NEW HISTOGRAM EQUALIZATION BASED DETAIL AND BRIGHTNESS PRESERVING TECHNIQUES FOR DIGITAL IMAGES

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NEW HISTOGRAM EQUALIZATION BASED DETAIL AND BRIGHTNESS PRESERVING TECHNIQUES FOR DIGITAL IMAGES

by

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LIST OF ABBREVIATIONS

AAMBE	Average Absolute Mean Brightness Error
AC	Average Contrast
AE	Average Entropy
AMBE	Absolute Mean Brightness Error
AMHE	Adaptive Modified Histogram Equalization
BBHE	Brightness Preserving Bi-Histogram Equalization
BHE	Bi-Histogram Equalization
BLS	Black Level Stretch
BPDHE	Brightness Preserving Dynamic Histogram Equalization
BPPLHE	Brightness Preserving Plateau Limit Histogram Equalization
BUBOHE	Bin Underflow and Bin Overflow Histogram Equalization
CDF	Cumulative Density Function
CF	Cumulative Frequency
CHE	Clipped Histogram Equalization
CLAHE	Clipped Limited Adaptive Histogram Equalization
DPPLHE	Detail Preserving Plateau Limit Histogram Equalization
DRHE	Dynamic Range Histogram Equalization
DHE	Dynamic Histogram Equalization
DSIHE	Dualistic Sub-Image Histogram Equalization
FPGA	Field Programmable Gate Array
GC-CHE	Gain Controllable Clipped Histogram Equalization
HE	Histogram Equalization
LHE	Local Histogram Equalization

MBE	Mean Brightness Error
MHE	Multi Histogram Equalization
MMBEBHE	Minimum Mean Brightness Error Bi-Histogram Equalization
MMLSEMHE	Minimum Middle Level Squared Error MHE
MPHE	Multi-Peak Histogram Equalization
MSSI	Mean Structural Similarity Index
MWCVMHE	Minimum Within-Class Variance MHE
PDF	Probability Density Function
PLHE	Plateau Limit Histogram Equalization
PSNR	Peak Signal to Noise Ratio
QBHE	Quantized Bi-histogram equalization
QPLBHE	Quantized Plateau Limits Bi-Histogram Equalization
RHE	Recursive Histogram Equalization
RMSHE	Recursive Mean Separate Histogram Equalization
RSIHE	Recursive Sub-Image Histogram Equalization
RSWHE	Recursively Separated and Weighted Histogram Equalization
RSWHE-D	median separation RSWHE
RSWHE-M	mean separation RSWHE
SAPHE	Self-Adaptive Plateau Histogram Equalization
SMBE	Scaled MBE
WCHE	Weight Clustering Histogram Equalization
WHE	Weighted Histogram Equalization
WLS	White Level Stretch

LIST OF SYMBOLS

α_i	Accumulative probability value
∂	Power law transformation function
β	Degree of mean brightness
B	Cluster Width Criteria
c_{BO}	Bin overflow
c_{BU}	Bins underflow
$C_n(i)$	*i*-th cluster
$Cost(k)$	Cost function with k sub-histograms
$c_Q(X_k)$	CDF of the quantized histogram
CR	Clipped rate
$c(X_k)$	Cumulative Density Function
c_{wt}	Weight and threshold CDF
D	Reminder
$Disc(k)$	Discrepancy function with k number of sub-histograms
ε	Amount of emphasis
$E_m(i)$	Number of bins in the corresponding partition in i-th cluster
$E(X)$	Input mean
$E(Y)$	Output mean
$F(X_k)$	Number of pixels occupied in intensity level k
$f(x,y)$	Intensity level at position (x,y)
G^G	Global gain
G_{high}^L	High local gain
G_{low}^L	Low local gain
G_{max}	Pre-set maximum gain

G^T	Total gain
H^B	Black-level corrected histogram
h_c	Clipped histogram
h_{ci}	Clipped histogram at *i*-th sub-histogram
$h_d(X_k)$	Different of the input histogram
H^G	Global gain corrected histogram
H_{h_L}	Peak value of lower sub-histogram
H_{h_U}	Peak value of upper sub-histogram
$h_{QL}(X_k)$	Quantized histograms at the intensity level *k* in the lower sub-histogram
$h_{QU}(X_k)$	Quantized histograms at the intensity level *k* in the upper sub-histogram
h_U	Upper sub-histogram
$h(X_k)$	Histogram of the input image
H^W	white-level corrected histogram
I_{height}	Height of input Image
I_{width}	Width of input Image
i_{start}	First intensity value at *i*-th sub-histogram
i_{end}	Last intensity value at *i*-th sub-histogram
K	Number of Discrete Gray Levels
k_T	Total number of gray level
L	Discrete Gray Levels
l	Number of power
L_{in}	dynamic range of the input image
L_{out}	Dynamic range of the output image
M_{adj}	Mean adjustment factor
M_{si}	Number of pixels in *i*-th sub-histogram

M_T	Total number of pixels
N	Number of sample image
$\check{n}$	Number of local minimal
ρ	Positive weighting constant
p_{AMHE}	PDF of AMHE
P_d^i	The plateau limit of i-th sub-histogram based on median
$P_{h_L}^i$	*i*-th plateau limit of lower sub-histogram
$P_{h_U}^i$	*i*-th plateau limit of lower sub-histogram
p_l	Lower threshold of PDF
P_L^i	*i*-th plateau limit at lower histogram
p_{max}	Highest PDF of input image
p_{mid}	Mean value of p_{min} and p_{max}
p_{min}	Lowest PDF of input image
p_{nw}	Normalized p_w
p_u	Upper threshold of PDF
P_μ^i	The plateau limit of i-th sub-histogram based on mean
p_w	Weighting PDF
p_{wt}	Weight and threshold PDF
R	Cluster Weight Ratio Criteria
range	Dynamic range
Span	Span
τ	Average Processing Time
v	Upper threshold normalized to p_{max}
W	Cluster Weight Criteria

$W_n(i)$	Cluster weight
X_d	Median of the input image
X_d^t	The *t*-th separating point based on median
X_f^j	Final intensity level at *j*-th sub-histogram
X_k	Intensity level at *k*
X_{kB}	Black level region
X_{kW}	White level region
X_l	The lowest intensity level in that sub-histogram
X_{li}	Lowest intensity level at i-th sub-histogram
X_{max}	Highest intensity value
X_{min}	Lowest intensity value
X_{mm}^j	Middle value of the *j*-th sub-histogram
X_s^j	Start intensity level at *j*-th sub-histogram
X_T	Intensity threshold level
X_μ	Mean brightness of input image
X_{ui}	Highest intensity level at i-th sub-histogram
X_u	The highest intensity level in that sub-histogram
X_μ^t	The *t*-th separating point based on mean
X_μ^j	Mean value of the *j*-th sub-histogram
¥	Amount of emphasis given on the frequency
$Y(X)$	Output image
σ	Standard deviation
ϕ	Enhancement rate factor
<<	Left shift operator

TEKNIK-TEKNIK PENGEKALAN PERINCIAN DAN KECERAHAN BERDASARKAN PENYERAGAMAN HISTOGRAM UNTUK IMEJ DIGIT

ABSTRAK

Penyeragaman Histogram “*Histogram Equalization*” (HE) adalah satu kaedah penyerlahan imej yang digunakan dengan meluas kerana keberkesanan dan kemudahan apabila diaplikasikan. Namun, kaedah HE menghadapi masalah-masalah seperti perubahan purata kecerahan, intensiti tepu dan pembesaran hingar. Oleh itu, dua kaedah pengekalan kecerahan baru yang dinamakan “*Brightness Preserving Plateau Limit Histogram Equalization*” (BPPLHE) dan “*Quantized Plateau Limit Bi-Histogram Equalization*” (QPLBHE) telah dicadangkan. Keadah pengekalan kecerahan amat diperlukan bagi produk-produk elektronik, terutamanya televisyen, monitor dan sebagainya. Daripada keputusan yang diperolehi, kedua-dua kaedah adalah lebih baik daripada kaedah-kaedah lain dengan menghasilkan keputusan visual yang lebih jelas, lebih sempurna dan kecerahan intensiti masih dikekalkan. Tetapi, kaedah-kaedah pengekalan kecerahan tidak dapat meningkatkan kecerahan latar belakang imej disebabkan oleh bahagian tepi sub histogram yang tidak dapat dikembangkan. Oleh itu, kaedah pengekalan perincian yang dinamakan “*Detail Preserving Plateau Limit Histogram Equalization*” (DPPLHE) telah dicadangkan. Daripada keputusan perbandingan, kaedah DPPLHE adalah paling berkesan dalam mengekstrak perincian imej daripada imej berkontras rendah. Keputusan simulasi membuktikan bahawa kaedah DPPLHE adalah paling berkesan, baik dari segi penilaian kualitatif mahupun kuantitatif. Ia amat sesuai diaplikasikan untuk imej-imej yang ditangkap di bawah suasana cahaya berkapasiti rendah, iaitu satu situasi yang sering dihadapi oleh produk-produk elektronik seperti kamera telefon bimbit.

NEW HISTOGRAM EQUALIZATION-BASED DETAIL AND BRIGHTNESS PRESERVATION TECHNIQUES FOR DIGITAL IMAGES

ABSTRACT

Although Histogram Equalization (HE) is widely used as a contrast enhancement method because of its effectiveness and simplicity, it tends to produce mean brightness change, intensity saturation problem and noise amplification. Therefore, two novel brightness preservation methods, namely Brightness Preserving Plateau Limit Histogram Equalization (BPPLHE) and Quantized Plateau Limit Bi-Histogram Equalization (QPLBHE) are proposed. The method for preserving mean brightness is under a very high demand, especially where consumer electronic products are concerned, like the television, monitor etc. The experiment results show that, both of the proposed methods outperform those conventional methods by producing clearer, more visually-enhanced images with brightness preservation ability. However, brightness preservation methods are usually incapable in enhancing the contrast of the low-lighted backgrounds of the images due to the non-expandable side sub-histogram. Therefore, a detail preservation method is proposed, namely Detail Preserving Plateau Limit Histogram Equalization (DPPLHE). From the results of the experiments, the proposed DPPLHE is the most robust method in extracting the details of the low contrast images. Observing from the simulation results obtained, the DPPLHE has produced the better performance from the perspectives of both qualitative and quantitative evaluations. It is suitable for images captured in low-lighted environment – unavoidable situation faced by many electronics products such as camera devices in cell phone.

CHAPTER 1

INTRODUCTION

1.1 Background

An enhancement process is normally applied to digital images in order to get more information or details from images. Due to insufficient illumination sources and improper focusing during the image acquisition process, these conditions are contributive towards images of low quality. Insufficient illumination makes the brightness in the images unevenly distributed. The aim of image enhancement is to improve the interpretability or perception of information in images for human, or provide better input image for other techniques or computer vision (Downtown & Jones, 1991).

Contrast enhancement is one of the enhancement processes of which the purpose is to provide a better visual as well as extract the detail of an image. Generally, this can be done by stretching the original narrow dynamic range of an image into its full dynamic range (Ekstrom, 1984).

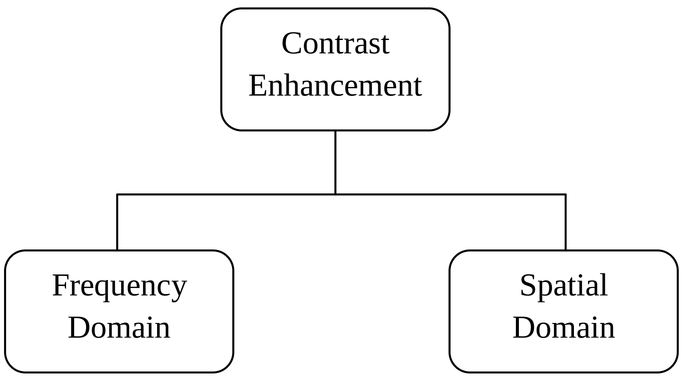


Figure 1.1 Domains of the contrast enhancement methods

Contrast enhancement can be categorized into frequency domain and spatial domain as shown in Figure 1.1. The process in the frequency domain requires the transformation of the digital image from the spatial domain to frequency domain. After the enhancement, the frequency domain is transformed back into its corresponding spatial image (Mohiy, 1999). In spatial domain, the enhancement process is applied directly to the pixels. Therefore, in order to reduce the complexity of the enhancement process, this study focuses on the spatial domain.

As shown in Figure 1.2, generally, contrast enhancement methods are categorized into 5 groups namely the histogram equalization (HE)-based (Teuber, 1993), power law-based (Gonzalez & Woods, 2002), intensity pair-based (Jen *et al.*, 2005; Kabir *et al.*, 2006, 2009) and standard deviation-based (Patrenhalli & Robert, 1981; Diagakis *et al.*, 1993) methods. However, this study only focuses on the HE-based method as other methods tend to have a high computational complexity or slower processing time. (Jen *et al.*, 2005; Kabir *et al.*, 2006, 2009).

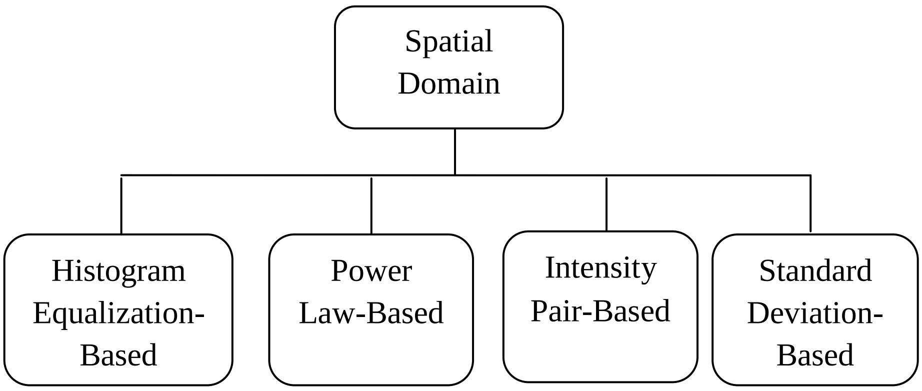


Figure 1.2 The contrast enhancement methods in spatial domain.

The simplicity and effectiveness of the HE-based methods have led them to be widely used in the medical field, satellite images, microscopic and real life photographic images, which usually tend to have poor contrast. The fundamental idea of the HE-based method will be discussed in the next sub-section.

1.2 Histogram Equalization (HE)

Contrast enhancement plays an important role to improve the visual quality of an image (Cadik *et al.,* 2006; Wang, 2004). Histogram Equalization is one of the well known methods to improve the contrast of an image. It is operated by remapping the gray levels of an image according to the distribution of the gray levels (Gonzalez & Woods, 2002). Generally, the density distribution is flatted and has an effect of stretching the dynamic range. As a result the contrast of the image is enhanced (Kawakami, 2009).

Let $\boldsymbol{A}$ as the original unenhanced image with $f(x,y)$ represents the gray level at position (x, y). Then the probability of the density function (PDF), $p(X_k)$ is defined as:

$$p(X_k) = \frac{F(X_k)}{I_{width} \times I_{height}} \tag{1.1}$$

where I_{width} and I_{height} are the image width and height respectively, $F(X_k)$ is the total number of pixels at level k, where k = 0, 1, 2,…, L-1 with L is the total number of gray levels. From Equation 1.1, the cumulative density function (CDF), $c(X_k)$ is created as given by Equation 1.2.

$$c(X_k) = \sum_{i=0}^{k} p(X_i) \tag{1.2}$$

CDF is used to be a transform function that maps the input image dynamic range to the dynamic range of the output image. The transform function is expressed as seen in Equation 1.3. An example of HE-ed image is shown in Figure 1.3. The details of the image can easily be seen as the HE method successfully increases the contrast of the image.

$$Y(X) = X_0 + c(X_k) \cdot (X_{L-1} - X_0) \tag{1.3}$$

where X_0 and X_{L-1} are the intensity level at values of zero and L-1 respectively.

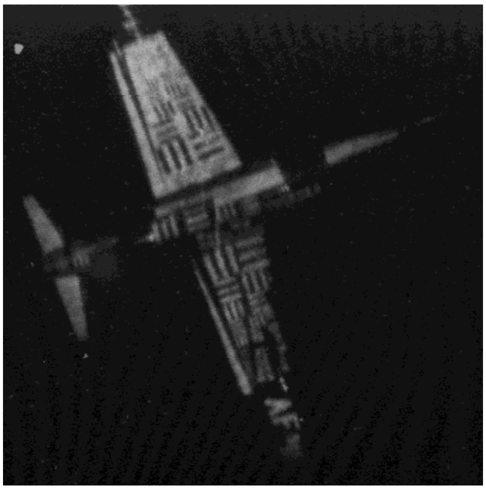
The applications of HE-based methods are found in many fields, most of them are used to extract the information of the image, for examples, in the medical image processing (Wang *et al.,* 2008; Ziaei *et al.,* 2008), color image processing (Trahanias & Venetsanopoulos 1992; Kim & Yang, 2006), texture synthesis (Soo *et al.*, 2004), texture clarification (Batista *et al.,* 2005), infrared image processing (Wang, *et al.,* 2005), sonar image processing (Cervenka & Moustier, 1993), satellite image processing (Demirel *et al.,* 2010; Attachoo & Pattanasethanon, 2008; Babu & Krishna, 1997; Deng *et al.,* 1995; Chanussot *et al.,* 2003), speech recognition (Suh & Kim, 2009; Shu *et al.,* 2007; Angel et.al., 2005), fog removal (Xu *et al.,* 2009), underwater image processing (Singhai & Rawat, 2007), CMOS vision sensor image processing (Chen *et al.,* 2005), near-sensor image processing (Anders *et al.,* 1998), holography image processing(Tushar & Chandra, 2002), optical image processing (Ryan *et al.,* 2002), watermarking system (Lee *et al.,* 2001) and motion detection (Hasanzadeh *et al.,* 2005)

Furthermore, the HE-based methods are also used as methods employed in the pre-processing for other applications such as when improving the edge detector performance (Gholamali & Houtan, 2007). The HE-based method is able to extract the special feature of the image and is also helpful for the edge detection in the computer vision applications such as the magnetic resolution image (MRI) and remote sensing. For the pre-processing task (i.e. image segmentation and recognition), the HE-based method has also been applied for image simplification (Wu *et al.*, 2006).

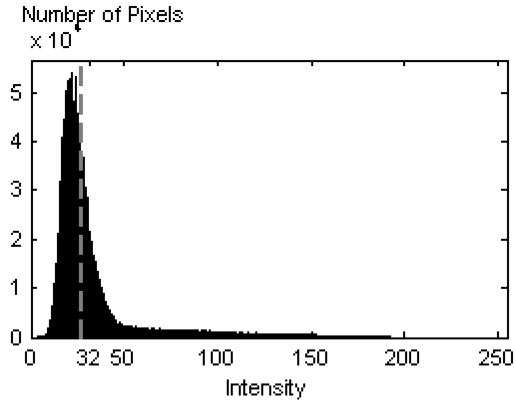
In the real system application, the HE-based method is used for consumer electronic products due to its easy implementation and shorter processing time. For example, it can be used as the backlight scaling for liquid crystal display (LCD) with significant power saving and effective distortion rate (Iranli *et al.,* 2005). Moreover, the HE method is fast, simple, and flexible with reasonable cost of its development hardware and thus it is adopted in the field programmable gate arrays (FPGAs) (Alsuwailem & Alshebeili, 2005).

1.3 Problem Statements

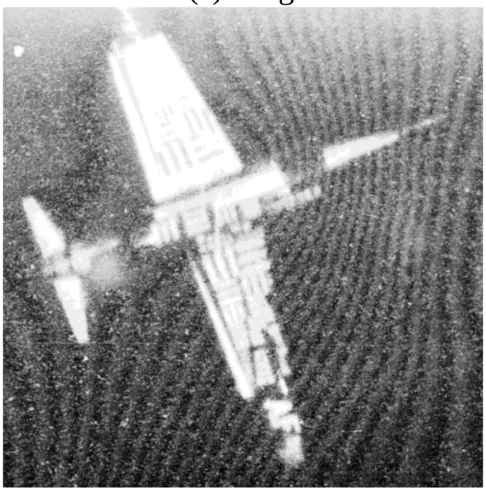
Although the HE method is widely used in many areas and applications as discussed previously, it may produce undesirable artifacts such as excessive brightness change, intensity saturation problem and noise amplification (Chen & Manjit, 2009). For the excessive brightness change, by comparing the original image and histogram of the input image to the HE-ed image and histogram of HE-ed image in Figure 1.3, the gray level of the background of the image is excessively changed. Furthermore, the mean of the input histogram is shifted from the intensity level 32 to 133 as represented by the dotted line.



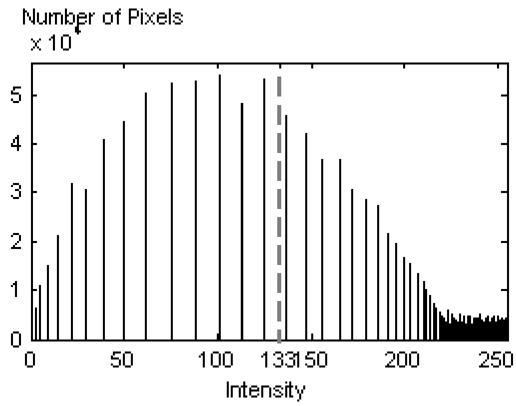
(a) Original



(b) Histogram of image (a)



(c) HE-ed Image (a)



(d) Histogram of image(c)

Figure 1.3 Example for HE-ed "U2" image

The undesirable detail loss caused by the heavy intensity saturation problem is presented in Figure 1.3(c), as can be clearly seen at the body of the jet. This phenomenon is caused by the uneven enhancement rate for large and small bins' regions of the output histogram. Reference can be made in Figure 1.3, where the original image is dominated by dark intensity pixels (i.e. large bins) while the small bins are dominated by white pixels. The large enhancement rate is usually applied to the regions of the large bins, while the small bins' regions of the output histogram are squeezed. This process is the one that causes the intensity saturation problem. In addition, the squeezed small bins are merged into a bin, which may lead to information loss.

In addition, the HE also tends to produce noise amplification artifact, especially when looking at the background of the image in Figure 1.3(c). This is due to the higher enhancement that has been applied to this region; resulting in each detail (including noise) becoming significantly enhanced. Although the HE method is able to enhance the contrast of the image, it tends to significantly degrade the quality of the image due to the previously mentioned artifacts.

To achieve original mean brightness preservation and overcome the mean shift problem of the HE method, numerous brightness preservation methods (i.e. Bi-Histogram Equalization (BHE), Recursive Histogram Equalization and Multi Histogram Equalization (Multi-HE)) have been proposed in literature. In order to maintain original mean brightness of the image, the brightness preservation methods usually determine the mean brightness of the input image, X_μ as presented in Figure 1.4. However, many of them tend to have neglected the issue surrounding the intensity saturation problem.

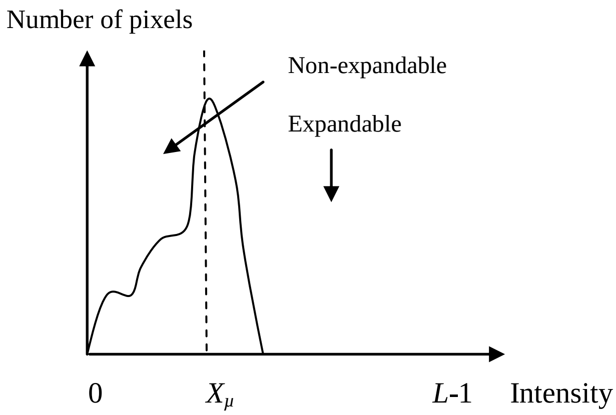


Figure 1.4 The example of the non-expandable side sub-histogram.

Furthermore, in the low-lighted cases, the conventional brightness preservation methods do not enhance the left side sub-histogram of Figure 1.4. Only the right side of the sub-histogram will be expanded. The left side of the sub-histogram is referred as the non-expandable side sub-histogram. Thus, the detail in this region will not be enhanced.

The non-expandable side sub-histogram phenomenon highly occurs in low-lighted environment images. The input histogram of such environment image is usually concentrated to the left hand side of the histogram with a small dynamic range, which is illustrated in Figure 1.4.

1.4 Objectives

Based on the abovementioned drawbacks, the objectives of this project are as follows:

1. To develop new HE-based brightness preservation methods with the capability to preserve the mean brightness of input image and reduce the intensity saturation problem.
2. To develop a new HE-based detail preservation method with the capability to reduce intensity saturation problem. In addition, the proposed method is specifically designed for images with low-lighted environment and overcome the problem of non-expandable side sub-histogram.

The scope of this project focuses on gray scale images. For the brightness preservation method, the proposed method focuses on poor contrast images, while the detail preservation method stresses on images captured in low-lighted environments. The development of the proposed algorithms will be implemented by using the C++ programming language.

1.5 Thesis Outline

There are a total of five chapters included in this thesis, which are the introduction, literature review, methodology, results and discussion, and conclusion.

In Chapter 2 some relevant theories about the brightness and detail preservation methods will be presented and summarized. The detail description of the conventional brightness and detail preservation methods are also given in this chapter. The description includes the fundamental concept, current works as well as advantages and disadvantages of these works. Some of these methods will be implemented as they are used as comparison with the proposed methods. The results and analysis on the reliability of the implementation done will be presented at the end of Chapter 2.

The content in Chapter 3 provides the methodology of proposed brightness and detail preservation methods. Details regarding the steps of implementation and explanation of these three methods will then be presented.

Next, Chapter 4 discusses the resultant images obtained from the proposed methods and some selected HE-based enhancement methods. All results are discussed, explained, analyzed and compared qualitative and quantitatively in order for the performance of each method to be able to be evaluated.

Chapter 5 is the last chapter, functioning to conclude and summarize the entire project. Suggestions for future works are also included in this final chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

It has been stated earlier that the enhancement process is normally applied to digital images in order to get more information or details from the image. Generally, insufficient illumination sources could result in the production of low quality images, other than the fact that these flawed sources can also unevenly distribute the brightness in the images and produce low contrast images.

An enhancement process is very useful for consumer electronic products where it serves as a pre-processing or post-processing tool for digital image processing. As discussed in Chapter 1, HE is a simple and effective method to enhance the contrast of the images. However, this method tends to produce unwanted and annoying artifacts. Thus, HE-based brightness and detail preservation methods have been proposed. The brightness preservation method is introduced to overcome the mean shifting problem of the HE method. The detail preservation method is proposed to overcome the non-expandable sub-histogram of the conventional brightness preservation method.

The fundamental concepts and examples of brightness preserving methods will be reviewed in Section 2.2. They are the Bi-Histogram Equalization (BHE)-based, Recursive Histogram Equalization (RHE)-based and Multi Histogram Equalization (Multi-HE)-based. In Section 2.3, methods concerning detail preservation will be discussed. Explanation will cover several well-known and recent methods namely the Plateau Limit Histogram Equalization (PLHE)-based, Weighted Histogram Equalization (WHE)-based and Dynamic Range Histogram Equalization (DRHE)-based. As those methods chosen to be compared with the proposed methods, the implementation of these conventional methods will be verified in Section 2.4. Finally, this chapter is summarized in Section 2.5.

2.2 Brightness Preservation Methods

The goal of the brightness preservation method is to overcome the problem of mean shift of the HE method, which is highly demanded for electrical items such as the television and monitor. The brightness preservation methods can be categorized in three main techniques; the BHE-based, the RHE-based and the Multi-HE-based methods, as shown in Figure 2.1. The BHE-based technique divides the input histogram into two sub-histograms, while the RHE-based technique recursively segments the input histogram into 2-based multiplication sub-histograms. On the other hand, the Multi-HE-based technique segments the input histogram into several sub-histograms based on the characteristics of the histogram. The methods underlying each technique are discussed in the following sub sections.

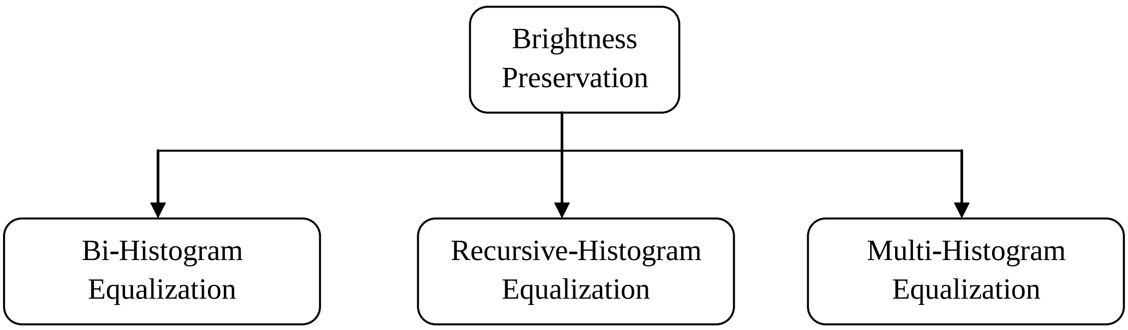


Figure 2.1 Different categories of brightness preservation methods.

2.2.1 Bi-Histogram Equalization (BHE)

The BHE-based method is the first technique introduced to overcome the input mean shifting, which is the drawback of the HE method. The BHE-based technique enhances the contrast of images while maintaining the input mean of the images. This can be done by dividing the histogram of the input image into two sub-histograms based on the information of the histogram, as shown in Figure 2.2. The histogram of the input image is divided by the intensity level of the threshold, whereby X_T with L is the number of gray levels.

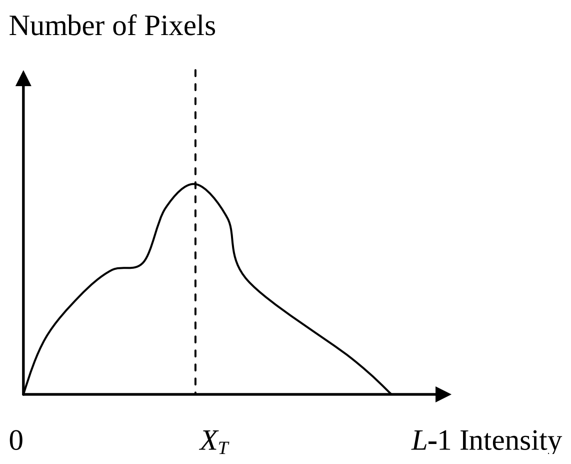


Figure 2.2 The cross-division of two sub-histograms

From the BHE family, the Brightness Preserving Bi-Histogram Equalization (BBHE) is proposed by Kim in 1997 (Kim, 1997a). Firstly, the BBHE divides the original histogram into two sub-histograms based on mean brightness, X_μ as defined by Equation 2.1.

$$X_\mu = \frac{\sum_{i=0}^{L-1} h(X_i)}{I_{width} \times I_{height}} \tag{2.1}$$

Where $h(X_i)$ is the histogram at i-th intensity level. One of the sub-histograms is assigned to the intensity level less than and equal to the mean of the input image. On the other hand, the other sub-histogram is the portion of the intensity level which is greater than the mean of the input image.

Then, the conventional HE technique is implemented independently in each sub-histogram. As a consequence, the mean brightness of the input image can be preserved due to the ability to maintain the original mean brightness. However, the BBHE method only provides effective mean brightness preservation on the histogram that has a quasi-symmetrical distribution around its mean (Wang & Ye, 2005). Otherwise, the BBHE fails to manipulate the mean shifting problem.

For real time system application, a method called the Quantized Bi-Histogram Equalization (QBHE) was introduced to reduce the hardware complexity of the BBHE as well as enhance the contrast and maintain the mean brightness of the input image (Kim, 1997b). For the essence of the histogram based implementation, cumulative function is needed. Thus, many components are required to construct a hardware structure, for example, comparators, counters and dividers. In order to avoid this complication, the QBHE utilizes the CDF of a quantized image. Hence, the QBHE can be more feasible in the real time implementation.

Generally, the QBHE is the extension of the BBHE method. Unlike BBHE, the QBHE quantizes the L discrete gray levels into K discrete levels. After quantization, similar to the BBHE, the QBHE method decomposes the input histogram into two sub-histograms based on the mean of the input image. To demonstrate an equalization property, a linear interpolation is used to evaluate the function value at every input gray level in certain manners. Finally, the quantized bi-histogram is equalized by using HE independently.

The parameter K regulates the degree of the enhancement in the QBHE method. For example, higher values of parameter K will lead to higher degrees of enhancement and vice versa. Moreover, in order to find an optimum result, the K

discrete levels require the perspective of the user. Thus, the QBHE method is not suggested in the real time implementation.

A novel method called the Dualistic Sub-Image Histogram Equalization (DSIHE) is proposed as the extension of the BBHE (Wang *et al.,* 1999). The DSIHE method adopts a similar concept to the BBHE method. It segments input histogram into bi-histogram. Unlike the BBHE, the DSIHE decomposes the input histogram into two equal areas, using the median of the input image, X_d as the separating point. Mathematically, the separating point can be calculated as follows:

$$X_d = k,\ if\ c(X_k) = 0.5 \tag{2.2}$$

The research claimed that this procedure would yield the maximum entropy of the enhanced image. Thus, more hidden details or information could be brought out from the original image. After segmenting the histogram of the input image, both bi-histograms are then equalized independently. The bins with less than or equal to the input median are mapped to $[X_0\ X_d]$ (X_0 is the intensity level with zero value), otherwise are mapped to $[X_{d+1}\ X_{L-1}]$. Finally, the two sub-images are composed as an enhanced image.

In some cases, the DSIHE method produces a better brightness preservation and entropy than the BBHE method. However, the DSIHE method produces significant brightness preservation only on the image with a large uniform background. Regardless of the circumstances, the DSIHE method tends to have a similar drawback as the BBHE, which requires a higher degree of brightness preservation to avoid annoying intensity saturation artifacts (Chen & Ramli, 2003a).

In order to optimize the mean brightness preservation, the Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) has been introduced (Chen & Ramli, 2003b; 2004). The MMBEBHE determines the Absolute Mean Brightness Error (AMBE) for each possible separating point X_T. The AMBE is defined as follows:

$$AMBE = \left| E(Y) - E(X) \right| \tag{2.3}$$

where *E(Y)* and *E(X)* are the output and input mean brightness respectively. The MMBEBHE is implemented following these three steps:

1. Calculate the AMBE value for each of the possible X_T.
2. Find the value X_T that yields the minimum AMBE value.
3. Divide the input histogram based on X_T and equalize the bi-histogram independently.

Lower value of the AMBE denotes better mean brightness preservation. Unfortunately, Step 1 of the MMBEBHE takes a lengthy computational time in finding the lowest value of the AMBE. For example, an 8-bits image requires 256 times of processing (*L*=256) to equalize the segmented bi-histogram. This condition is probably the major drawback of the MMBEBHE method due to a large value of *L*.

The output of the mean brightness of the bi-histogram can be estimated as follows:

$$E_T(Y) = \left(\frac{X_0 + X_T}{2}\right)\left[\sum_{i=0}^{T} p(X_i)\right] + \left(\frac{X_T + X_{L-1}}{2}\right)\left[\sum_{i=T+1}^{L-1} p(X_i)\right] \tag{2.4}$$

After some simplified steps, Equation 2.4 can be defined as follows:

$$E_T(Y) = \frac{1}{2}\left[X_T + L\left(1 - \sum_{i=0}^{T} p(X_i)\right)\right] \tag{2.5}$$

Then, the mean of the output image with the threshold level set as X_{T+1}:

$$E_{T+1}(Y) = \frac{1}{2}\left[X_T + 1 + L\left(1 - \sum_{i=0}^{T+1} p(X_i)\right)\right]$$

$$= \frac{1}{2}\left[X_T + 1 + L\left(1 - \sum_{i=0}^{T} p(X_i) - p(X_{T+1})\right)\right]$$

$$= \frac{1}{2}\left[X_T + L\left(1 - \sum_{i=0}^{T} p(X_i)\right)\right] + \frac{1}{2}\left[1 - Lp(X_{T+1})\right]$$

$$= E_T(Y) + \frac{1}{2}\left[1 - Lp(X_{T+1})\right] \tag{2.6}$$

The calculation is initiated with the simplest solution, where X_T=X_0=0.

$$E_0(Y) = \frac{1}{2}[L(1 - p(X_0))]$$

$$E_1(Y) = E_0(Y) + \frac{1}{2}[1 - Lp(X_1)]$$

....

$$E_{T+1}(Y) = E_T(Y) + \frac{1}{2}[1 - Lp(X_{T+1})] \tag{2.7}$$

where $E_T(Y)$ is the output mean brightness at the threshold level X_T. For the performance evaluation, the Mean Brightness Error (MBE) is employed. The MBE assumes the X_T starts at zero. The MBE can be expressed by:

$$MBE = E(Y) - E(X)$$

$$MBE_0 = E_0(Y) - E(X)$$

$$MBE_1 = E_1(Y) - E(X)$$

$$= E_0(Y) + \frac{1}{2}\left[1 - Lp(X_1)\right] - E(X)$$

$$= MBE_0 + \frac{1}{2}\left[1 - Lp(X_1)\right]$$

....

$$MBE_T = MBE_{T-1} + \frac{1}{2}\left[1 - Lp(X_T)\right] \tag{2.8}$$

The MBE can be further simplified to:

$$MBE_0 = \frac{1}{2M_T}[L(M_T - F(X_0) - 2\sum_{i=0}^{L-1} iF(X_i)] \tag{2.9}$$

Where M_T is the total number of pixels and $F(X_i)$ is the number of pixels occupied in an intensity level i. In order to remove the divider $2M_T$, the scaled MBE (SMBE) is expressed as follows:

$$SMBE = (2M_T)MBE \tag{2.10}$$

$$SMBE_0 = (2M_T)MBE_0$$

$$= L(M_T - F(X_0)) - 2\sum_{i=0}^{L-1} iF(X_i) \tag{2.11}$$

Where, the equation for each SMBE threshold level is given by:

$$SMBE_1 = (2M_T)MBE_1$$

$$= 2M_T \left[MBE_0 + \frac{1}{2M_T}[M_T - LF(X_1)] \right]$$

$$= 2M_T(MBE_0) + [M_T - LF(X_1)]$$

$$= SMBE_0 + [M_T - LF(X_1)]$$

....

$$SMBE_T = SMBE_{T-1} + [M_T - LF(X_T)] \tag{2.12}$$

The number of L is usually the base multiplication of the 2. In order to reduce the complexity of computation of the multiplier, L could be replaced by a basic shift operator. For $L = 2^l$, Equations 2.9 and 2.10 are expressed as Equations 2.11 and 2.12 respectively.

$$SMBE_0 = [(M_T - F(X_0)) << l] - \left(\sum_{i=0}^{L-1} iF(X_i) \right) << l \tag{2.13}$$

$$SMBE_T = SMBE_{T-1} + [M_T - (F(X_T) << l)] \tag{2.14}$$

where l is the number of bits of the input image, << is the l bits left shift operator.

After the threshold level is chosen by enumeration, the MMBEBHE method is then combined with the Steps 2 and 3 to compose the enhanced sub-images.

2.2.2 Recursive Histogram Equalization (RHE)

The fundamental concept of the Recursive Histogram Equalization (RHE)-based suggests dividing the input histogram into 2^r sub-histograms with r being the recursion level. Higher number of recursion level ensures the mean brightness of the output image to converge to the mean brightness of the input image. Thus, better brightness preservation ascertains that an enhanced image will be obtained.

The recursion level of the RHE-based technique plays an important role in brightness preservation. For example, the RHE-based technique is equivalent to HE for r=0. Hence, without any separating point, the RHE-based technique does not preserve the original mean brightness of the image. For r=1, it is equivalent to the BBHE or the DSIHE if the separating point is equal to the mean and median of the input image respectively. Generally, a novel method is introduced when the recursion level is higher than one (i.e. $r \geq 2$).

A RHE-based method was proposed by Chen and Ramli, called the Recursive Mean Separate Histogram Equalization (RMSHE) (Chen & Ramli, 2003a; 2004). Firstly, the RMSHE method deals with the first separation level (r=1) on the input histogram based on the mean of the input image. Next, the second separation level (r=2) is applied to decompose the histogram of the input image into four sub-histograms. This technique is executed recursively until the desired number of sub-histograms is met. Finally, each sub-histogram is equalized independently by using the HE method similar to the BBHE, DSIHE and MMBEBHE methods.

It is also worth mentioning that another similar recursive technique based on median histogram segmentation has also been proposed, named as the Recursive Sub-Image Histogram Equalization (RSIHE) (Sim *et al.,* 2007). Unlike the RMSHE, the RSIHE method uses the median of the input image as the recursive separating points instead of the mean. Generally, the DSIHE is repeatedly used in the RSIHE method. The procedure is claimed to be able to obtain more information and generate a better quality image by referring to the higher values of the Mean Structural Similarity Index (MSSI) and Peak Signal to Noise Ratio (PSNR).

A novel RHE method has been proposed by modifying the input histogram before HE is performed, named the Recursively Separated and Weighted Histogram Equalization (RSWHE) (Kim & Chung, 2008). Unlike, the RMSHE and the RSIHE methods, a weighting function is applied to the input histogram in the RSWHE method, which could avoid over-enhancement on the high probabilities' regions of the gray levels and improve the important visual details on the regions of low probabilities. There are three modules in the RSWHE method, namely the histogram segmentation, histogram weighting and histogram equalization as illustrated in Figure 2.3.

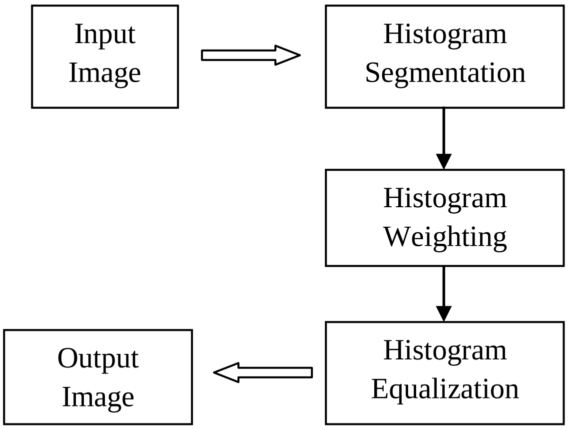


Figure 2.3 The Process of the RSWHE method.

The modules shall be discussed:

1. Histogram Segmentation

   Similar to the RMSHE and RSIHE methods, the RSWHE method also segments the input histogram recursively based on the mean and median of the input image. The mean separation and median separation are defined as the RSWHE-M and RSWHE-D respectively.

2. Histogram Weighting

   After the histogram of the input image is segmented into 2^r sub-histograms, the RSWHE method changes the shape of the histogram of the input image by modifying the PDF of the sub-histograms as follows:

   (a). Determine the highest PDF, p_{max} and the lowest PDF, p_{min} referring to the following equations:

$$p_{\max} = \max_{0 \le k \le L-1} p(X_k) \tag{2.15}$$

$$p_{\min} = \min_{0 \le k \le L-1} p(X_k) \tag{2.16}$$

   (b). Accumulative probability value, α_i is calculated as in Equation 2.17, where the total value of all α_i's is equal to one.

$$\alpha_i = \sum_{k=l_i}^{u_i} {}_{0 \le k \le L-1}\, p(k) \tag{2.17}$$

$$\sum_{i=0}^{L-1} \alpha_i = \alpha_0 + \alpha_1 + \alpha_2 ... + \alpha_{L-1} = 1 \tag{2.18}$$

   (c). The weighting function is then applied to each sub-histogram using the following equation:

$$p_w(k) = p_{\max} \cdot \left(\frac{p(k) - p_{\min}}{p_{\max} - p_{\min}} \right)^{\alpha_i} + \beta, (X_{li} \le k \le X_{ui}) \tag{2.19}$$

   where β is a constant with the value more than zero. β is used to manipulate the degree of the mean brightness and contrast enhancement. From the experiment results, the optimal value β is calculated as follows:

$$\beta \approx p_{\max} \cdot \left| X_{\mu} - X_{d} \right| / (X_{max} - X_{min}) \tag{2.20}$$

where X_μ and X_d are the mean and median of the input histogram respectively. X_{max} and X_{min} are the greatest and least gray levels of the input image respectively.

(d). Having been modified the input histogram, all p_w are no longer equal to one. Therefore, the normalization is done by using Equation 2.21.

$$p_{wn}(k) = \frac{p_w(X_k)}{\sum_{j=0}^{L-1} p_w(j)} \tag{2.21}$$

3. Histogram Equalization

HE is executed in all of 2^r sub-histograms. All of the decomposed sub-images are then combined to form a final output image.

The goals of the RSWHE method lie in the brightness preservation and contrast enhancement. Thus, the results of the RSWHE method are demonstrated in qualitative and quantitative forms (i.e. AMBE, PSNR and Entropy analyses). Both of the qualitative and quantitative results suggest that the mean-based, RWSHE-M method is the best and this is followed by the median-based, RWSHE-D.

Although the RSWHE method can preserve the mean brightness more accurately than the RMSHE and the RSIHE methods, in some cases it tends to have an unnatural enhancement. This phenomenon is the major drawback of the RHE method, due to the non-expandable narrow sub-histograms which decreases its effectiveness.

2.2.3 Multi-Histogram Equalization (Multi-HE)

The Multi-Histogram Equalization (Multi-HE)-based technique is intended to decompose the input histogram into several sub-histograms. Abovementioned methods in previous section are segmented based on the mean or median of the input image, but on the other hand, the Multi-HE-based technique decomposes the input histogram based on the local maximum, local minimum and histogram clustering. As proven, the Multi-HE-based technique restricts the over-enhancement phenomenon as well as maintaining the input mean brightness of the original image (Wongsritong *et al.*, 1998).

The first Multi-HE-based technique is introduced to reduce the intensity saturation problem while maintaining its mean brightness, named the Multi-Peak Histogram Equalization (MPHE) (Wongsritong *et al.*, 1998). However, the input histogram usually fluctuates, suggesting some difficulties in the peak detection. Moreover, some intensity levels disappear in the input histogram. Thus, a linear interpolation is applied to ensure that all the intensity levels are filled. Next, a smoothing process is implemented to remove the fluctuated bins in the input histogram.

Nevertheless, in order to detect the local minimums of the histogram, the smoothing process does not sufficiently handle the fluctuated input histogram. Hence, the algorithm changes the three consecutive signs from (+-+) to (+++) and (-+-) to (---). After the sign-changing process, the local minimums are detected on the smoothed histogram when four successive negative signs followed by eight successive positive signs are found. Next, the detected local minimums are then used to act as the separating points in the input histogram. Finally, each sub-histogram is equalized independently.

Although Wongsritong et al (1998) claimed that the performance of the MPHE method yields a better brightness preservation method than the BBHE method, it does not significantly maintain the mean brightness of the input image by segmenting using local minimums (Ibrahim & Kong, 2007). Therefore, a local maximum detection method has been proposed, called the Brightness Preserving Dynamic Histogram Equalization (BPDHE) (Ibrahim & Kong, 2007; Kong & Ibrahim, 2008).

Generally, the BPDHE method is the extension of the MPHE method. Firstly, the BPDHE method smoothes the input histogram by using the Gaussian filter. Then, the sign-changing process and local maximums detection are applied. Unlike the MPHE method, the local maximums are detected when four successive negative signs are followed by eight successive positive signs. Next, the algorithm segments the input histogram by the local maximums instead of the local minimums as in the MPHE method.

The algorithm then assigns a new dynamic range to each sub-histogram. This step avoids the insignificant enhancement due to the small non-expandable sub-histograms. Next, the BPDHE method equalizes all the sub-histograms independently. In order to maintain the original mean brightness of the input image, the brightness normalization process is applied. The ratio of brightness normalization plays an important role in order to ensure the output mean brightness is almost similar to the original mean brightness. Thus, the original mean brightness is able to be preserved.

Although the BPDHE method has the smaller value of AMBE than the MPHE method, it tends to have wash-out appearance on some images (Sengee *et al.*, 2009). Furthermore, the ratio of the brightness normalization may produce an annoying problem of intensity saturation (i.e. if ratio value is more than one) and insignificant contrast enhancement (i.e. if ratio is less than 0.1).

Another novel Multi-HE-based method was introduced by dividing the histogram of the input histogram into several sub-histograms based on its discrepancy function (Menotti et. al., 2007). The authors suggest two discrepancy functions to decompose the input histogram. The goals of the method are to enhance the contrast of the images while preserving the mean brightness of the input image and to further produce a natural appearance for the output images.

The first method finds the optimal threshold that minimizes the decompose error of the input histogram with the minimum intra-class variance, called the Minimum Within-Class Variance Multi-HE (MWCVMHE). In order to produce natural-looking images with a brightness preservation capability, another discrepancy function is employed in order to minimize the brightness shifting in each sub-histogram. This method is named the Minimum Middle Level Squared Error Multi-HE (MMLSEMHE). At the beginning of the process, the MWCVMHE and the MMLSEMHE manually decompose the histogram of the input image into *k* number of sub-histograms. Both discrepancy functions are defined as the Equations 2.22 and 2.23 respectively.

$$Disc(k) = \sum_{j=1}^{k} \sum_{l=X_s^j}^{X_f^j} (l - X_\mu^j)^2 \cdot p(X_l) \tag{2.22}$$

$$Disc(k) = \sum_{j=1}^{k} \sum_{l=X_s^j}^{X_f^j} (l - X_{mm}^j)^2 \cdot p(X_l) \tag{2.23}$$