Mathematical Thinking Assessment (MaTA) Framework: A Complete Guide

Hwa Tee Yong
Universiti Sains Malaysia
P-PD0023/06 (R)

Supervisor
Associate Professor Dr. Lim Chap Sam

CONTENT

Section 1

Mathematical Thinking Assessment (MaTA) Framework

Introduction

Mathematical thinking is important particularly in the process of acquiring mathematical concepts and skills. However, teachers in schools are not aware of the importance of thinking in mathematics and hence do not emphasize it in the development of students‟ intellectual growth (Ministry of Education Malaysia, 1993). Thus, many students fail to engage thinking skills in solving complex real life problems. In the words of Von Glaserfeld (1995):

> "[Educators] have noticed that many students were quite able to learn the necessary formula and apply them to the limited range of textbook and test situation, but when faced with novel problem, they fell short and showed that they were far from having understood the relevant concepts and conceptual relations." (p. 20)

One of the causes of this phenomenon is the assessment format. The current standardized tests format does not require students to demonstrate their thinking during problem solving processes; instead they encourage students to regurgitate facts that have been memorized. As commented by Nickerson (1989), standardized tests inclined towards giving emphasis to recall content knowledge, and hence provide little indication about students‟ level of understanding or quality of thinking. For this reason, students do not practice any act of cognition during the assessment since they only memorize what is imparted to them by their teachers. On top of this, "students are bombarded with exercises, which function only to give them training on the rules or procedures that they have just learnt. They give students no training in calling to mind possible strategies for a solution and discriminating between them." (Lau et al, 2003, p. 3).

Beyer (1984b) claimed that most of the tests on thinking skills suffer from two flaws: conceptual inadequacy and inadequate definition of the components of the skills that are tested. He commented that most of the tests "measure discrete skills in isolation, ignoring, by large, students‟ ability to engage in a sequences of cognitive operation." and in many circumstances, "items on tests of thinking skills bear no relation to the skills these tests suppose to evaluate" (p. 490). Therefore, an effective assessment framework is needed to promote students‟ mastery of mathematical thinking through the classroom learning. Without appropriate assessment and grading system in assessing mathematical thinking, we cannot know how effective and efficient a teacher is at teaching mathematical thinking or how competent a student is at mathematical thinking. Nevertheless, we also do not know what needs to be attended to in order to promote the teaching and learning of mathematical thinking in the classroom.

Definition of Mathematical Thinking

What is mathematical thinking? According to Lutfiyya (1998) and Cai, (2002) there is yet to find a well defined meaning or explanation of mathematical thinking. To make the situation worse, the educators from different countries seem to define differently the meaning of mathematical thinking with respect to their mathematics curricula. Hence, a well define meaning of mathematical thinking should be established first before any study or research related to mathematical thinking can be conducted.

The word "mathematical thinking" is not used or stated explicitly in the Malaysian primary and secondary levels mathematics curriculum. However, a related phrase, "to think mathematically" was used in the write-up of the main aim of secondary school mathematics curriculum:

> "The Mathematics curriculum for secondary school aims to develop individuals who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decision." (Ministry of Education Malaysia, 2005, p.2)

The above statement denotes that mathematical thinking should be promoted in the Malaysian mathematics classroom if we are to produce future students who can think mathematically. Nonetheless, a closer analysis of the intended aim of secondary school mathematics curriculum shows that there are three components which constitute to the construction of mathematical thinking framework: content knowledge (mathematical knowledge), attitudes or disposition (effectively and responsibly) and mental operations (problem solving and decision making). These three components are found able to fit and incorporate into both the primary and the secondary school mathematics curriculum documents as display in Table 1.

Table 1:
Comparison of Mathematics Objectives between Primary School Curriculum and Secondary School Curriculum

<table>
<tr><th rowspan="2">Component</th><th rowspan="2">Primary School Mathematics Curriculum (MOE, 2003)</th><th colspan="2">Secondary School</th></tr>
<tr><th>Mathematics Curriculum (MOE, 2005)</th><th>Additional Mathematics Curriculum (MOE, 2004)</th></tr>
<tr><td rowspan="4">Mathematical Content Knowledge</td><td>Objective 1:
know and understand the concepts, definition, rules and principles related to numbers, operations, space, measures and data representation</td><td>Objective 1:
understand definition, concepts, laws, principles and theorem related to Number. Shape and Space, and Relationships</td><td rowspan="4">Objective 1:
widen their ability in the field of numbers, shapes and relationships as well as to gain knowledge in calculus, vector and linear programming</td></tr>
<tr><td>Objective 2:
master the basic operations of mathematics: addition; subtraction; multiplication; division</td><td rowspan="2">Objective 2:
widen application of basic fundamental skills such as addition, subtraction, multiplication and division related to Number. Shape and Space, and Relationships</td></tr>
<tr><td>Objective 3:
master the skills of combined operations</td></tr>
<tr><td>Objective 4:
master basic mathematical skills, namely: making estimates and approximates;
measuring; handling data; representing information in the form of graphs and
charts</td><td>Objective 3:
acquire basic mathematical skills such as: making estimation and rounding; measuring and constructing; collecting and handling data; representing and interpreting data; recognizing and representing relationship mathematically; using algorithm and relationship; solving
problem; and making decision.</td></tr>
<tr><td rowspan="7">Mental Operations</td><td>Objective 6:
use the language of mathematics correctly</td><td>Objective 4:
communicate mathematically</td><td>Objective 7:
debate solutions in accurate language of mathematics</td></tr>
<tr><td rowspan="6">Objective 8:
apply the knowledge of mathematics
systematically, heuristically, accurately
and carefully</td><td rowspan="4">Objective 5:
apply knowledge and the skills of mathematics in solving problems and making decisions</td><td>Objective 2:
enhance problem solving skills</td></tr>
<tr><td>Objective 4:
make inference and reasonable generalization from given information</td></tr>
<tr><td>Objective 3:
develop the ability to think critically, creatively and to reason out logically</td></tr>
<tr><td>Objective 6:
use the knowledge and skills of mathematics to interpret and solve real-life problems</td></tr>
<tr><td rowspan="2">Objective 6:
relate mathematics with other areas of knowledge</td><td>Objective 5:
relate the learning of mathematics to daily activities and careers</td></tr>
<tr><td>Objective 8:
Relate mathematical ideas to the needs and activities of human beings</td></tr>
<tr><td rowspan="3">Mathematical Disposition</td><td>Objective 5:
use mathematical skills and knowledge to solve problems in everyday life effectively
and responsibly.</td><td>Objective 8:
cultivate mathematical knowledge and skills effectively and responsibly</td><td rowspan="3">Objective 10:
Practice intrinsic mathematical values</td></tr>
<tr><td>Objective 9:
Participate in activities related to mathematics</td><td>Objective 9:
Inculcate positive attitudes towards mathematics</td></tr>
<tr><td>Objective 10:
appreciate the importance and beauty of
mathematics</td><td>Objective 10:
appreciate the importance and beauty of
mathematics</td></tr>
</table>

- One objective related to the use of ICT in mathematics is excluded from each curriculum document.
- MOE – Ministry of Education Malaysia

Table 1 shows that all the three components of mathematical thinking are implicitly incorporated in both levels of Malaysian school mathematics curricula. For the primary mathematics curriculum, there is a higher emphasis on basic mathematical skills as compared to the problem solving skills and appreciation of mathematical values. In comparison, the emphasis is more on complex mathematical skills such as problem solving, decisions making, communication and extension of mathematical abstraction as well as positive attitudes toward mathematics rather than the basic mathematical skills for the secondary mathematics curriculum (Lim & Hwa, 2006). Further investigation shows that secondary additional mathematics curriculum places greatest emphasis on complex mental skills rather than basic mathematical skills and disposition toward mathematics, where seven out of ten objectives of the curriculum fall into this component.

Since mathematical thinking is ill defined (Lutfiyya, 1998, Cai, 2002) and no detailed description of the words "mathematical thinking" in most of the national mathematics curriculum documents (Isoda, 2006), different perspectives on mathematical thinking have evoked. For examples, Katagiri (2004) defined mathematical thinking as the ability to think and to make judgments independently while solving mathematics problems. As for Mason, Burton and Stacey (1982), they defined mathematical thinking as a dynamic process enabling one to increase the complexity of ideas he or she can handle, and consequently expands his or her understanding. Alternatively, Schoenfeld (1992) proposed that there are five important aspects of cognition involved in the inquiries of mathematical thinking and problem solving, namely (a) the knowledge base; (b) problem solving strategies; (c) monitoring and control; (d) beliefs and affects; and (e) practices (p.348). And most recently, Wood, Williams and McNeal (2006) defined mathematical thinking as the mental activity involved in the abstraction and generalization of mathematical ideas.

Although all the above descriptions were not totally similar, they seem to highlight three major domains of mathematical thinking: (a) mathematical knowledge; (b) mental operations; and (c) mathematical dispositions. Mathematical knowledge refers to mathematical concepts and ideas that one has acquired or learnt, while mental operations can be illustrated as cognitive activities that the mind needs to perform when thinking (Beyer, 1988). As for mathematical dispositions, it refers to the tendency or predilection to think in certain ways under certain circumstances (Siegel, 1999). Examples of mathematical dispositions include reasonableness, thinking alertness and open-mindedness, as well as beliefs and affects.

In view of the above discussion, mathematical thinking should include the following characteristics:

- it involves the manipulation of mental skills and strategies
- it is highly influenced by the tendencies, beliefs or attitudes of a thinker
- it shows the awareness and control of one"s thinking such as metacognition
- it is a knowledge–dependent activities (Lim & Hwa, 2006)

Base on these characteristics, this study defined mathematical thinking as mental operations which are supported by mathematical knowledge and certain kind of dispositions toward the attainment of solution to mathematics problem.

The conceptual of mathematical thinking in the study is supported by Concept of Thinking (Beyer, 1988), Dimensions of Thinking (Marzano et al, 1988) and Critical and Creative Thinking - KBKK (Ministry of Education Malaysia, 1993). As defined in this study, Mathematical Thinking Model comprises of three components, namely mathematical knowledge, mental operations and mathematical dispositions. The interrelationships among these components are shown in Figure 1.

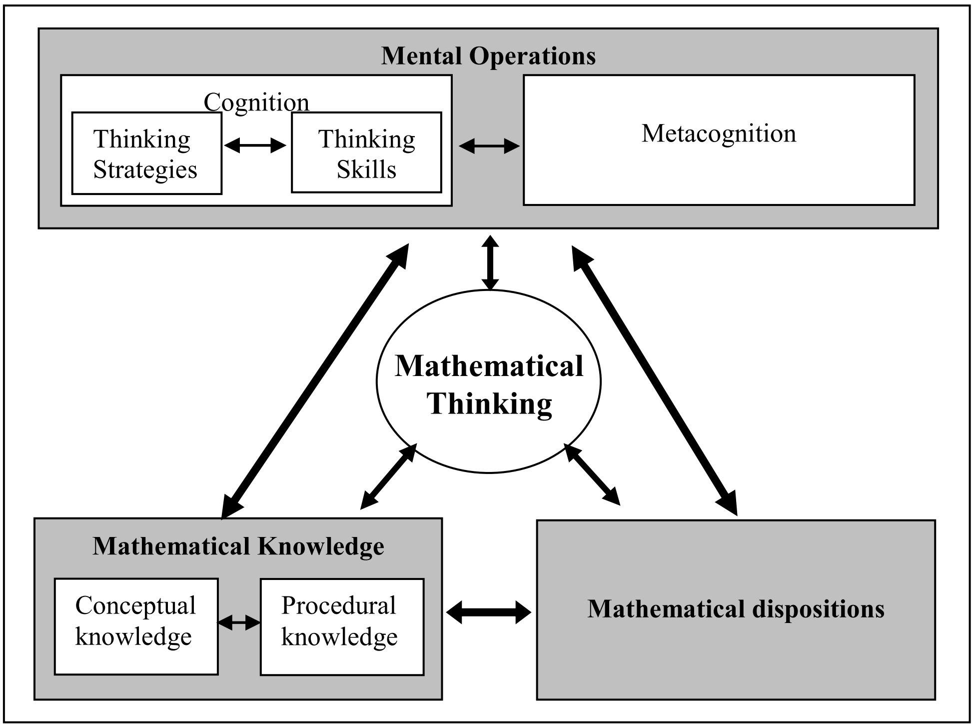


Figure 1: Mathematical Thinking Model

Mathematical Knowledge

According to Schoenfeld (1992), mathematical knowledge refers to a set of mathematical concepts and procedures that can be used to execute the solution to a problem reliably and correctly. It is difficult to distinguish between conceptual knowledge and procedural knowledge; however, understanding the differences of these two types of knowledge will provide significant insights into mathematics learning (Hiebert & Lefevre, 1986). Conceptual knowledge, as defined by Hiebert and Lefevre (1986), is the linking relationship which connects all the discrete existing bits of information, whereas procedural knowledge is composed of using formal mathematics language or symbol representation to carry out an algorithm while attempting to complete a mathematical task.

Mental Operations

Cognition is usually synonymous to mental activities and it involves a series of processes by which knowledge is acquired and manipulated (Bjorklund, 1989). Beyer (1988) pointed that these mental activities can be illustrated in terms of operations that the mind seems to perform when thinking exists. There are two general types: cognition and metacognition. Cognition engages a variety of complex strategies in an overall plan, such as problem solving or decision making, to produce a thinking product. Another aspect of cognitive operation involves more discrete processing skills, such as organizing, analyzing, generating as well conjunction with other similar operations to guide and execute a thinking strategy (Perkins, 1986; Beyer, 1988).

Metacognition, as defined by Flavell (1976), is "one"s knowledge concerning one"s own cognitive processes and products or anything related to them… Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of theses processes." (p. 232). Beyer (1988) commented that metacognition "consists of those operations by which we direct and control these meaning making strategies and skills… Any act of thinking involves a combination of operations designed to produce meaning (cognitive operations) and to direct how that meaning is produced (metacognitive operations)" (p. 47). He further claimed that metacognition is also associated closely to the knowledge, cognitive operation and dispositions that make up to the thinking activities.

Mathematical Dispositions

A thinking disposition is a tendency or predilection to think in certain ways under certain circumstances (Siegel, 1999). According to Perkins, Jay and Tishman (1993), it comprises of three central elements: abilities, sensitivities and inclination**.** They postulate that abilities refer to the capabilities and skills required to carry through on the behavior, whereas sensitivities refer to being alert for appropriate occasions for modeling the behavior. Finally, inclination deals with the tendency to actually behave in a certain way. These arguments seem similar to how NCTM (1989) defines mathematical dispositions: "mathematical dispositions are manifested in the way they approach tasks--whether with confidence, willingness to explore alternatives, perseverance, and interest--and in their tendency to reflect on their own thinking." (p. 87)

Each domain of mathematical thinking is interrelated and complements one another (Figure 2). For this reason, any effective mathematical thinking act will involve the orchestration of components in these three domains. Acquisition of mathematical knowledge is the basis to engage in mathematical thinking. Understanding of subject matter will support and guide one to choose the appropriate cognitive skills and strategies according to the problem situation. However, the acquisition of knowledge requires one to explore, inquire, seek clarity, take intellectual risks, and think critically and imaginatively (Tishman, Jay, & Perkins, 1993). Hence, the right attitudes or dispositions toward attainment of mathematical knowledge are very important and serve as the ground force to execute cognitive skills and strategies in mathematics problem–solving. Schoenfeld (1992) argued that "core knowledge, problem solving strategies, effective use of one"s resources, having a mathematical perspective, and engagement in mathematical practices – are fundamental aspects of thinking mathematically." (p. 335). Hence, to become a successful and effective mathematical thinker, one needs to possess and internalize all these three domains: mathematical knowledge, cognitive skills cum strategies and thinking dispositions.

The component of Mathematical Thinking Assessment (MaTA) Framework

Thus far, there is yet to find an assessment framework that could be used by school teachers to assess students" thinking in our Mathematics Curriculum. Hence, the framework of Mathematical Thinking Assessment (MaTA) is established with the aims to assess students" mathematical thinking effectively and reliably. This framework consists of four components:

(a) performance assessment, (b) Metacognition Rating Scale, (c) Mathematical Dispositions Rating Scale, and (d) Mathematical Thinking Scoring Rubric. The MaTA will be implemented by teachers in the school context to assess students" mathematical thinking: the performance assessment will be administered to elicit students" thinking process (conceptual knowledge, procedural knowledge, thinking strategies and thinking skills) while solving the mathematical problem; the Metacognition Rating Scale will be used to specify students" awareness, such as monitoring and reflection, during the problem solving process; the Mathematical Dispositions Rating Scale will be used to indicate students" predisposition toward learning of mathematics; whereas the Mathematical Thinking Scoring Rubric will be used to score and grade students" mathematical thinking according to the domains defined in this study. The conceptual framework of MaTA is illustrated in Figure 2, whereas Figure 3 shows the summary of how this framework could be implemented in the school context. The detailed descriptions of each component of MaTA are presented at the following chapters.

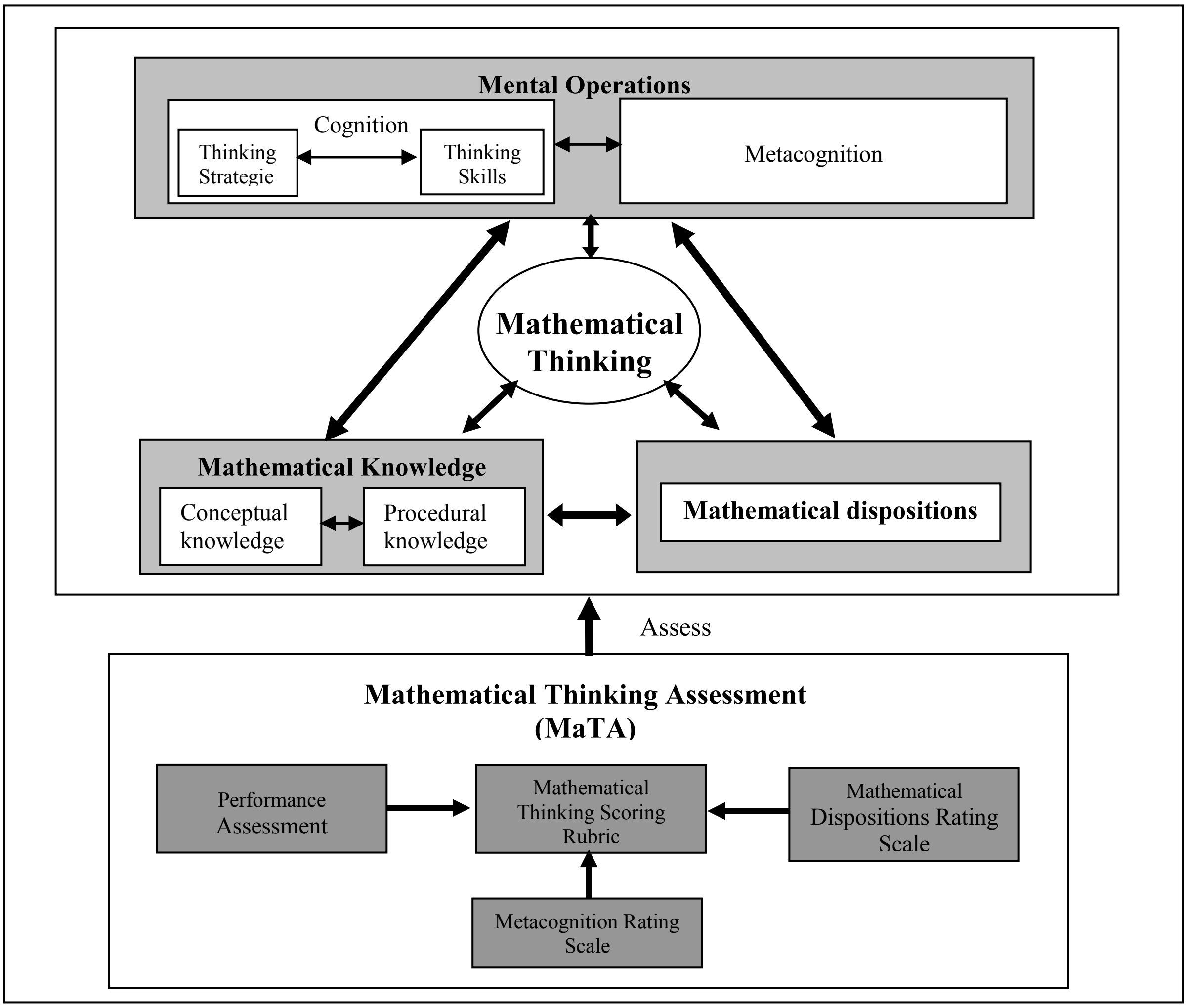


Figure 2: Conceptual Framework of Mathematical Thinking Assessment (MaTA)

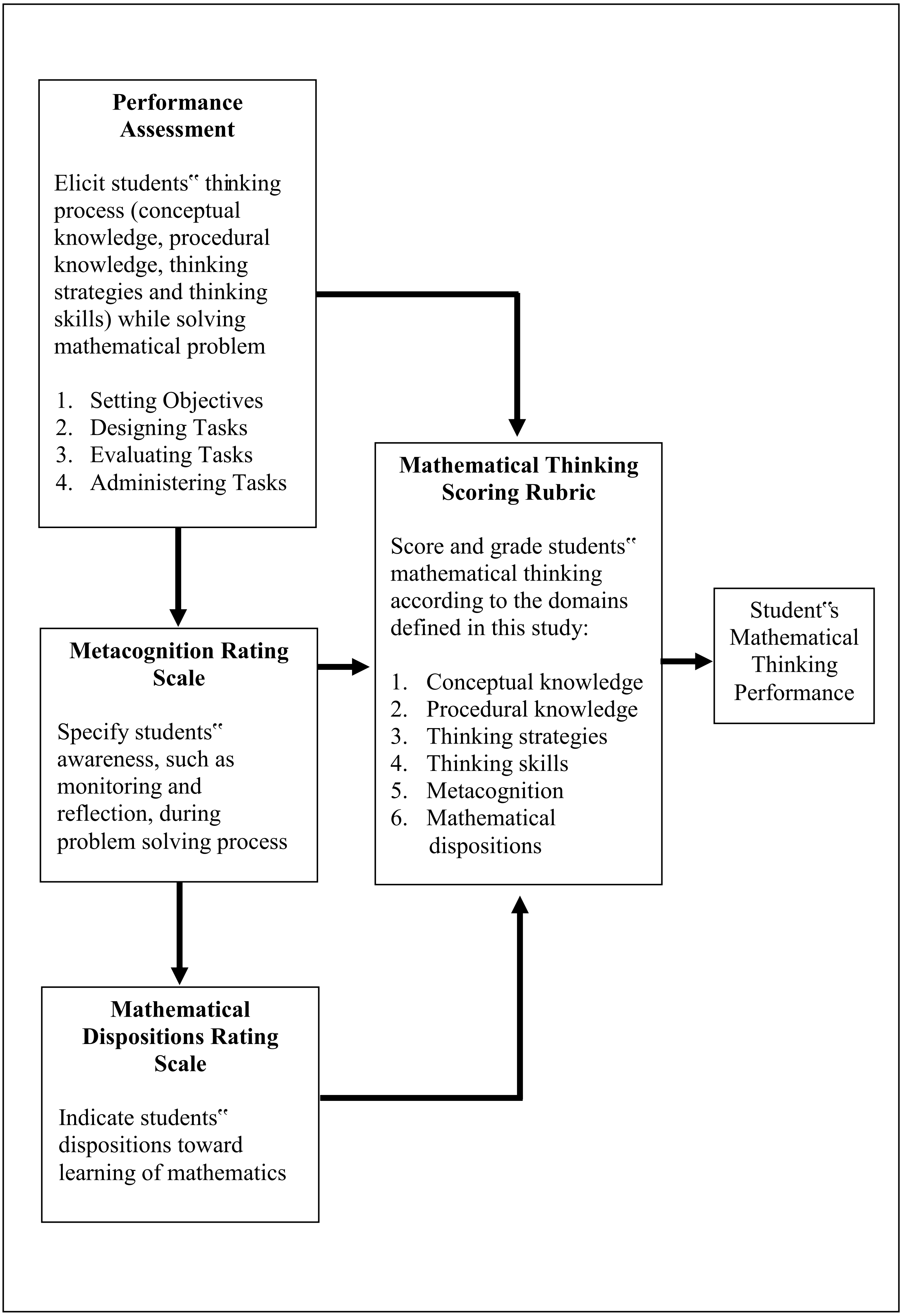


Figure 3: Summary of Implementing Mathematical Thinking Assessment (MaTA)

Section 2
Performance Assessment

Introduction

Performance assessment is a type of school-based assessment which allows the students to demonstrate their skills and knowledge in real life situation. Through the demonstration of problem solving strategies, students‟ mathematical thinking could be revealed. Hence, it is very important to design and select the performance tasks that are able to elicit students‟ mathematical thinking. Performance tasks which are carefully designed and selected will determine the success of implementing performance assessment in the school context. Figure 4 below illustrates how to plan a valid and reliable performance assessment that could be used to assess students‟ mathematical thinking.

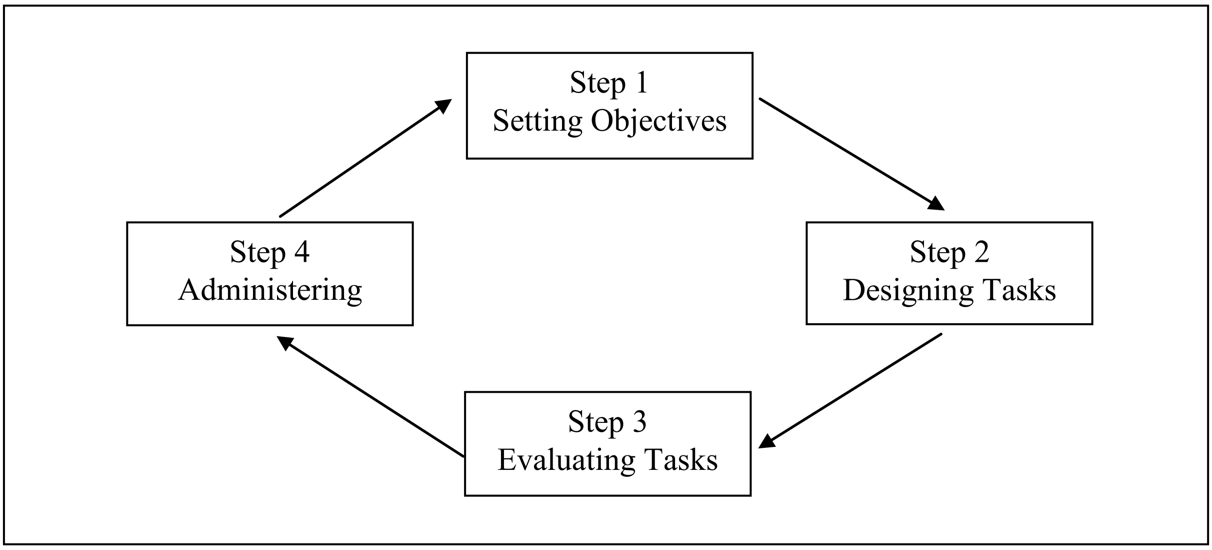


Figure 4: Planning Performance Assessment

Step 1: Setting Objectives for Performance Assessment

When planning performance assessment, it is important to set the objectives of the assessment. By setting the objectives, teachers will be able to know exactly what are the learning outcomes anticipated from their students. Furthermore, these objectives will guide the teachers in selecting valid and reliable tasks that meet the expectation and the objectives of the assessment. The following are the steps proposed:

(a) Identifying learning objectives – Teachers can identify the learning objectives of each mathematics topic by referring to Mathematics Curriculum Specification, published by Ministry of Education.

(b) Identifying learning outcomes that correspond to the learning objectives – Similarly, teachers can also identify the learning outcomes that correspond to the learning objectives through the Mathematics Curriculum Specification.

(c) Identified intended skills and knowledge – After identifying the leaning objectives and learning outcomes, the following three questions can be used as a guide to set appropriate objectives of the performance assessment:

  (i) What is (are) the expected outcome(s)?

  (ii) Is (are) the outcome(s) measurable?

  (iii) What is (are) the evidence(s) that indicates students possess the intended knowledge and skills?

(d) Set the objectives of the performance assessment – Once the intended skills and knowledge of the mathematical topics being identified, teachers can use the question cues proposed at Table 2 (Bloom Taxonomy Cognitive Domain) to state the objectives of the performance assessment.

The following procedures (Figure 5) could be used to identify and set the goals or objectives of performance assessment for Chapter 1: Standard Form of Form Four Mathematics.

Procedure	Action
Identify learning objectives.	Learning Objectives of Chapter 1: Standard Form 1.1 understand and use the concept of significant figure; 1.2 understand and use the concept of standard form to solve problem Extracted from Mathematics Curriculum Specifications (MOE, 2004)
Identify learning outcomes that correspond to the learning objectives.	Learning outcomes of learning objective 1.1: (i) round off positive numbers to a given number of significant figures when the numbers are: (a) greater than 1; (b) less than 1; (ii) perform operations of addition, subtraction, multiplication and division, involving a few numbers and state the answer in specific significant figures; (iii) solve problem involving significant figures; Learning outcomes of learning objective 1.2: (i) state positive numbers in standard form when the numbers are: (a) greater than or equal to 10; (b) less than 1; (ii) Convert numbers in standard form to single numbers; (iii) perform operations of addition, subtraction, multiplication and division, involving any two numbers and state the answers in standard form; (iv) solve problems involving numbers in standard form. Extracted from Mathematics Curriculum Specifications (MOE, 2004)
Identify the intended skills and knowledge: (a) What is (are) the expected outcome(s)? (b) Is (are) the outcome(s) measurable? (c) What is (are) the evidence(s) that indicates students possess the intended knowledge and skills?	Skills and knowledge intended: 1. Round off positive number to 3 significant figures. 2. Perform operations of multiplication and division involving more than two numbers; 3. Convert the answer in standard form.
Set the objectives of the performance assessment	Teachers can use the question cues proposed at Table 2 of Blomm Taxonomy Cognitive Domain to state the objectives of assessment, for example: 1. Students are able to **analyze** and **solve** the task by **demonstrating** the operations of multiplication and division involving more than two numbers; 2. Students are able to **round off** the answer to 3 significant figures; 3. Students are able to **convert** the answer to standard form; 4: Students are able to **justify** their solution.

Figure 5: Procedure for Setting Objectives for Performance Assessment

Table 2
Bloom's Taxonomy Cognitive Domain

Competence	Skills Demonstrated
Knowledge	• observation and recall of information • knowledge of dates, events, places • knowledge of major ideas • mastery of subject matter • *Question Cues:* list, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
Comprehension	• understanding information • grasp meaning • translate knowledge into new context • interpret facts, compare, contrast • order, group, infer causes • predict consequences • *Question Cues:* summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
Application	• use information • use methods, concepts, theories in new situations • solve problems using required skills or knowledge • *Questions Cues:* apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover
Analysis	• seeing patterns • organization of parts • recognition of hidden meanings • identification of components • *Question Cues:* analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, infer
Synthesis	• use old ideas to create new ones • generalize from given facts • relate knowledge from several areas • predict, draw conclusions • *Question Cues:* combine, integrate, modify, rearrange, substitute, plan, create, design, invent, what if?, compose, formulate, prepare, generalize, rewrite
Evaluation	• compare and discriminate between ideas • assess value of theories, presentations • make choices based on reasoned argument • verify value of evidence • recognize subjectivity • *Question Cues* assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize

Source: In Bloom, B. J. (1984), Taxonomy of Educational Objectives

Step 2: Designing Performance Tasks

Performance tasks should be designed with open-ended format which allow alternative interpretations or solutions that ask for explanations and reasoning. Hence, it is important to start designing the performance tasks by referring to questions or problems that are well established, such as from textbooks, reference books, internet resources or assessment institutions such as TIMSS, NAEP and PISA. While selecting the performance tasks, teachers will have to always keep in mind that the tasks must be able to achieve the objectives of performance assessment as set in Step 1.

Since most of the questions and problems from textbooks and reference books are classified as traditional assessment tasks, it is important for teachers to note the differences between traditional assessment and performance assessment, and to know how to modify traditional assessment tasks into performance assessment tasks that suit the Malaysian Mathematics Curriculum. Table 3 and Table 4 show samples of task in traditional assessment and the task in performance assessment respectively, whereas Table 5 illustrates how traditional assessment tasks could be adapted and modify to become performance tasks.

Once the performance tasks have been designed, teachers will have to check the suitability of the tasks. These can be done by investigating the characteristics of good and effective performance tasks, as highlighted below:

(a) The tasks are open-ended in nature.
(b) The tasks are authentic and real-life-based.
(c) The tasks can be solved by using multiple approaches or solutions.
(d) The tasks adequately represent the skills and knowledge you expect students to attain.
(e) The tasks must match specific instructional intentions, such as the learning objectives that are specified in each of the mathematics topic.
(f) The tasks require students to explain/reason in words how they derived the solutions.

Therefore, it is very important for teachers to examine the designed task carefully so that it meets all the criteria mentioned above. This is to ensure that the task is challenging and is able to elicit students‟ mathematical thinking while they try to solve the given task. The procedure of how to design performance task for Chapter 1: Standard Form of Form Four Mathematics is illustrated in Figure 6.

Table 3:
Samples of Current Traditional Assessment Tasks

Tasks	Comment
1. (SPM 2004) $\frac{4.86\times 10^{-2}}{(3\times 10^{-3})^2} = ?$	• Not an open-ended problem. • Not a real life problem. • Can be solved by using direct approach/algorithm, little thinking required. • No explanation/justification given. • No application, the teachers will know very little whether the students are able to demonstrate the skills and knowledge learned.
2. (SPM2003) The area of a rectangular nursery plot is 7.2km^2. Its width is 2400m. The length, in m, of the nursery plot is **A** 3×10^3 **C** 4.8×10^3 **B** 3×10^4 **D** 4.8×10^4	• Not an open-ended problem because the alternative answers are provided. • It is a real life problem. • Can be solved by using direct approach/algorithm, moderate level of thinking required. • No explanation/justification given. • It involved application of concepts and the teachers will know little whether the students are able to demonstrate the skills and knowledge learned.

Table 4:
Samples of Performance Assessment Tasks

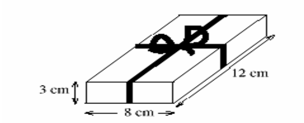
Tasks	Comment
1. The diagram below shows that a box is wrapped with some ribbon around and has 25 cm left to tie a bow. How long a piece of ribbon does he need if two boxes of this size are to tie together? **Show your reasoning how you solved this problem** 	• It is an open-ended problem. • It is a real life problem. • Can be solved by using multiple approaches/ solutions, complex thinking required. • Students are required to explain and reason how they solved the problem • Its involved application of concepts and the teachers will know better whether the students are able to demonstrate the skills and knowledge learned.
2. The length and width of a rectangular farm are 210m and 150 meter respectively. If the pepper trees are to be planted 4 m apart at the farm, and on average each pepper tree produces 2 kg of pepper in a month. Find the total amount of pepper produced by the farm each month. Round off the answer to three significant figures and state your answer in standard form. **Explain in words how you solved this problem**	• It is an open-ended problem. • It is a real life problem. • Can be solved by using multiple approaches, complex thinking required. • Students are required to explain and reason how they solved the problem • Its involved application of concepts and the teachers will know better whether the students are able to demonstrate the skills and knowledge learned.

Table 5:
Adapt and Modify Traditional Assessment Tasks to Become Performance Tasks

Traditional Assessment Task	Performance Task	Comment
(SPM 2004) 20 coupons with serial number 21 to 40 are put in a box. One coupon is drawn at random. The probability of drawing a coupon with a number which is not multiple of 5 is **A** $\frac{1}{5}$ **C** $\frac{3}{5}$ **B** $\frac{2}{5}$ **D** $\frac{4}{5}$	**A: Weak Performance Task** 20 coupons with serial number 21 to 40 are put in a box. One coupon is drawn at random. Find the probability of drawing a coupon with a number which is not multiple of 5. Explain how you solved this problem.	This question meets most of the criteria of designing a performance task. However, it can be solved directly by identifying the number of coupons whereby their number are not multiple of 5.
	B: Good Performance Task A number of coupons with serial number 21 to 40, 54 to 70 and 105 to 125 are put in a box. One coupon is drawn at random. Find the probability of drawing a coupon with a number which is not multiple of 5. Explain how you solved this problem.	Since the total number of coupons is not given directly, students are required to perform more complex thinking before he/she could obtain the answer.
	C: Excellent Performance Task A number of coupons with serial number 21 to 40, 54 to 70 and 105 to 125 are put in a box. One coupon is drawn at random. Find which multiple of number having higher chance to be picked, number which is multiple of 4, or number which is multiple of 5. Explain how you solved this problem.	This question is more challenging whereby the students not only are required to identify the total number of coupons, the students also have to identify the total number of multiple of 4 and the total number of multiple of 5, and analyze and compare the results before they could make the conclusion.

Procedure | **Action**

Procedure: Keep in mind the objectives of the assessment

Action:

Objectives of the assessment:

1. Students are able to **analyze** and **solve** the task by **demonstrating** the operations of multiplication and division involving more than two numbers;
2. Students are able to **round off** the answer to 3 significant figures;
3. Students are able to **convert** the answer to standard form;
4. Students are able to **justify** their solution.

↓

Procedure: Select the appropriate task by referring to textbooks, reference books, internet resources or assessment institutions, adapt and modify it to become the performance tasks

Action:

Multiple-choice (SPM 2006, Paper 1):
A rectangular floor has a width of 2400cm and a length of 3000cm. The floor will be covered with tiles. Each tile is a square of side 20cm. Calculate the number of tiles required to cover the floor fully.

A 1.8×10^3 **C** 3.6×10^4
B 1.8×10^4 **D** 3.6×10^5

↓ Modify

Performance Tasks:
A rectangular floor has a width 14m and a length of 10m. The floor will be covered with rectangular tiles. Each rectangular tile has a width of 20cm and a length of 30cm. Calculate the number of tiles required to cover the floor fully. Round off the answer to 3 significant figures, and state your final answer in standard form. **Explain in word how you obtained the solution.**

↓

Procedure: Check the suitability of the task:

(1) Is it open-ended?
(2) Is it real-life-based?
(3) Is it multiple approaches/ solutions enabling?
(4) Is it able to elicit intended skills and knowledge?
(5) Does it match the learning objectives?
(6) The students are asked to explain/reason their solutions?

Action:

The task:

(1) Open-ended because it is a word problem/answer not provided.
(2) Allow the students to demonstrate their skills and knowledge in real life context.
(3) Enabling multiple approaches/ solutions – many ways to get the answer.
(4) Able to elicit students understanding of significant figures and standard form
(5) It matches the learning objectives:
 1.1 understand and use the concept of significant figure;
 1.2 understand and use the concept of standard form to solve problem
(6) The students are required to explain/reason how they derived the solutions.

Figure 6: Procedure for Designing Performance Tasks

Step 3: Evaluating Performance Tasks

Once the tasks are designed, teacher could engage the following steps (Figure 7) to examine and evaluate the suitability of the tasks in meeting the objectives of the assessment.

(a) Perform or solve the task yourself - Teachers will have to perform or solve the task before administering it to their students. During the self-check problem-solving, teachers are encouraged to produce as many as possible the approaches or solutions to the task, as shown in Figure 8 and Figure 9.

(b) List the important aspects of performance which are related to the objectives of the assessment - From the solutions, teachers will have to identify the important aspects of performance and list them according to the objectives of the assessment set in Step 1.

(c) Examine performance criteria - Teachers will have to identify the performance criteria which are observable (Table 7, p. 34) and arranged them in the following order:

   (i) Conceptual knowledge

   (ii) Procedural knowledge

   (iii) Thinking Strategies

   (iv) Thinking Skills

(d) Seek second opinion to improve and refine the quality of the performance task – Teachers could seek comments from other teachers on the suitability of tasks, or pilot test the task to a few selected students.

Teachers could also use the following checklist (Table 6) to counter check whether the performance task designed exhibit the desire credibility.

Table 6
Checklist for Evaluating Performance Tasks

Item	Description	Check
1.	Identify the performance task and perform it yourself	
2.	The solution(s) is reasonable and according to the syllabus.	
3.	List the important aspects of the performance which are related to the objectives of the assessment	
4.	Make sure the performance criteria can be expressed in terms of observable student behaviors or product characteristics.	
5.	Make sure the performance criteria are arranged in the order in which they are likely to be observed.	
6.	Seek second opinion to improve the performance tasks, such as asking other teachers to solve and comments on the same tasks or pilot test the tasks to a few selected students.	

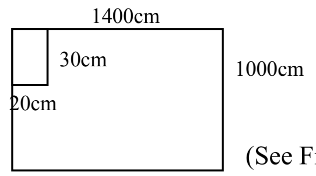
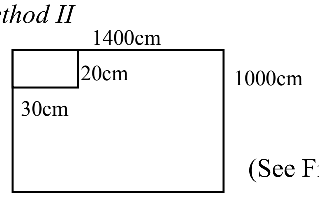
Procedure	Action
Perform or solve the task yourself	At least two solutions: *Method I*  (See Figure 9) *Method II*  (See Figure 10)
List the important aspects of performance which are related to the objectives of the assessment	The important aspects of performance: (1) Analyze the task – at least two possible solutions (see Figure 9 and Figure 10); (2) Perform the operations of multiplication and division; (3) Round off the answer to 3 s. f.; (4) Convert the answer to standard form; (5) Justify the solution.
Examine performance criteria: (a) Behaviour or product are observable (b) Arranged in order in which they are likely to be observed.	The performance criteria: (1) Conceptual knowledge – apply the concepts and get the answer correctly (2) Procedural knowledge – select /execute appropriate procedure & justify each step of the procedure (3) Thinking strategies – plan carefully, use appropriate strategy and check correctness of the answer. (4) Thinking skills – link mathematical idea(s) to real life situation, using correct terms/notations and show logical/mathematical sense
Seek second opinion to improve and refine the quality of the performance task	(1) Obtain comments from other teachers on the suitability of tasks, or (2) Pilot test the task to a few selected students.

Figure 7: Procedure of Evaluating Performance Tasks

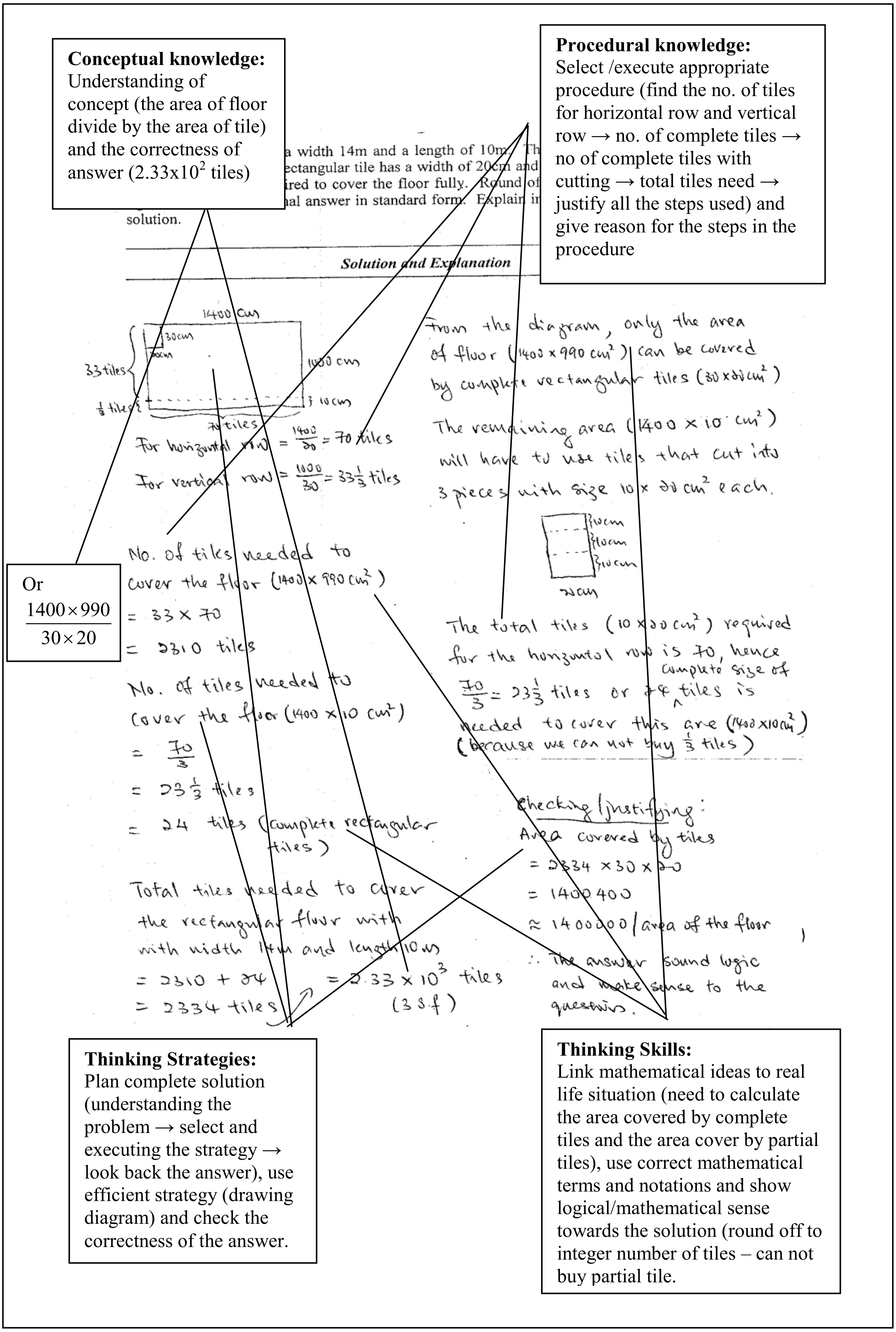


Figure 8: Example of Solution (1)

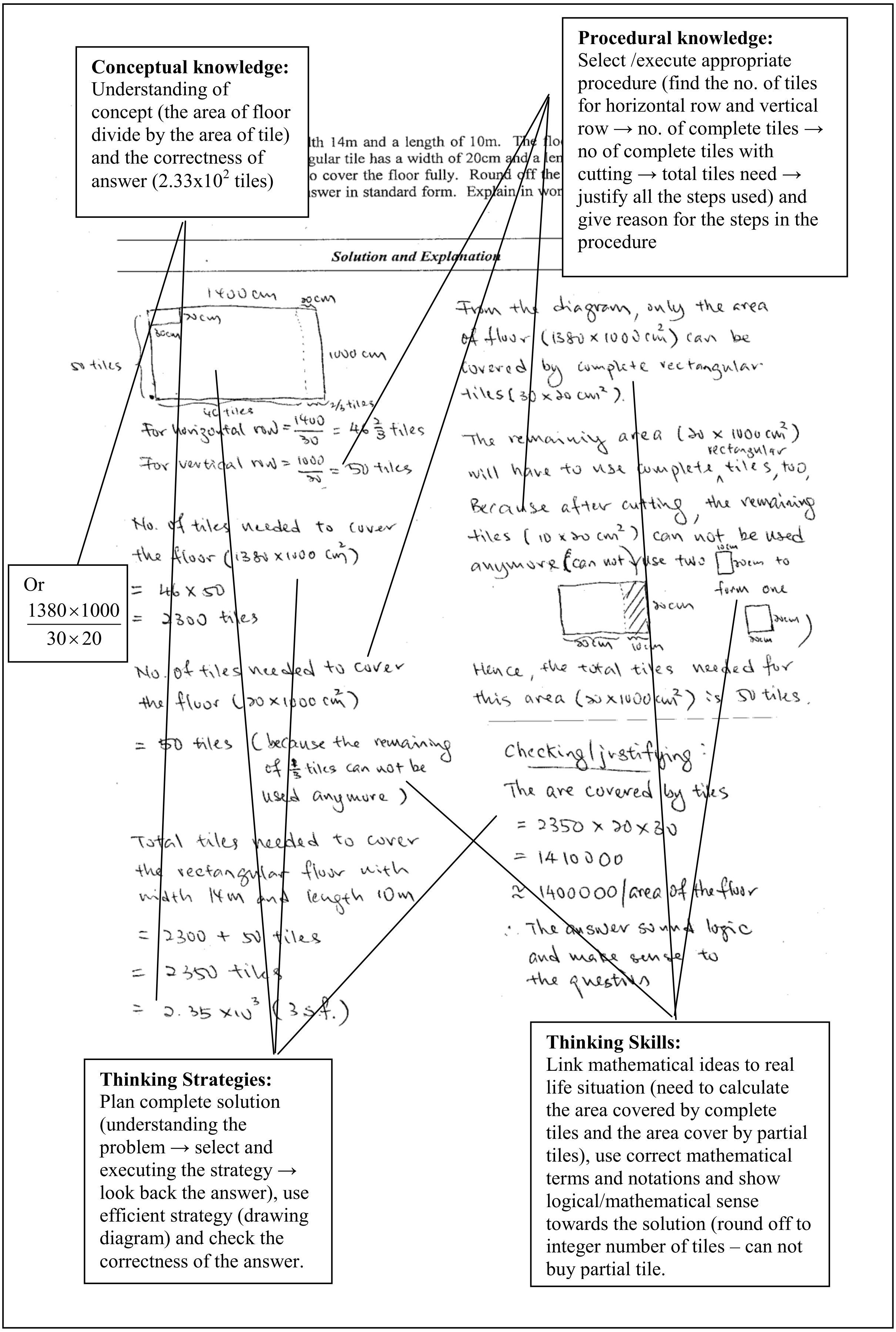


Figure 9: Example of Solution (2)

Step 4: Administering Performance Assessment

Before the performance assessment is being administered, make sure that the students are aware of the evaluation criteria specified in the Mathematical Thinking Scoring Rubric (Figure 14 on page 35). This can be done by:

(a) Providing Mathematical Thinking Scoring Rubric to each of the students.

(b) Discussing with the students each of the performance criteria and the levels of performance specified in this scoring rubric.

(c) Discussing with the students how their mathematics written solutions are being assessed through this scoring rubric (use the examples from this framework)

(d) Discussing with the students different approaches that could be used in attempting the same task in the performance assessment.

(e) More importantly, constantly promoting performance assessment during teaching and learning in the classroom by giving them real life problems to solve; asking them to reason and verify their solutions; and reminding them whether they have achieved the satisfactory levels of performance in the Mathematical Thinking Scoring Rubric.

Once the students are aware of the evaluation criteria, teachers can begin to train them on how to solve the performance tasks (Appendix A: Sample of performance task). After the student are ready and familiar with the solution/explanation to performance tasks, teachers could administer the performance assessment that aim to elicit students‟ mathematical thinking. The ideals number of performance tasks given for each assessment is **three (3)** tasks. This is because students are unable to complete many tasks within the class hours. Teachers will have to make sure that ample time is allocated for students to solve all the performance tasks. After the assessment, teachers will collect all the students‟ written responses and score them according to the evaluation criteria stated in the Mathematical Thinking Scoring Rubric.