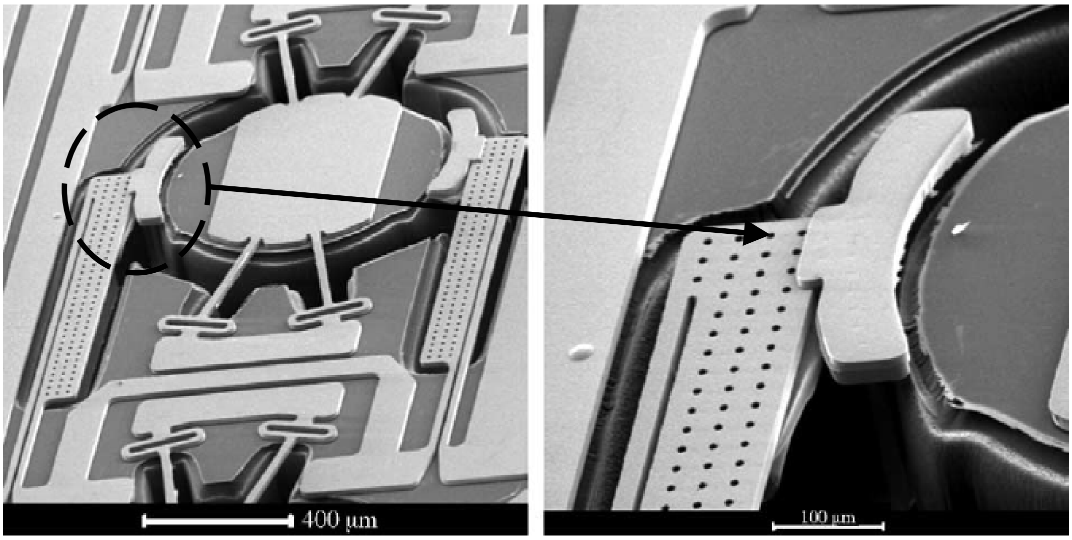

Figure 1.2: SEM image of tactile pixel switch which consists of a pair of U-shaped electro-thermal micro-actuators (Enikov and Lazarov, 2005).

In addition, electro-thermal micro-actuator is also used to provide actuation mechanism for micro-gripper. In Figure 1.3, a microgripper has been designed using two U-shaped electro-thermal micro-actuators with extended prongs. When these two micro-actuators are heated, their structures will bend due to asymmetrical thermal expansion and hence this results in a gripping or holding motion at the end of the prongs. Thin layer of gold is also deposited on top of the hot arm to allow current flow in and out from the hot arm only, which in turn further increases the deflection at given power. Besides that, Figure 1.4 also shows the application of a single V-shaped electro-thermal micro-actuator for optical fibre alignment. Optical fibre alignment tasks are technologically challenging due to the high-accuracy alignment involved. The required positioning accuracy for the fibre tip with respect to the laser diode is approximately ± 0.1 µm, whereas the required actuation stroke is typically in the order of tens of micrometres, and the required force in the milli-Newton range. With such requirements, electro-thermal micro-actuators are highly suitable for performing this task because of its ability to deliver large forces in combination with large actuator displacements. In summary, the applications of electro-thermal micro-actuator are unlimited and it is highly crucial in enabling the operation of various micro-devices.

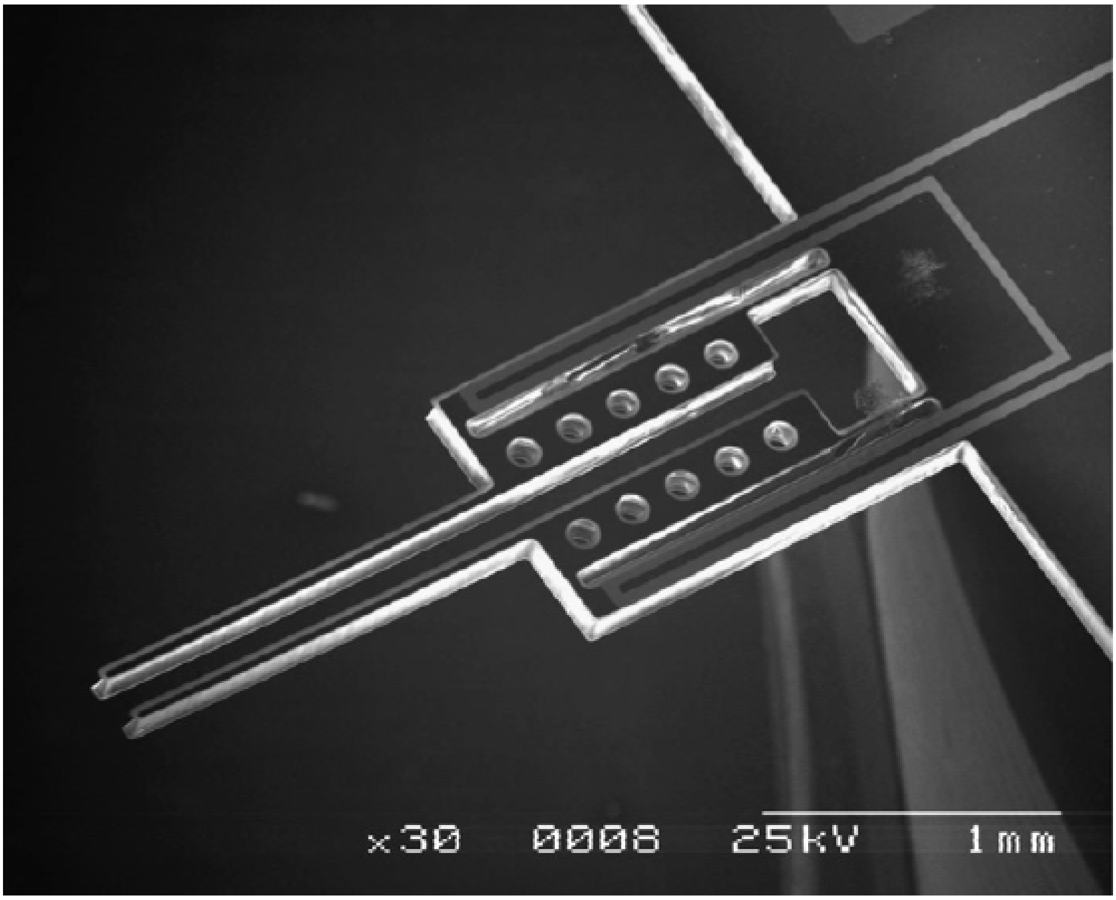

Figure 1.3: SEM image of microgripper which consists of a pair of U-shaped electro-thermal micro-actuators (Solano and Wood, 2007).

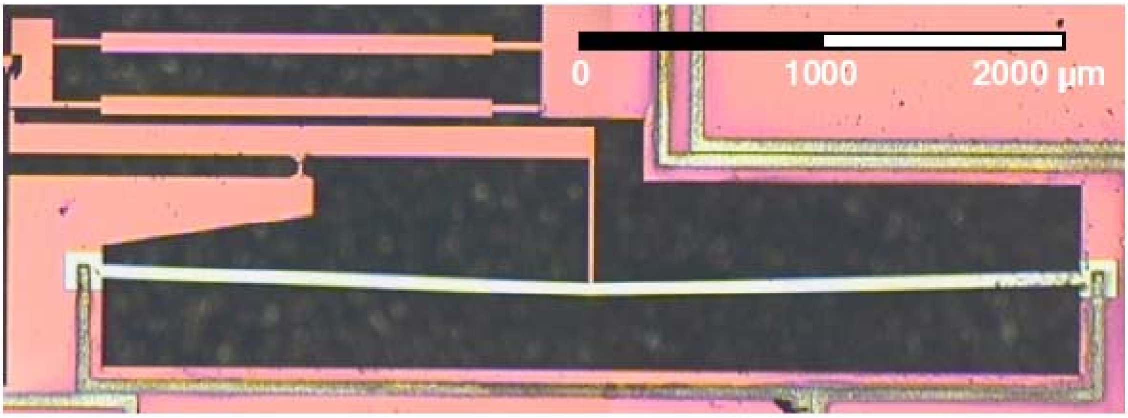

Figure 1.4: Image of a single V-shaped electro-thermal micro-actuator used for optical fibre alignment (Sassen et al., 2008a).

1.2 Steady-state modeling of electro-thermal micro-actuators

The main function of a micro-actuator is to provide controlled motion or displacement, which functions as actuation mechanism. Hence, it is essential to characterize and understand its multi-physics behaviours before using it in real application. Without prior knowledge on the displacement response, the required optimum geometrical size, the required input voltage or power, and also its efficiency in term of displacement per unit power will be unknown. Hence, numerical simulation comes as a handy tool to provide initial assessment and also optimization on electro-thermal micro-actuator prior to prototyping.

The multi-physics behaviours behind the operation of electro-thermal micro-actuator can be accurately simulated using finite element analysis software. To avoid costly characterization through testing and measurement on prototypes, finite element analysis software have been widely employed as efficient and effective tool for evaluating the performance of micro-actuator during the design and development phase. The complex operation of electro-thermal micro-actuator involves the coupled physics of electrical conduction, heat transfer and structural mechanics. The good news is that modern-day finite element software is capable to simulate the interactions among these three physics, and hence providing detailed insight of the current density, temperature distribution as well as the resulted mechanical deformation. Simplified analytical equations have also been derived by some researchers to efficiently model the multi-physics behaviour of various electro-thermal micro-actuators. Since not all material non-linearity is accounted for, the simulated response at high input voltage or power is usually not accurate.

Using finite element analysis software, the current density, current flow and voltage distribution throughout the micro-actuator structure due to voltage difference can be simulated. Then, the calculated current density is used to compute the amount of Joule heating, which acts as volumetric heat generation in thermal analysis. The resulted temperature increase and distribution along the structure of micro-actuator can then be determined. Since electrical resistivity and thermal conductivity properties can be significantly temperature-dependent, a coupled electrical-thermal analysis is needed to ensure equilibrium between these two physics. The calculated temperature distribution is then used in structural analysis to determine the material thermal expansion and also the resulted mechanical deformation of the structure micro-actuator, which is specifically engineered to provide controlled motion or displacement.

In short, finite element analysis is capable of providing detailed and accurate prediction on the multi-physics behaviour of electro-thermal micro-actuator. However, it lacks of the computational speed provided by simplified analytical equation, and hence is unable to provide rapid parametric study and design optimization. In this thesis, this deficit will be addressed by introducing the simulation methodology of using novel beam model to replace typical full-detailed three-dimensional finite element model for efficient and accurate steady-state simulation of various electro-thermal micro-actuators.

1.3 Dynamic analyses of MEMS devices

Microelectromechanical system (MEMS) is also known as micro-machine, where it usually involves multiphysics sensing and actuation principles. Its device size generally ranges from micrometers to a millimeter. Some examples of these devices are micro-accelerometer, micro-hotplate gas sensor, micro-pump and micro-resonator. Experimental characterization on the prototypes of MEMS devices is usually costly and also sometimes may not be feasible due to space constraint, hence leading to widespread use of finite element analysis (FEA) software in designing and optimizing MEMS devices.

FEA is capable of addressing many critical steps in the development of MEMS devices effectively, such as optimizing sensor sensitivity, reducing device size and even improving power consumption. Moreover, the increasing computational power of modern computers enables steady-state analysis involving large degree of freedoms to be computed at reasonable amount of time. However, this may not be the case for dynamic analysis as almost all commercial FEA software, such as ANSYS® and ABAQUS®, employ conventional time integration methods, where the dynamic solution is computed at each increment of small time step. Moreover, increasingly large and complex model may also consequently lead to unreasonable computational time, especially at the pace of shorter development cycle time. This has also become a bottleneck especially for optimization involving dynamic response.

In view of this deficiency in using FEA software for dynamic analyses of MEMS devices, many researches have been carried out focusing on creating compact dynamic model for efficient numerical characterization, as well as for optimization and control of the dynamic response. Commonly, researchers simplify the analysis domain into one-dimensional model and then employ simplified analytical equations to simulate the dynamic characteristics of analyzed MEMS device. For examples, the harmonic response of a micro-accelerometer is derived analytically by using lumped representations for the mass and stiffness, while the analytical solution for transient temperature response of micro-actuator is also derived by using variable substitution method. However, analytical solution may not be easily obtained when the model becomes complex, and the solution accuracy is also sometimes compromised due to the incapability to account for all material non-linearity.

Another popular method is to form a R-C (Resistance-Capacitance) network or lumped model to represent the dynamic behaviour of the analyzed MEMS device. This method has been widely used in the electronic packaging field. As for the application in

MEMS devices, R-C method is successfully used to simulate the transient temperature response of a micro-hotplate. Besides that, lumped dynamical model is also created for electro-thermal micro-actuator by using steady-state finite element analyses to calculate the system averaged electrical conductance, thermal conductance, effective stiffness and effective mass. Both methods above are indirect and require a number of FEA computations to extract the appropriate coefficients for the RC network or lumped model. In recent years, lots of works have also been done on developing reduced-order model for MEMS devices using Krylov-subspace method. It is reported that this method is relatively faster and also accurate as compared to the solution from commercial finite element package ANSYS®.

In this thesis, Asymptotic Waveform Evaluation (AWE) method will be employed to develop the reduced order models for three MEMS devices, namely micro-actuator, micro-hotplate and micro-accelerometer. The reduced-order models will be derived from the original finite element models built in ANSYS®, and then the solutions from these reduced-order models are benchmarked against ANSYS® solutions in term of accuracy and computational time. Since it is generated from the finite element approximation of the original solid model, it is expected that there will be no significant deviation from the original response. Instead, significant computational speed gain is expected since the dynamic solution is not computed at each increment of time step as in ANSYS®.

1.4 Problem statement

Numerical simulation is a critical phase in the development of MEMS devices. It is the most cost-effective mean for upfront evaluation and optimization of new MEMS devices before prototyping. Nowadays, commercial finite element analysis (FEA) tools, such as ANSYS® and ABAQUS®, are commonly employed to numerically simulate the multi-physics behaviours of MEMS devices. Generally, this FEA tools are able to produce reasonably accurate results by accounting for all analysis details, but they lack of the computational efficiency to allow for rapid optimization, particularly for dynamic simulation. Various analytical methods have also been introduced to achieve computational efficiency, but most of them lack of the solution accuracy as provided by FEA tools, mainly due to limitation in accounting for material non-linearity. Hence, the challenge is to increase the computational efficiency of MEMS device simulation without compromising on the solution accuracy. This research will address this challenge by focusing on the development of simulation methodology to achieve efficient and accurate steady-state analysis of electro-thermal micro-actuator specifically, and also to achieve efficient and accurate dynamic analyses of any MEMS devices in general.

1.5 Project objectives

For this project of *"Fast steady-state and dynamic analyses of MEMS devices"*, there are four objectives to be achieved. They are as below:

- To develop a simplified finite element model, which is applicable for efficient and also accurate electrical-thermal and structural analyses of electro-thermal micro-actuators.
- To conduct parametric study to determine the effects of geometrical dimensions on yielding the optimum displacement per unit power for various electro-thermal micro-actuators.
- To develop reduced-order model using the Asymptotic Waveform Evaluation (AWE) method for fast transient thermal characterization of micro-actuator and micro-hotplate.
- To develop reduced-order model using the Asymptotic Waveform Evaluation (AWE) method for fast transient and harmonic structural analyses of micro-accelerometer.

1.6 Thesis outline

This thesis is presented in six chapters, which includes introduction, literature survey, ANSYS® simulation of electro-thermal micro-actuators and formulation of finite element method, formulation of the Asymptotic Waveform Evaluation (AWE) method, results and discussion, and finally conclusions. The First Chapter gives a brief introduction to electro-thermal micro-actuators and also the coupled physics involved in modeling its steady-state behaviour. On top of that, the simulation methodologies used to model the dynamic behaviour of MEMS devices are also discussed. Besides that, the problem statement and also objectives to be achieved in this thesis are also presented in this chapter. In Chapter Two, literature survey is presented on the steady-state modeling of electro-thermal micro-actuators in particular, on the dynamic modeling of MEMS devices in general, and also on the AWE method. It shows the past and current research works in these three areas. The application of a finite element tool to solve coupled electrical-thermal and structural analyses of electro-thermal micro-actuators will be discussed in Chapter Three. It gives detailed description on constructing three-dimensional solid finite element model of electro-thermal micro-actuator, as well as on modeling novel beam model for efficient computation. In addition, the general finite element formulations as well as the conventional numerical time integration methods used for transient thermal analysis and linear structural dynamics analysis are also presented in Chapter Three. Then in Chapter Four, the algorithms of AWE method as an alternative method to efficiently compute these analyses are presented accordingly. Results for Chapter Three and Four are presented and discussed in Chapter Five. These include verification with published numerical and experimental results, accuracy and computational time comparison between solid model and beam model, as well as between ANSYS® and AWE, and also parametric study on the output efficiency of various electro-thermal micro-actuators. Finally, this thesis is concluded in Chapter Six with a summary of work done as well as its findings.

CHAPTER 2
LITERATURE SURVEY

2.0 Overview

Literature reviews on scope of this thesis, which are focusing on fast steady-state and dynamic simulation of MEMS devices, are presented in this section. They are divided into three topics as below:

- Steady-state modeling of electro-thermal micro-actuators
- Dynamic modeling of MEMS devices
- Asymptotic Waveform Evaluation (AWE) method

2.1 Steady-state modeling of electro-thermal micro-actuators

Micro-actuator is a key component to provide actuation of mechanism in a micro-electro-mechanical system. It commonly functions as micro-switch, micro-gripper, micro-motor, micro-mirror and many more. Among different types of actuation mechanism, electro-thermal micro-actuator receives the most attention as it can provide large displacement with relatively high force at common operating voltage of electronics. As such, many efforts were put in to characterize the steady-state behaviour of this device, both numerically and experimentally. The main interest on an electro-thermal micro-actuator is its steady-state displacement response, while electrical and temperature responses are secondary.

From the available literature, numerical characterization can be divided into two categories, which are analytical modeling and finite element analysis (FEA) respectively. In analytical modeling, the closed-loop analytical solution is typically derived from the governing equation over a simplified one-dimensional domain so that efficient prediction of the steady-state behaviour is obtained. Meanwhile, commercial finite element software, such as ANSYS® and ABAQUS®, are commonly employed to obtain accurate simulation result through detailed three-dimensional finite element model.

Que et al. (1999) used FEA approach to simulate the displacement as well as the generated force for single and cascaded bent-beam electro-thermal actuators. From their FEA work, they showed that the increase in displacement achieved by cascaded actuator is quite substantial, about 4 times more than single actuator even without optimization. Experimental measurement of displacement was also conducted, but its comparison to FEA result is not available. Instead of using FEA, Huang and Lee (1999a) pioneered the analytical modeling work to approximate the steady-state response of a simple laterally-driven electro-thermal micro-actuator. They used one-dimensional governing equation to analytically calculate the temperature response as well as the electrical current and voltage behaviour. Then, they used virtual work method to form the analytical equation to approximate the displacement response. Their analytical modeling work produces results that are just within acceptable range from the FEA result.

Huang and Lee (1999b) continued their analytical modeling work on a laterally-driven micro-actuator with non-uniform cross-section, where a portion arm of this micro-actuator has larger width. Besides that, they also took into account the inclusion of linear temperature-dependent electrical resistivity in their analytical work. The predicted electrical current and displacement correlates well with the measurement results at low input voltage. At higher input voltage, the predicted results started to deviate from the measurement data, indicating the effect of temperature on thermal and mechanical material properties becomes more prominent and this material non-linearity has to be taken into account in order to achieve higher accuracy. Nevertheless, Huang and Lee (2000) continued to extend their analytical modeling work on approximating the driving force of this micro-actuator. Their derived equation only managed to predict the driving force within acceptable range from the measured result.

Lott et al. (2001) employed finite element software, ANSYS®, to simulate the steady-state displacement of a thermomechanical in-plane micro-actuator when subjected to constant input current. In addition to using temperature-dependent electrical resistivity, their analysis showed that temperature-dependent thermal conductivity also need to be considered in order to achieve accurate result as compared to experimental measurement. When constant thermal conductivity is used, the simulated displacement is offset from the measurement result. Mankame and Ananthasuresh (2001) also employed the finite element software, ABAQUS®, to study the effect of imposed boundary condition on the yielded output displacement of electro-thermal-compliant micro-actuator. Their numerical study shows that restricting the heat loss by conduction to the substrate via the device anchors can help to yield an average 66% more displacement.

Dong et al. (2003) extended the analytical modeling work by Huang and Lee (1999b) to predict the steady-state behaviour of a two-hot-arm electro-thermal micro-actuator. Due to the complex geometry as well as some approximation on the imposed boundary condition, the predicted result is not very close to the measured data. More accurate prediction to measurement result is achieved when finite element analysis is employed. On the other hand, Wang et al. (2003) also presented on how FEA was employed in designing a RF micro-relay with electro-thermal actuation. FEA achieved reasonable accuracy when simulation parameters were adjusted to account for material properties changes due to thermal heating.

Geisberger et al. (2003) showed that temperature-dependent properties for electrical resistivity, thermal conductivity and thermal expansion have to be considered in order to achieve good accuracy. Their work also showed that finite element result deviates from experimental result if these temperature-dependent properties are neglected. On the other hand, Enikov et al. (2005) presented their analytical modeling work on a single V-shaped thermal micro-actuator, where their analytical solution agrees well with FEA result. Their analytical prediction is also comparable to the experimental measurement, but its accuracy starts to degrade at higher input voltage. This is because their analytical solutions can only accounts for linear temperature-dependent electrical resistivity, with other material properties assumed as constant over temperature range. At higher input voltage, induced temperature is high enough to cause significant changes on the initial material properties, and has to be considered in order to achieve good accuracy.

Continuing the work by Enikov et al. (2005), Zhang et al. (2006) worked on formulating the analytical solution for modeling cascaded V-shaped thermal micro-actuator, which is a combination of three single V-shaped thermal micro-actuators. Using one-dimensional equation to model the heat transfer behaviour, as well as using unit-load method to model the mechanical response, their analytical solution is comparable to the solution produced by three-dimensional FEA. Besides that, Venditti et al. (2006) used FEA software known as FEMLAB® as aid to design a bi-directional in-plane micro-actuator. Since temperature-dependent material properties are not considered, there is significant difference in the simulated steady-state displacement as compared to experimental measurement.

Atre (2007) presented his FEA work on the effect of shape factor and material property variation on modeling the response of thermal micro-actuator. His work shows that using heat transfer coefficient and shape factor to model the heat loss to substrate is sufficiently good. There is no significant difference between this method and modeling the surrounding air using full FEA model. His work also showed that temperature variations of material properties have to be considered in order to achieve close approximation to experimental measurement. His worked also indicated that using constant value assumption for all material properties can indirectly lead to close approximation of the actual micro-actuator's behaviour because they somehow balance out the inaccuracy resulted from each other.

Girbau et al. (2007a) employed FEA software as the analysis tool in designing a RF MEMS switch, which is actuated by a V-shaped thermal micro-actuator. In order to ensure simulation accuracy, the electrical resistivity property used in FEA simulation was derived from current-voltage measurement over the micro-actuator material. With that, the simulated displacement is close to measurement result. Besides that, Girbau et al. (2007b) also presented their analytical work on modeling a low-power-consumption out-of-plane electrothermal micro-actuator, which is used as a driving element for parallel-plate capacitor. Their analytical solution is comparably close to FEA result in term of out-of-plane displacement, but with the assumption that all material properties are constant over the temperature. This assumption was not investigated as the accuracy comparison to the measurement data is not presented.

Sassen et al. (2008b) have employed FEA as upfront evaluation tool in designing an improved V-beam micro-actuator. They have widened the segment at the midspan of the micro-actuator arm. This allows more uniform heating, which leads to more uniform temperature distribution along the arm, and hence yielding more thermal expansion. With this, the improved V-beam micro-actuator has almost 50% more work output as compared to conventional one at the same input power. In addition, Chen et al. (2008) also used FEA to study the effects of geometrical dimensions on the out-of-plane displacement of a single-layer step-bridge thermal micro-actuator. Their simulation approach was to use three-dimensional solid finite element model, with the assumption of constant material properties over the temperature. Direct comparison of simulation and measurement result was not given, but their numerical study on the effects of geometrical dimensions agrees well with experimental observation.

From the available literatures, finite element analysis is commonly used to analyze the steady-state response of electro-thermal micro-actuator. Its prediction is accurate with temperature-dependent material property easily taken into account, but it lacks of computational efficiency. In contrast, analytical solution has the computational efficiency to enable quick parameter study and optimization, but its accuracy is often compromised due to the limitation of only considering temperature-independent material property. Hence, the application of simplified finite element model, termed as beam model, will be introduced in this thesis to achieve both solution accuracy and computational efficiency,

2.2 Dynamic modeling of MEMS devices

Finite element analysis software, such as ANSYS® and ABAQUS®, become common tools used to simulate and optimize the design of MEMS devices upfront. These tools are effective and accurate, but with the bottleneck of requiring large computational time, especially when the model size get larger due to the inclusion of all details. It is worsened when dynamic analysis is conducted because conventional time integration method is normally employed to calculate the transient solution at each increment of small time step. In view of this, various efforts have been put in by researchers to address this issue while maintaining the accuracy of dynamic simulation results. Basically, these efforts can be categorized into three scopes, namely simplified governing equation, electrical analogy network and model order reduction.

In the scope of simplified governing equation, researchers reduce the analysis domain from three-dimension to one-dimension and hence simplifying the process and also computational resource to solve the involved equations. Yu and Lan (2001) successfully used the transfer function for a simple one-dimensional mass-spring-damper structure to capture the frequency response of a three-dimensional micro-accelerometer. The results difference as compared to FEA is less then 0.2%, but significantly faster to compute.

On the other hand, Lott et al. (2002) have used one-dimensional heat transfer equation to model the transient thermal response of a thermomechanical in-plane micro-actuator and solve it using finite difference method. As the number of degree of freedom to be solved is greatly reduced, favorable computational time is attained. Similarly, Hickey et al. (2003) also attempted to model the transient thermal responses of various U-shaped and V-shaped micro-actuators using one-dimensional heat transfer equation. Instead of solving the involved equations, they aimed to make quick approximation to the thermal response time through simplification, by removing the differential term with respect to space.

Researchers also work on using electrical analogy network to model the transient response of MEMS devices. Rencz et al. (2003) used Foster type RC linear network to model the transient temperature responses on the heater and sensor resistors of a MEMS hotplate. Part of the circuit network is pre-calculated and stored, hence further speeding up the total simulation time. Besides that, Borovic et al. (2005) have developed the methodology to model the transient coupled electrical-thermal-mechanical behaviour of a micro-actuator using a state-space control model. Non-linear behaviour due to material non-linearity can also be accounted as the coefficients of this control model are derived from steady-state finite element analysis. In short, electrical analogy method is accurate, but significant efforts are indirectly required to compute the network coefficients.

Nevertheless, model order reduction is the most favourable technique as simplification of analysis domain or governing equations are not involved, nor does the need to use any equivalent electrical network. This technique works on the mathematical basis of reducing the original model into a reduced order model, which has equivalent characteristics but at smaller dimension size. Hence, reduced order model will produce almost equivalent result as original model but at much higher computational speed. Rudnyi and Korvink (2002) pioneered the work of applying model order reduction technique in transient MEMS analysis. They presented an overview of the model order reduction concept and the available methods. They also recommended the use of the Arnoldi algorithm for model reduction of large system as it is easier to implement as well as more computationally stable.

Bechtold et al. (2003) presented the first application of model order reduction technique in MEMS device analysis. They used the Arnoldi algorithm to build a reduced model from the original two-dimensional axisymmetric finite element model of a micro-thruster and compute the transient thermal response. The computed result is comparable to finite element result, with lower error if higher order approximation is used in the reduced model. Later, Bechtold et al. (2004) extended their work to build a reduced order model for a large three-dimensional finite element model of a micro-hotplate with total 73955 nodes. By comparing the solution time, the reduced order model built using the Arnoldi algorithm is about 10 times faster than ANSYS®.

Similarly, Yang and Yu (2004) also adopted Arnoldi algorithm to build reduced order models for a thermal micro-actuator and also an infrared imager cell. However, they built the original three-dimensional models using finite difference method, instead of finite element method. They also showed that the transient result difference is less than 1% and the computational speed gain is more than three orders. Very significant speed gain is obtained likely because the author did not include the initial computational time used to compute the reduced order model when calculating the speed gain.

Bechtold et al. (2005) continued to work on the application of model order reduction on MEMS devices through the Arnoldi algorithm, but this time, they showed that optimization of dynamic response is made possible as the computational time for each dynamic analysis is significantly reduced using model order reduction. Using a micro-hotplate as case study, it was demonstrated that simulated transient temperature result can be matched to experimental result by changing the assumed material properties through optimization. In this case, thermal conductivity and capacitance were adjusted accordingly and close approximation of simulation result to measured result was obtained after 35 cycles of optimization. Without using model order reduction, the total computational time will be enormous and hence it becomes not practical to be carried out.

As for linear structural dynamics analysis, Han et al. (2005) also applied Arnoldi algorithm to compute the reduced order model for a micro-accelerometer. The original model is three-dimensional finite element model, which is computed using ANSYS®. As compared to ANSYS®, reduced order model shows almost equivalent results for both transient and frequency responses. After verification, they proceeded to use the reduced order model for optimizing the output sensitivity. Using ANSYS® for this purpose would be impractical as the total computational time will be enormous.

Tin et al. (2006a) reduced the computational time of running pre-stressed harmonic analysis of a two-port clamped-clamped lateral beam micro-resonator using model order reduction through Arnoldi algorithm. their work showed that the overall frequency response is accurately close to the ANSYS® result, including the peak value. In term of computational time, reduced order model is approximately 30 times faster than ANSYS®. Besides that, Tin et al. (2006b) also extended their work onto the harmonic analysis of a disk-type resonator. Similarly, the computational speed gain is large and the relative error is small, with less than one percent different than the original finite element model result.

Zhang et al. (2008) have presented a generalized procedure to generate reduced order model for MEMS devices using modified block Arnoldi algorithm. To demonstrate the validity of the proposed method on second-order structural dynamics problems, reduced order models were generated for butterfly gyroscope and linear-drive mutilmode resonator, and then used for harmonic and transient analyses. The generated reduced order models produced almost equivalent result as original model, but with significant improvement in computational time. Reduced order model was also generated for a micropyros thruster to show its applicability in first-order heat transfer problem. Similarly, accurate transient temperature response is achieved with significantly less computational time as compared to original model.

From the above literature survey, simplifying the governing equation from three-dimensional to one-dimensional is computationally efficient and most of the time accurate as well. However, this method may not be applicable to problems with complex structure. Electrical analogy method is feasible to model dynamic behaviour of complex model, as well as accounting for material non-linearity. But significant effort is needed to form a suitable network and to obtain its appropriate coefficients. On the other hand, model order reduction is the more favorable method as it is directly derived from the original three-dimensional finite element model. The produced result has equivalent accuracy as original model and with significant computational time improvement. Hence, many research works have been carried out in this area, focusing on using the Arnoldi algorithm. In this thesis, the Asymptotic Waveform

Evaluation (AWE) method will be employed instead to generate reduced order models for various MEMS devices.

2.3 Asymptotic Waveform Evaluation (AWE) method

The inspiration of the Asymptotic Waveform Evaluation (AWE) concept first came from the research of Rubinstein et al. (1983). They were doing the research in the RC-trees network simulation by using the efficient Elmore delay (first moment of the impulse response) estimate approach. However, this estimate approach does not always produce an accurate result because there are a lot of limitations in doing a transient analysis. So, the major effort of the early work was to find a solution scheme for transient analysis.

The work from McCormick (1989) gave a second spark to this concept. From his research in interconnect circuit simulation, he has shown that the circuit moments (the coefficient of expansion of circuit driving point of transfer function in a Maclaurin series about s=0 in the frequency domain) could lead to lower circuit models and reasonably accurate transient response results. The works of these researchers led to the formalization and generalization of the algorithm, where eventually n^{th} order extension of the first order Elmore delay approximation was developed and named as AWE.

Pillage and Rohrer (1990) then have demonstrated the capability of AWE in transient analysis, where it was used to capture the delay effect of interconnect using a simplified RC tree model. Reasonable results were produced as compared with SPICE simulation, but AWE is two or three orders faster than SPICE. From their work also, AWE is shown to be applicable with or in floating node, linear controlled sources, finite input rise time, charge sharing, bipolar circuitry, interconnect timing estimation and MOS circuit analysis.

On invitation, a tutorial paper on the literature of AWE was published (Raghavan et al., 1993). This invited paper gives an overview of the algorithm of AWE, starting with the definition of moments. This paper gives the details on how the time domain moments of a signal, f(t), are related to the Taylor series coefficients about s=0 (Maclaurin series) of the signal `s Laplace transform, F(s). It also elaborates that the algorithm of AWE can be divided into two main parts, namely moment computation and moment matching. The formulation given was for equation in the form of first order linear differential equation, which is equivalent to that derived from a RC model.

In the same year, AWE is also used to compute the time response of an arbitrary 3-D interconnect structure (Kumashiro et al., 1993). It has been implemented in software called 3DAWE. To facilitate the application of AWE to a 3D RC mesh network model, AWE formulation was re-derived based upon a nodal analysis approach. Using this software, a typical transient response of a reasonably large 3-D RC network could be obtained within a few minutes on a 15 MIPS computer. On the following year, Ratzlaff and Pillage (1994) developed Rapid Interconnect Circuit Evaluator (RICE) to specifically analyze RC and RLC interconnect circuit. RICE focuses on the passive interconnect problems by applying AWE to yield large gain in run-time efficiency over conventional circuit simulation without sacrificing accuracy. The application of RICE was successfully demonstrated on several VLSI interconnect and off-chip microstrip models.

Slone et al. (2001) proposed to couple Galerkin method and AWE to solve FEM equations in electromagnetic radiation analysis. The proposed method shows good agreement with LU decomposition method. Condon and Brennan (2004) also used AWE method to obtain efficient solution for electromagnetic scattering problems over a wide frequency band. Their proposed technique involves extracting high-frequency phase information from the surface current to generate a residual problem, which is solved over the frequency band using AWE method. The phase information is then reinserted to get good estimates for the surface current at all frequencies along the band.

Liu et al. (2006) applied the AWE method to conventional eigenmode expansion method for characterizing a power/ground plane pair and analyzing the simultaneous switching noise on this plane pair in printed circuit board. AWE was applied to avoid large number of iterations in computing the impedance frequency response of a power/ground plane pair structure. Hence, it reduces the computational time without compromising on solution accuracy. Peng and Sheng (2008) proposed an efficient and flexible bandwidth estimation approach based on the AWE method for scattering problems. This approach is validated against finite element result for frequency and angle sweep of radar cross section (RCS) for a conducting sphere, coated sphere and a conducting brick with deep cavity. Good accuracy was achieved with significant improvement on computational speed.

The application of AWE in mechanical engineering discipline was first published by Da et al. (1995), where transient thermal analysis of a printed circuit board was solved using AWE scheme. Using electric-thermal network analogy method, the simplified governing equations for the thermal behaviour of printed circuit board was reduced into a set of linear differential equations. This set of equations was solved with AWE and then the solution was compared with HSPICE solution. They have shown that the application of AWE to solve transient thermal analysis of printed circuit boards often resulted in two order speed-up, yet retaining comparable accuracy. However, the response formulation, as well as the poles and residues used to predict the transient temperature response for the first order ordinary equation seem to be incorrect and there is no detail to describe the incorporation of boundary conditions.

Then, Ooi et al. (2003) have presented a general formulation of AWE for solving first and second order linear differential equations, which are typically encountered in transient thermal and vibration analyses respectively. The incorporation of boundary conditions has also been clearly described by using the concept of Zero State Response (ZSR) and Zero Input Response (ZIR), which is typically used in control system. In their work, the finite element method (FEM) has been used to formulate the transient temperature behaviour of a simple one-dimensional fin, as well as a simplified micro-channel heat exchanger system. Then, AWE was applied to efficiently and accurately obtain the transient temperature response. Similarly, they have also used AWE to solve the vibration response of a simple one-dimensional cantilever beam.

From the available literatures, the application of AWE in solving mechanical problems is still at development stage, where so far most of the research works and applications are conducted on electrical simulations. Ooi et al. (2003) have successfully coupled finite element method with AWE to solve transient thermal and vibration problems. However, the demonstrated problems are just simple one-dimensional problems and are not representative of practical engineering application. The size of equations involved is also minimal, and hence the actual computational efficiency of AWE is not correctly quantified. The accuracy of AWE against commercial finite element software is also not numerically studied, and the feasibility of AWE to account for various boundary conditions that are encountered in practical engineering problems is also not investigated. All the above will be addressed by the novel work in this thesis to verify the AWE capability in solving practical mechanical engineering problems.

2.4 Summary

From the available literature survey, it is apparent that three-dimensional finite element analysis (FEA) is commonly employed to simulate the steady-state and dynamic behaviours of MEMS devices. While this approach often produces accurate