ASSESSING MATHEMATICAL THINKING LEVELS OF YEAR SIX STUDENTS USING PERFORMANCE TASKS

NURULHIDAYAH LUCY BINTI ABDULLAH

UNIVERSITI SAINS MALAYSIA

2010

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MENAKSIR TAHAP PEMIKIRAN MATEMATIK PELAJAR-PELAJAR TAHUN ENAM MENGGUNAKAN TUGASAN PERFORMA

ABSTRAK

Kajian ini menumpukan perhatian kepada menaksir dan mengenal pasti tahap pemikiran matematik pelajar-pelajar Tahun Enam dengan menggunakan tugasan performa. Selanjutnya, kajian ini meninjau tahap pemikiran matematik pelajar berdasarkan empat domain pemikiran matematik iaitu, pemahaman konseptual, perwakilan matematik, kelancaran prosedural dan penjelasan matematik. Kajian ini bertujuan membekalkan maklumat sama ada terdapat perkaitan antara domain dan penggunaan masalah matematik yang berbeza memberi kesan kepada tahap pencapaian pelajar. Profil matematik pencapaian rendah dan tinggi pelajar melibatkan empat domain pemikiran matematik dikenal pasti.

Kajian ini melibatkan sejumlah 155 orang pelajar Tahun Enam dari empat buah sekolah. Respons pelajar dianalisa untuk mengenal pasti tahap pemikiran dan mendiagnos aspek-aspek kelemahan pelajar. Respons pelajar bagi setiap masalah matematik dianalisa secara berasingan dengan menggunakan tujuh rubrik pengskoran yang spesifik terdiri daripada empat tahap skor. Hasil kerja penulisan pelajar, temu bual dan helaian refleksi merupakan instrumen utama untuk pengumpulan data. Kebolehpercayaan antara pemeriksa dikira untuk memastikan pemeriksaan antara pemeriksa adalah konsisten.

Keseluruhan analisa menunjukkan peratusan pelajar yang paling tinggi dikenal pasti berada pada “Tahap Satu” (tidak berkaitan) bagi keempat-empat domain iaitu pemahaman konseptual, perwakilan matematik, kelancaran prosedural dan penjelasan matematik. Pada tahap pemikiran yang lebih rendah, peratusan pelajar adalah hampir sama dalam pemahaman konseptual, perwakilan matematik dan kelancaran prosedural kecuali bagi penjelasan matematik di mana peratusan pelajar adalah lebih tinggi. Dapatan kajian juga menunjukkan pelajar yang dikenalpasti berada pada “Tahap

Empat" (mahir) untuk pemahaman konseptual juga mempamerkan pencapaian yang lebih rendah dalam domain yang memerlukan mereka menyediakan penjelasan. Dapatan kajian mendapati kelemahan utama pelajar adalah dalam domain penjelasan matematik. Dapatan kajian yang sama dipamerkan oleh kumpulan pelajar berprestasi tinggi dan rendah. Keseluruhannya, pencapaian pelajar menunjukkan adanya hubungan positif yang signifikan antara semua domain pemikiran matematik.

Hasil kajian juga menunjukkan bahawa tahap pemikiran pelajar dipengaruhi oleh kandungan pelajaran dan kumpulan tugasan dengan kompleksiti yang sama. Dapatan kajian mendapati pelajar menunjukkan tahap pemikiran matematik yang lebih tinggi bagi soalan bersifat sederhana. Pelajar juga menunjukkan tahap pemikiran yang lebih tinggi bagi soalan yang berkait dengan tajuk "luas". Dari aspek profil pelajar didapati pelajar berpencapaian rendah dan tinggi mempunyai beberapa persamaan seperti kebergantungan mereka kepada rumus, pendekatan cuba-jaya dan kebergantungan kepada kata kunci atau rajah yang dibekalkan dalam tugasan. Walau bagaimanapun pelajar tidak menunjukkan kepelbagaian strategi dalam menyelesaikan masalah. Kekurangan utama pelajar ialah penjelasan matematik. Hal ini kerana pelajar tidak dapat menguasai terminologi matematik yang relevan semasa membuat penjelasan.

Kata Kunci: Tahap pemikiran matematik, Penilaian, Tugasan Performa, Pemahaman konseptual, Perwakilan matematik, Kelancaran prosedural, Penjelasan matematik

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ABSTRACT

This study aimed at assessing and identifying Year Six students' mathematical thinking level using constructed response tasks. This study further explored students' mathematical thinking levels in relation to the four mathematical thinking domains of conceptual understanding, mathematical representations, procedural fluency and mathematical explanation. It provided information whether the domains were associated and whether the different tasks used affect students' levels of performance. The profiles of mathematical thinking involving the four mathematical thinking domains were identified for low and high achievers.

A total of 155 Year Six students enrolled in four schools participated in this study. Students' responses were examined to identify mathematical thinking levels and diagnose areas of deficiency. Students' responses for each task were rated independently using seven specific scoring rubrics with four score levels. Students' written works, interviews and reflective sheets were the main tools used for data collection. Inter-raters reliability was computed to determine the consistency of judgment between raters.

Overall analysis showed that highest percentages of students were assigned to 'Level 1' (not related) for all domains: conceptual understanding, mathematical representation, procedural fluency and mathematical explanation. Percentages of students assigned to lower levels were almost equal for conceptual understanding, mathematical representation, and procedural fluency except mathematical explanation which was relatively higher. Finding also revealed that even students assigned to 'Level 4' (proficient) for conceptual understanding exhibited lower performance in mathematical explanation of the same level. Therefore students' main deficiency was in the mathematical explanation domain. The same finding was obtained among the low and high achievers. Overall students' performances also showed statistically significant positive relationships between all mathematical thinking domains.

Findings also showed that students' performances were determined by content area of the tasks and group of tasks with similar complexity. Students' performances were significantly higher for group of moderate tasks. Students also performed higher in tasks related to the topic of 'area'. Low and high achievers exhibited similar profiles as both groups of students rely on memorizing formulas, trial-and-error approach and supportive words and diagrams to solve the tasks. Their solutions also showed lack of various strategies. Their main deficiencies were for mathematical explanation as they were not able to grasp relevant mathematical terminologies to present their explanation.

Keywords: Mathematical thinking levels, Assessment, Performance tasks, Conceptual understanding, Mathematical representation, Procedural fluency, Mathematical Explanation

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CHAPTER ONE

INTRODUCTION

This study aims at assessing and identifying Year Six students' level of mathematical thinking performance in solving constructed response tasks. This study explores students' levels of performance and its association to mathematical thinking domains such as how students demonstrate conceptual understanding, mathematical representations, procedural fluency and mathematical explanation. This will provide information related to students' mathematical thinking levels. It will also provide information whether the domains are associated and whether tasks of different complexity used effect students' levels of performance. This chapter presents the background of this study, research problem, research objectives, research questions, significance, limitations and finally the working definitions of this research.

1.1 Mathematics Curriculum in Malaysian Schools

Mathematics has been universally accepted as part of the school curriculum and it is an important tool to cope with everyday life and life-long education. Malaysia is one of the countries that implement one mathematics curriculum to all students (Mullis, Martin, Gonzalez & Chrostowski, 2003). In Malaysia, the Mathematics Curriculum has been centrally designed by Ministry of Education and Curriculum Development Centre. As a school subject in the Malaysian education curriculum, mathematics consists of area of knowledge that is regarded as able to train students to think logically and systematically in solving problems and making decisions (Ministry of Education, 2001). Mathematics curriculum provides knowledge and mathematical skills to be acquired by children of various backgrounds and levels of ability ever since they enter into the primary schools. The primary school mathematics syllabus consists of two levels; Level 1 (Year One to Three) emphasizes the mastering of basic concepts of primary numbers and the four basic operations while Level 2 (Year Four to Six) focuses on application of the basic mathematical skills in solving problems. The primary mathematics curriculum (Ministry of Education, 2006) stressed that teaching and learning of mathematics at all levels should involve the main aspects such as (1) knowledge and concepts building, (2) application of skills, (3) inculcation of positive values towards the ability to appreciate mathematics, (4) problem solving strategies, (5) mathematics communication, (6) understanding of mathematical thinking e.g. constructs of mathematical connections between conceptual and procedural and (7) applications of technology. These reflected that primary mathematics curriculum in Malaysia has been designed to facilitate the acquisition of knowledge, implementation of basic skills, and to enhance students' mathematical thinking to enable them to face and cope challenges in life and for continual life-long education.

1.2 Reforms in Mathematics Curriculum

Living in the information age requires changes in Malaysia educational goals. Thus, Malaysian mathematics curriculum had undergone several reforms. First was the 'The Modern Mathematics Programme' introduced during the early 70's and in the 80's. Second was the 'Integrated Curriculum for Primary School' (ICPS) or *Kurikulum*

Bersepadu Sekolah Rendah (KBSR) which was introduced in 1982. The KBSR emphasize on the understanding of number concept and acquiring basic counting skills and memorization of facts related to the basic computation skills. Both conceptual understanding and mastery of computation skills should be emphasized in the teaching and learning of mathematics in primary classroom (Ministry of Education, 2001). In 2000, ICPS was reviewed and known as the KBSR *edisi tahun* 2000 which can be found in circular issued by the Ministry of Education No KP(BS) 8591/Jld.XVIII (2) (Ministry of Education, 2002). This reform highlighted the need for the development of thinking skills as the dominant elements, which is said to include students' competence in explaining mathematical ideas, solving problems and presenting mathematical arguments orally or through writing using accurate mathematical language implemented in the teaching and learning of primary mathematics. In addition, it also emphasized mathematical reasoning, relationship and communication in mathematics. The importance of mathematical thinking elements in the teaching and learning of mathematics is clearly shown in the statement from the KBSR syllabus as following:

> " The learning of mathematics at all levels involves more than just the basic acquisition of concepts and skills. It involves, more importantly, an understanding of the underlying mathematical thinking, general strategies of problem solving, communicating mathematically and inculcating positive attitudes towards an appreciation of mathematics as an important and powerful tool in everyday life".
>
> *(Ministry of Education, 2006)*

Thus, classroom teaching and learning process of mathematics in primary school should emphasized conceptual understanding, mastery of skills, understanding of the underlying mathematical thinking, strategies in solving problems and mathematical communication.

1.3 Importance of Mathematics Curriculum Reform

The learning of mathematics is a complex and dynamic process (Noraini, 2006). The ability to think mathematically and to use mathematical thinking to solve problems is an important goal of schooling as it will support science, technology, economic life and development in an economy (Stacey, 2007). Therefore, today's world and the world of the future demand effective and skillful thinkers. These visions can be achieved through knowledgeable, thinking and competent citizens. It is important to build and produce thinking citizens of Malaysia that are able to communicate and interact effectively, able to think creatively and critically in facing changes and future challenges. Knowing the content of mathematics and producing correct answers alone do not help students to face challenges in today's global competition. Thus the reforms in the Malaysian curriculum system have placed more emphasis on problem solving, communications, mathematical reasoning, connections and representations in the teaching and learning of mathematics towards producing students that can think creatively, critically, and able to solve problems.

Solving mathematical problems and communicating mathematical ideas with understanding require students to explain their mathematical thinking, that connect to other related mathematical elements such as acquisition of concepts, representing mathematical ideas, executions of strategies and explanation on decision making (Ministry of Education, 2006). The ability to solve problems with understanding and communicate mathematical ideas must be nurtured from an early age so that children acquire the skills to evaluate information, to compare, to make decisions and justify effectively (Pugalee, 2004). According to Ginsburg, Jacobs and Lopez (1998), the demands for mathematical thinking proficiency such as mathematical reasoning and communicating mathematical ideas at the early age seem to be important for students to take responsibility or make decision of their own learning processes.

Since 2003, Malaysian Primary Mathematics Curriculum is inclined towards producing thinking students with more emphasis on mathematical processes (Cheah, 2010). Students need deep conceptual understanding, application of knowledge and reasoning in solving mathematical problems in order to link them to real life situations. Citizens who cannot reason mathematically are cut off from the whole realms of human endeavour (Kilpatrick, Swafford, & Findell, 2001). Hence, learning how to think mathematically is an extremely important issue in mathematics education (Ma'Moon, 2005). Students should move from learning mathematics based on rules that need to be memorized to expressing their mathematical understanding.

1.4 Contemporary Views and Components of Mathematical Thinking

There are many different ways through which mathematical thinking can be viewed (Stacey, 2007). The following section presents various views of mathematical thinking and components of mathematical thinking.

1.4.1 Contemporary Views of Mathematical Thinking

Mathematical thinking is seen as an important branch of cognition which involves a highly complex activity in solving problems (Stacey, 2007). Stacey finds that mathematical thinking is important as goal of schooling, way of learning and teaching mathematics. Dunlap (2001) has also regarded mathematics as ‘a way of thinking’ implemented in problem solving activities that challenges thinking and develop an understanding of the processes of mathematics that brings meaningful learning rather than a collection of mechanical rules. Stenger (1999) and Thompson (1971) have viewed mathematical thinking as skills that involved active engagement of students in building an understanding of mathematical thinking process which can be used to assess individual differences. Stenger's view was supported by Posamentier and Stepelman (2002) who have regarded mathematical thinking as a process which students were given direct experience to develop their own thought, to build and gain increased understanding of mathematics.

1.4.2 Components for Mathematical Thinking

Thompson (1971) viewed mathematical thinking as skills consists of using words, images or asking oneself tasks in silent. Siegler (1991) viewed mathematical thinking as a process which obviously involves the higher mental processes such as problem solving, reasoning, creativity, conceptual understanding, remembering, symbolizing, producing solutions, and comprehending language. Stenger (1999) viewed mathematical thinking as skills in conceptual and procedural knowledge, reasoning, strategy used and written solution process while Posamentier and Stepelman (2002), viewed mathematical thinking as process through direct experiences, reasoning, problem solving, exploring and communicating. Mathematical thinking is also viewed as a process consisting of highly complex activity in solving problems (Stacey, 2007).

1.5 Mathematical Thinking as Process and Behaviour

Information gathered from the contemporary views and components for mathematical thinking obviously linked mathematical thinking as process and behaviour in solving problems. Considering mathematical thinking as process, Ginsburg, Jacobs and Lopez (1998) had earlier seen the importance of creating thinking oriented classroom. Through thinking oriented classroom, students could construct their own learning instead of being told about related concepts or skills. According to Hiebert and Carpenter (1992), mathematical knowledge gained through active engagement of mathematical thinking process could be easily understood and remembered by student. Therefore, a very important goal in the thinking classroom is that students learn to reflect their own learning experience. Thus it is important to understand how students solve mathematical problems because solving mathematical problems are related to thinking process, mathematical skills and cognitive development (Chamberlin, 2002). Solving mathematical problems enables individuals to create new knowledge, build ideas and create connections within ideas.

1.6 Mathematical Thinking as Process in Solving Problem

Most of the contemporary views above have regarded mathematical thinking as a process in problem solving. Lesh and Zawojewski (1992) have defined a problem as a situation in which the solver wants to find a solution and for which the solver does not have an immediate means at hand to find a solution. It means problem solving is a directed process intended to find a solution when no obvious method is available. Problem solving as a process is completely internalized (Newell & Simon, 1972). In order to make any inference on the internalized information, it is necessary to make the internal processes explicit or by observing behaviour (Gavin, 1998). Posamentier and Stepelman (2002) stated that problem solving should become an integral part of students' mathematical thinking processes such as: (a) finding a pattern, (b) making a drawing (visual representation), (c) organizing data and (d) logical reasoning. According to Anghileri (1995) and Pandey (1990), problem solving is known as the heart of all mathematics. The problems posed will allow spacious space for the development of mathematical thinking. According to Anghileri, when solving problems, pupils are involved in creative processes such as searching for alternative methods and experimenting with different ways of communicating mathematical ideas. It provides content and process in daily life. Content consisted of concept, principle, problem, fact and definitions while 'process' consisted of thinking skills, thinking strategy, meta-cognition, process skills and social skills. Anghileri's view supported Pandey's as he stated that focus should not only be given on the content standards but also on the processes that are the heart of mathematics.

Many educators believed that by encouraging students to develop and construct their own solution processes will allow them to think mathematically and develop their mathematical thinking and mathematical understanding. These would involve transformation in the instructional style such as seeking solutions and exploring patterns and not just memorizing procedures and formulas. Thus it would require knowing how to solve problem with understanding. Knowing how to solve problem with understanding is more important than only acquiring information (Koay, 2007). Skemp (1978) referred solving problems with understanding to 'knowing how without why' (instrumental understanding) and ‘knowing how and why’ (relational understanding). Skemp’s view of solving problems with understanding could be adapted to Hiebert and Lefevre (1986) definition of conceptual understanding and procedural knowledge. Hiebert and Lefevre defined conceptual understanding as ‘knowledge that is rich in relationships’ and procedural knowledge as ‘formal language or symbol representation and algorithms’ for completing mathematical tasks. As for Hiebert and Carpenter (1992), understanding mathematical ideas, procedure or fact involves recognizing relationships between pieces of information that build internal networks of connected knowledge.

However students often have misconception that solving mathematical problems requires memorizing procedures and formulas and doing exercises (Suzuki, 1998). According to Suzuki, students should move from solving mathematical problems based on memorized rules to mathematical explanation which requires them to express mathematical understanding, and justify a solution or decision in the language of mathematics. Therefore, in solving mathematical problems students should be able to communicate and provide explanation of their mathematical thought while generating the output. Ability to transform and communicate mathematical thoughts could extend their mathematical thinking and help to explain conceptual understanding (Cheah, 2007). Therefore this study focuses on assessing and identifying students’ mathematical thinking levels that would reflect whether students had constructed their own solution processes with understanding or sense making when they were given the chance to exhibit their conceptual understanding, transforming of mathematical ideas, performing computations and providing mathematical explanation in their written works. Therefore, in this study students' mathematical thinking levels would be assessed based on their thinking process and behaviours reflected in their written works as according to Namukasa (2005), thinking as process and acting are inseparable.

1.7 Assessing Mathematical Thinking

As mentioned earlier, solving mathematical problems involve students engaging in different ways of communicating mathematical ideas that provide content knowledge and solution processes (Anghileri, 1995). This would require students to demonstrate their mathematical thinking processes and provide explanation to their mathematical ideas. Furthermore, today's school children need to learn to reason and communicate using mathematical ideas (Schoenfeld, 2002). According to Schoenfeld, mathematics assessment should not merely focus on the giving of the right or wrong answers but must focus on children's ability to understand mathematical concepts and application of concept, procedural, relationships, and reason logically. These are the process involved that help to display the thinking process applied by students. The right answer may not imply understanding because it may indicate mere memorization or parroting. At the same time, the wrong answer may not imply the lack of knowledge. Therefore it is important to look for the meaning underlying students' written work.

Many educators believed that by encouraging students to develop and construct their own solution processes will allow them to think mathematically, develop and communicate their mathematical thinking (Pugalee, 2004; Schoenfeld, 1985). As for Pugalee, learning to provide written solution is very useful for teachers to assess how well students express their thinking, strategies they employ, the quality of their answers and the time required to think when given a problem. This can inform and improve the quality of students' learning and mathematical thinking in the classroom as well as in developing students' ability to demonstrate their mathematical thinking processes.

However, Pugalee (1997) stated that students often require time to think when given a problem before they were able to gain insight into possible solutions. Furthermore, time required by the students will be different because of the individual differences in cognition. For example, there are students who can make decisions more quickly, can perceive relationships between variables more clearly and can think more deeply than others. Differences in students' abilities of learning mathematics resulted differences in students' mathematical performance (Pyke, 1998). Therefore, the individual differences in thinking can best be understood by appreciating how they communicate and display their thought processes (Bjorklund, 1989).

1.8 Issues in Mathematics Education

Following are issues and challenges in promoting mathematical thinking in mathematics education from the aspects of classroom and assessment practices.

1.8.1 Current Classroom Practice and Achievement

Solving mathematical problems with understanding can be a powerful tool for teachers to recognize and understand students' conceptual understanding and procedural fluency underlying mathematical thinking (Schoenfeld, 1985). The National Council of Teachers of Mathematics (NCTM, 2000) stressed the importance of building mathematical understanding by stating that "learning with understanding is essential to enable students to use what they learn to solve the new kinds of problems they will inevitably face in future" (p.21). According to Schoenfeld (2002), teaching and learning practices in the classroom is closely related to building individual with mathematical knowledge, and mathematical thinking. These would require students' ability to think, apply mathematical knowledge in solving problem, present knowledge and communicate either verbally or non-verbally.

However, report of Trends in International Mathematics and Science Study (TIMSS) by Mullis et al. (2003; 2008) showed that the approaches practiced in the mathematics curriculum of schools in Malaysia are still towards the collection of facts and mastering the basic skills in mathematics instead of mathematical understanding. TIMSS –R (Ministry of Education, 2000) and TIMSS 2003 (Mullis, et al., 2003) reported that Malaysian students found lecturing to be the main mode of instruction in the classroom followed by teachers modeling how to solve mathematics problems correctly. Students' roles are to listen to their teachers' explanations. Teachers too stated that students should follow rules to solve problems rather than explaining the rule or reasoning for their answers. Teachers' main focus is still on drilling, applying formula and finally producing the correct answers. These reports reflected the practiced of procedural knowledge or instrumental understanding in the mathematics classroom.

The 2003 and 2008 TIMSS reports also indicated insufficient focus had been given in developing students' explaining and reasoning abilities in mathematics. There was inadequate guidance to allow students to discover and correct their misconceptions. These might be the reasons why the Malaysian Grade Eight participants' average percent accuracy in all five mathematics content areas as reported in 2003 TIMSS is significantly lower compared to the 1999 TIMSS. Table 1.1 shows the difference in average percent accuracy for five mathematics content area assessed during the 1999 and the 2003 TIMSS. These could be the reason why Malaysia has had successively lower average achievement with each assessment since 1999. Malaysia average achievements also continuously decline since 1999 (Mullis et al., 2008). Even though reports by TIMSS were related to Form Two students in Malaysia, it could also reflect similar practice at the primary schools due to the centralized curriculum and centralized teacher training programme. In this present study, researcher intends to analyze primary students' ability in making connections among the various pieces of information in their attempts to make sense of their solution processes in different tasks.

Table 1.1
Summary of Percent Correct Answers from TIMSS-R 1999 and TIMSS 2003 for Grade Eight Participants in Malaysia.

	No of items	1999 TIMSS-R % correct	2003 TIMSS % correct	Difference
Number	25	62	57	-5
Algebra	16	46	42	-4
Measurement	16	51	45	-6
Geometry	12	53	51	-2
Data	10	68	67	-1

1.8.2 Current Assessment Practice in Mathematics

In the *Ujian Penilaian Sekolah Rendah* (UPSR) (*Primary School Evaluation Test*) students are required to answer 40 multiple choice questions (MCQ) within one hour in Paper One and 20 questions that requires short answers within 45 minutes in

Paper Two which contributes 60% and 40% respectively to the overall mathematics score. The MCQ in practice typically test for students' fact-based and procedural knowledge which require direct application of concepts and computation skills. On the other hand, the type of tasks consist in Paper Two require students to perform some computations, arrive at the answer, and write the answer in a box or space provided. Typically in order to earn full credit on mathematics questions in Paper Two, students need to perform accurate computations that arrive at the correct answer, and write the answer in the space provided. Usually in UPSR assessment, teachers focus on 'compute fast to get the right answer'. Students are not required to explain or justify the procedure that they used in solving problems for the final answers unless they are required to do so. Questions that emphasized on selecting the correct answer without inquiry may have limited measures of students' mathematical concepts, mathematical relationship and explanations. It may not help in the understanding of how students think.

The fact that students are able to do mathematics without truly understanding its basic logic and concepts is a great concern for many educators (Noraini, 2006). Assessments in practice are not in line with the curriculum that emphasized on the ability of students to think mathematically, to understand and communicate detailed levels of solution processes. These assessments do not reinforce students' understanding of the solution processes for final answers since they are not required to clarify and justify their approach to a particular question. They too were not given the chance to communicate their mathematical thinking or mental processes. As stated earlier, focus should not only be given on the correct answer but also on the processes that are the heart of mathematics (Pandey, 1990).

1.9 Purpose of Assessment

Malaysia primary school classroom practices are mostly inclined to summative assessment. This kind of assessment is used for the purpose of grading students' achievement and placement within the school. Students' scores in mathematics act more like a filter for classifying and comparing students' achievement and performance. The classroom assessment mimics the purpose of assessment observed by the national standardized examinations or public examination. The UPSR results have been used solely as the basis for selection and grading of students and for placement into selected residential schools such as science residential schools and MARA Science Junior Colleges (MRSM). Hence, this examinations results have become increasingly high-stakes due to such important decision making (Cheah, 2010). The results from the national standardized examinations too have been used to determine performance of schools. Therefore, it is common for the school administrators to use students' results as a yard stick to evaluate teachers' teaching (Lim, 2006). Consequently, this would strengthen teachers focus on drilling so that students are able to obtain excellent results in their examinations at school or national level.

Hence, the norm reference test and evaluation systems in schools are more on stimulating and promoting students to perform well in examinations. Even though the standardized test provides important feedback about students' performance but it might lead to misinformation on student's actual mathematical thinking performances because children vary widely in their way of thinking (Bjorklund, 1989; Pyke, 1998; Sternberg, 1994). Scores obtained by students do not reveal a clear picture or explain to teachers the various levels of mathematical thinking performances of the students. It is important to realize that students are not all of the same abilities (Pyke, 1998). Bjorklund's (1989) analogy of children's thinking has been compared to fingers of the right hand. He stated that "No two fingers are alike" (p200). This reflects the individuality of children. They learn and assimilate knowledge differently and at varied pace. They even have diverse learning styles. With the uniqueness of students' characteristics, it is important to identify students' level of mathematical thinking domains. It is hope that this will help to provide a better picture of individual differences specifically with regard to their mathematical thinking domains.

1.10 Problem Statement

Primary education has been the first step in formal education processes in Malaysia. Therefore the elements in the Integrated Curriculum Syllabus for Primary Mathematics (ICSP) are of great importance. Since the adoption of ICSP, emphasis has been given on the mastery of number concepts, and the understanding and applications of the basic computation skills of mathematics (Ministry of Education, 2003). The ICSP required the development of mathematical thinking elements in the process of teaching primary mathematics as it places more emphasis on problem solving, communications, mathematical reasoning, connections and representations. However the practices by teachers in the classroom contradict with the intended requirement of the new syllabus (Cheah, 2007; Jamaliah, 2001; Ruzlan, 2008). The primary mathematics curriculum recommends various activities with regards to the mathematical thinking elements however it did not provide specific examples which teachers can use as guidelines in preparing mathematics lesson plans (Cheah, 2007). Hence, most teachers revert back to traditional approaches despite the reform syllabus that required the development of thinking skills in the teaching of primary mathematics.

Research by Jamaliah (2001) found that schools allocated five to ten minutes of each mathematics lesson for doing mental calculation. Focus was on memorization and recall of basic facts. Jamaliah's study reported that Malaysian primary mathematics classroom teaching demonstrated a more traditional teaching style when she indicated that, "the teacher will present the day's lesson in the form of giving task to get the right answers, or present a brief explanation of the topic through examples, either taken from textbooks or workbooks, followed by drill exercises" (p.164). This was supported by earlier research conducted by Lim and Chan (1993). Their research found that the practice in the mathematics classroom was mainly drilling that focused on memorizing the content and formula and not on the thinking process. Pupils were not required to engage in high-level mathematical thinking tasks or challenging tasks. Noraini (2006) also has the view that memorization has been the aspect being emphasized in teaching mathematics. Teachers have been giving more attention to their students' ability to get answer quickly which required students to memorize, recall and apply routine procedures.

Research by Ruzlan (2008) revealed that primary school teachers' lesson plans were segmented into four distinct phases, namely consolidation, core-content, rehearsing and lesson closure. Ruzlan's findings on primary schools mathematics teachers' actions within their classroom showed that in all the four phases of the lesson plans, whole class teaching tended to be dominated by teachers. Teachers gave a brief review of definitions on specific topic followed by computational modeling of procedural knowledge. Pupils were generally inactive and discussion seldom occurred unless teachers gave tasks that required pupils to respond. The type of tasks appeared to be requiring only pupils' ability to recall the related procedures used to solve the worked examples rather than encouraging them to think. The questions framed by teachers were typically 'closed ended' type with closed procedural questions forming between 50% and 60% that exercise on component skills. There was almost complete absence of open-ended questions. The main focus of primary mathematics classroom teaching seemed to be on procedural competence and less concerned with the constructions and transfers of mathematical knowledge. Development of pupils' mathematical thinking was within a narrow frame due to the used of close procedural tasks used by the teachers (Ruzlan, 2008). Research by Ruzlan also revealed that primary school classroom teaching always ends up with teachers doing most of the talking despite being exposed to new curricular concepts and student-centered teaching approaches.

Teaching practices and mathematics classroom scenario revealed by Ruzlan (2008) and Jamaliah (2001) were supported by the Trend International Mathematics and Science Study (TIMSS). The students involved in TIMSS were from lower secondary schools and findings were based on lower secondary teachers' activities in their mathematics classroom, but this could provide the scenario of the primary mathematics classroom too since students participated in the study had spent six years learning in primary schools and only two years in secondary schools. In the Third International Mathematics and Science Study-R (TIMSS-R), (Ministry of Education, 2000), it was shown that 88% of Malaysian teachers' practice was focusing on students mastering of computation skills. TIMSS –R report reflected the practice of procedural knowledge or instrumental understanding. It was also reported that 92% of Malaysian teachers focused on showing rules to solve problems rather than explaining the rule or having students explaining their reasoning for the solution processes. Lecturing and listening to teachers explanations has been the main mode of instructions in the classroom followed by teachers demonstrating how to solve mathematics problems correctly (Mullis, et al., 2003; 2008). Only 23% of Malaysian mathematics teachers asked their students to give reasons during their solution processes. TIMSS-R report (Ministry of Education, 2000), TIMSS 2003 and 2007 (Mullis, et al., 2008) showed insufficient focus had been given in developing students' explaining and reasoning abilities in mathematics. This illustrated that mathematics classroom teaching did not emphasize students' ability to think mathematically, to explain ideas in solving problems, to link conceptual to procedural knowledge and to develop reasoning. These mathematical thinking elements in ICSP have not been developed in the primary mathematics classroom.

Another important constraint to assess mathematical thinking elements in the primary mathematics classroom was the examination-oriented culture that reinforced teachers' belief about teacher-centered classrooms and procedural competency (Lim, 2006). This typical classroom scenario further enforced teachers' beliefs that their teaching priority is to give clear explanations to students rather then allowing them to construct their own mathematical thinking processes so as to ensure students obtaining excellent results in the examinations. Students' performances in examinations have been used as the sole yard sticks for teachers to measure mastery on the mathematical concepts or skills. Teachers' responsibility was to make sure that they covered all the content area in the syllabus. Due to emphasis on examination results, teachers' belief and time constraint, students have undergone learning mathematics with a lot of drilling and were exposed to mostly MCQ before UPSR. The test results were mainly used to classify individuals and ranking them. The importance of the standardized tests results cause teachers to ignore students' conceptual and cognitive understanding that in turn leads to students' despair in mathematical learning when they obtained low scores or grades.

Several researches with regard to basic mathematics problem solving through memorization of algorithm, application of rules and routine procedures were carried out by Biggs (1990), Booker (2005), Chacko (1999), Lim and Chan (1993) and Noor Azlan (1996). Koay (2007) conducted a research that assessed the journey of pre-service teachers' solution path as they discussed and reacted to tasks given till they arrived at the answers. Similar research to assess meta-cognitive thinking of secondary schools students' solution path has also been examined by Lee (2002). Recent research by Ruzlan (2008) observed the relationship between teaching and learning and its association on primary students' knowledge and skills in mathematics classroom.

All the studies reported above consistently pointed out that students were not given the chance to demonstrate their thinking processes either in the form of writing or answering teachers' tasks in mathematics classroom. Students habitually worked on routine exercises that were limited to close-ended tasks and multiple choice questions. There were hardly any tasks that tested pupils' thinking abilities (Jamaliah, 2002;

Mullis et al., 2003; Ruzlan, 2008). Researches looking into students' performances in mathematical thinking domains through analyzing features of written works produced by the students were rarely being carried out. Thus, this study will adopt constructed response items to assess and identify primary students' mathematical thinking levels and performances reflected from students' behaviours transformed in written works, reflection sheets and interviews. Students' verbal and non-verbal responses would reflect whether they had solve problems with understanding or sense making when they exhibited their conceptual understanding and, transform mathematical ideas into different mode of representations, performing computations and use the language of mathematics to provide explanation on decision made.

1.11 Rationale of the study

In higher education, thinking skills are widely use not only in mathematics but also in other fields such as physics, economics and environmental science. Students thinking skills are needed when dealing with geographical matters, world affairs, history, social and cultural affairs (Pakade, 1996). According to Schoenfeld (2002) thinking skill such as making decision in one's personal life, on the job, and in matters of public interests calls increasingly for quantitatively sophisticated reasoning. Even for social studies it is important to help students to think, form accurate concepts and generalizations and make informed judgments that will be useful to students in understanding events and conditions in other countries. More than ever before, today's students need to learn to reason and communicate using mathematical ideas. Tertiary level textbooks such as "Calculus" by Anton (1999) and "Thinking Mathematically" by

Blitzer (2005) let students explore more real world phenomena as exercise problems. These books present real world problems as examples after introducing a concept and as exercise problems at the end of each chapter. Students should be able to develop heuristics of problem solving as part of thinking process. Genuine problems such as designing a bridge, and determining moon eclipse require abstract formulation of ideas on the part of the problem solver (Kline, 1973). The need for strategy in solving such problem requires the gathering of information not given as part of the problem. It also requires reasoning processes to infer and deduce facts that are tied closely to the study of the physical world. Eventually students will have to give intelligent reasoning about its possible solution.

There is significant discontent among the population in general with the lack of creativity in the schools and the difficulty that individuals have in expressing their own ideas (Kazuko, 2005). Therefore, study on primary students' mathematical thinking processes is important to gain information related to students thinking ability and their levels of performance in various mathematical thinking domains such as conceptual understanding, mathematical representations of ideas, procedural fluency and abilities to explain answers or decision made. It is important to ascertain students' proficiency in the various mathematical thinking domains and to examine the association between mathematical thinking domains and types of tasks associated to students' levels of mathematical thinking performances. This information can help teachers to predict students' performance in relation to various mathematical thinking domains and the domain of thinking that students are proficient or not proficient. In addition, teachers can utilize information provided in this study to improve instruction in primary mathematics classroom as well as to help sequencing of mathematics contents in a more effective manner to prepare students to solve real life mathematical problems.

1.12 Research Objectives

Mathematical thinking has been regarded as a process that involves active engagement of students in solving problems (Siegler, 1991; Posamentier & Stepelman, 2002; Stacey, 2007). Mathematical thinking has also been viewed as skills in solving problems (Thompson, 1971; Stenger, 1999). Mathematical thinking has also been defined as the ability to think while solving problems (Katagiri, 2004).

In Malaysia, mathematical thinking is generally recognized as one of the important components in the teaching and learning of mathematics in Primary Mathematics Curriculum (Ministry of Education, 2006). In fact, Malaysia classroom teaching and learning process of mathematics in primary school has been emphasizing conceptual understanding, mastery of computation skills, competence in explaining mathematical ideas, and presenting mathematical ideas through writing in solving problems. Writing solution processes in problem solving is an important focus of instructions that provides opportunity for students to acquire ways of thinking, exhibit their conceptual understanding, presenting mathematical ideas in various modes, producing solutions and enhance reasoning abilities using mathematical languages. Writing as communication tool to assess students' level of mathematical thinking with regards to making the mathematical thinking apparent at primary school level were not being widely practiced. Writing solution processes can be used as catalyst in the learning process to explore and analyze students mathematical thinking (Noraini, 2006;

Pugalee, 2005). This would expose how students express their thought processes using mathematical language rather than merely perform routine computations with correct or incorrect answers. Students of high and low achievers mathematical performances could be compared based on different mathematical thinking domains. This would require an assessment process that is able to describe students' mathematical thinking levels of performance for multiple mathematical thinking domains. Thus, assessment rubrics provide directions for teachers in assessing written responses and activities by specifying and describing indicators that would help teachers to make decisions about students' levels of performance (Pugalee, 2004).

Therefore it is important to make mathematical thinking apparent at primary school level through written responses in problem solving. It is also important to understand how students solve mathematical problems since according to Chamberlin (2002), solving mathematical problems are related to thinking process, mathematical skills and cognitive development. In this study, mathematical thinking is viewed in four mathematical thinking domains: conceptual understanding, mathematical representation, procedural fluency and mathematical explanation exhibit in students' written and oral responses. Analytic rubrics for constructed response tasks that describe these four domains were used to identify students' mathematical thinking levels.

Therefore, the purpose of this study is to identify Year Six students' mathematical thinking levels in relation to conceptual understanding, mathematical representations, procedural fluency and mathematical explanations that the students exhibit in their writing while solving constructed response tasks. This study also intends to determine the relationships among all mathematical thinking domains as reflected in students' responses and the effects of complexity of tasks on students' mathematical thinking levels. The low and high achievers were also compared by examining their written works of the solution processes as well as interview responses, to illustrate differences of mathematical thinking profiles when solving tasks of different complexity. In addition, students' responses in the reflective sheet were used to understand the omissions of certain skills in students' answer scripts, their approach in solving each task and to determine their existing content knowledge related to the tasks.

1.13 Research Questions

The research questions for this study are:

1. What are students' levels of mathematical thinking?
2. What are students' levels of performance for each mathematical thinking domain?
3. Are there significant relationships among the different mathematical thinking domains?
4. Do complexity of tasks affect students' mathematical thinking domains?
5. What are low and high achievers mathematical thinking levels for each mathematical thinking domain?
6. Do complexity of tasks affect low and high achievers' mathematical thinking levels?
7. What are the mathematical thinking profiles of the low and high achievers?