DYNAMIC STIFFNESS ANALYSIS OF ENGINE RUBBER MOUNTS

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BY

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TABLE OF CONTENTS

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ANALISIS DINAMIK KEKAKUAN PELAPIK GETAH ENJIN

ABSTRAK

Kekakuan dinamik dan faktor kehilangan pelapik getah enjin merupakan sifat dinamik penting yang digunakan untuk mewakili prestasi sistem pelapik enjin. Pengukuran redaman dan kekakuan memainkan peranan penting dalam pengukuran sifat dinamik untuk sistem pelapik getah enjin. Ujian impak merupakan teknik pengukuran yang mudah dan baik untuk pengukuran redaman. Perkembangan terkini ialah ujian impak untuk mengukur kekakuan dinamik titik memacu. Dalam kajian ini, ujian impak dikembangkan dengan lebih dalam untuk mengukur kekakuan dinamik pindaan dimana tukul impak digunakan untuk menggantikan penggoncang sebagai sumber untuk menggocangkan sistem tersebut. Rumus matematik dan prosedur eksperimen untuk mengukur kekakuan dinamik titik memacu dan kekakuan dinamik pindaan pemindahan untuk pelapik enjin dengan menggunakan uji impak dibentang. Hasilnya menunjukkan bahawa kekakuan dinamik titik memacu hanya boleh digunakan untuk mewakili kekakuan dinamik pindaan untuk frekuensi yang lebih rendah. Jisim antara pengasing dan transduser masukan akan mempengaruhi had frekuensi ini. Faktor kehilangan pelapik enjin juga dianggarkan dalam fungsi frekuensi. Anggaran fungsi untuk faktor kehilangan (η_{11}) menunjukkan sistem pelapik enjin bergantung secara linear kepada frekuensi dan faktor kehilangan (η_{21}) menunjukkan elemen bergantung secara tidak linear kepada frekuensi. Dengan menggunakan anggaran faktor kehilangan ini, kekakuan dinamik titik memacu dan kekakuan dinamik pindaan dihasilkan semula dengan tepat dan dibandingkan dengan keputusan yang diukur. Keputusan yang diperolehi dengan menggunakan ujian impak akhirnya disahkan dengan keputusan yang diperolehi dengan menggunakan penggoncang.

DYNAMIC STIFFNESS ANALYSIS OF ENGINE RUBBER MOUNTS

ABSTRACT

Dynamic stiffness and loss factor for engine rubber mount are important dynamic behaviour to represent the performance of an engine mount system. Damping and dynamic stiffness measurement play important role in the characterization of dynamic properties for the engine rubber mount system. Impact test is simple and powerful measurement technique for damping measurement. The recent development of impact technique is the application on dynamic driving point stiffness measurement. In this thesis, the impact technique is further developed for dynamic transfer stiffness measurement where the impact hammer is used to replace shaker as the source of excitation in damping measurement. The mathematical formulation and the experimental procedures to measure dynamic driving point stiffness and dynamic transfer stiffness for engine rubber mount by using impact test are presented. The results showed that the dynamic driving point stiffness can only be used to represent dynamic transfer stiffness for the lower range of frequency. The mass between the test isolator and the input force transducer will influence this range of frequency. The loss factor of the engine mount is also estimated in the function of frequency. The curve fitted function of loss factor (η_{11}) showed linear dependency of the engine rubber mount system on the frequency and loss factor (η_{21}) showed non-linear dependent of the resilient element on the frequency. By using this curve fitted loss factor, the dynamic driving point stiffness and dynamic transfer stiffness are accurately reproduced and compared to the measured results. The results obtained by using the impact technique are finally validated with the results obtained by using shaker.

CHAPTER 1
INTRODUCTION

1.0 Brief introduction

Engine vibrations are normally unwanted disturbance which will influence passenger comfort. Over the years, researchers have put a lot of effort to solve the problem. Engine mounting system is needed to act as vibration isolation system to isolate unwanted dynamic motion to improve passenger comfort. There are three vibration isolation systems for engine vibration reduction. These are active isolation, semi- active isolation and passive isolation. Engine rubber mounts system is one of the passive isolation which is the focus of this study.

The engine is always connected to the chassis by rubber mounts. Dynamic analysis of engine rubber mount is important to determine the effect of vibration transmitted from engine to the structure. The development of dynamic model for engine mount is needed so that response at speed near the system natural frequencies can be captured accurately. Engine mount which function as vibration isolator can reduce the response of the vehicle at low speed which is near to the system natural frequencies. In this sense, the design of engine mounting system is critical. (Ashrafiuon et al., 1992)

Snyman et al. (1995) analyzed the vibration transmission force from engine to the structure of a four cylinder engine mounting system by developing a mathematical model which describes the motion of the mounted engine with the individual balancing masses and the phase angles as design variables. The objective function was the motion at the engine mounting positions.

A review of the development of automotive engine mount system was presented by Yu et al. (2001). The study explained the function of engine mount system as vibration isolator to isolate engine disturbance force to passengers. The importance of considering dynamic stiffness and damping as frequency and amplitude dependent properties for engine mount were pointed out. Recent development of engine mounting systems has concentrated on improvement of the characterization of these properties. The discussion by the authors included the conventional elastomeric mount. The advantages of semi-active and active isolation were also listed together with optimization work for engine mounting system. The limitation of the optimization work and the need to include nonlinearities and variations in the elastomer properties for different types of mounting systems were also highlighted.

Sjoberg et al. (2003) studied the function of engine rubber mounts as vibration isolator to reduce transmission of engine vibrations to the chassis while the vehicle is driven through a rough road surfaces which will induce large movement of the engine. The nonlinear dynamic properties were also measured. The authors found that single harmonic excitation performance has strong amplitude dependence. Payne effect where stiffness is high for small excitation amplitudes and low for large amplitudes while damping displays a maximum at intermediate amplitudes was also demonstrated.

In certain application, rubber engine mount may need to operate in frequency as high as 5000 Hz. Commercial high frequency test machines are only capable of testing at frequencies up to 1000 Hz. In order to understand the dynamic properties of rubber at high frequency, Vahdati et al. (2002) proposed a high frequency test machine which can study the performance of engine rubber mount at frequencies up to 5000 Hz. The related mathematical model related to high frequency was also described.

Lin et al. (2005) presented an experimental method to measure the frequency dependent stiffness of engine rubber mount by applying the impact test. The results showed the stiffness as a function of frequency is a better representation of the properties of engine mount compared to constant stiffness. The constant stiffness can only predict the response at resonant frequency but not for non- resonant frequency.

In this research, impact technique is applied to measure the dynamic transfer stiffness of engine rubber mount and compared with the measurement of dynamic driving point stiffness. The estimation of loss factor in function of frequency is also carried out.

1.1 The engine rubber mount

Solid engine rubber to metal mount with a diameter of 15mm and 20mm length is selected as the resilient element test object for this experimental investigation as shown in figure 1.1. These engine rubber mounts are used for mounting of a 2.72 kg small utility two stroke engine of a backpack type grass trimmer as shown in figure 1.2. Total three engine rubber mounts from the grass trimmer are taken and used in this study which is in the same condition as the installation in the grass trimmer.

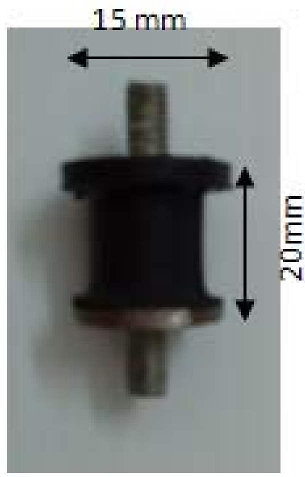


Figure 1.1: Engine rubbers mount from grass trimmer

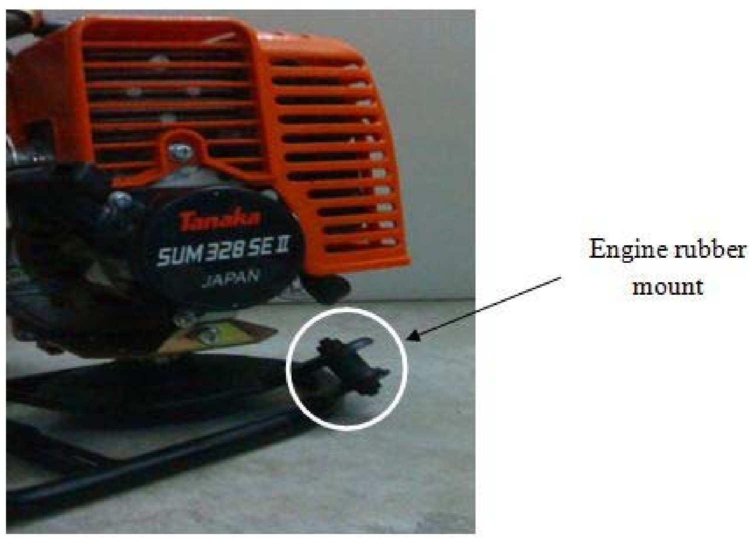


Figure 1.2: Mounting of engine rubber mount on grass trimmer

1.2 Motivation of the work

The research work on the dynamic analysis of the engine mount is motivated by the need to better characterize of engine mount to improve its performance. The application of impact technique on the damping measurement of engine mount is relatively new. Since the impact technique is simple and more economical, successful development of impact technique can help to measure performance engine mount in real application condition.

1.3 Objectives

There are three main objectives to be achieved in this study:

- To further develop the application of the impact technique for the dynamic transfer stiffness measurement.
- To compare the behaviour of dynamic driving point stiffness and dynamic transfer stiffness.
- To measure and analyze the dynamic properties of engine rubber mount.

1.4 Contributions

There are two main contributions of the overall of research:

- The developments of impact technique for engine mount system in the dynamic transfer stiffness measurement.
- The determination of the frequency range where the dynamic driving point stiffness can be used to represent dynamic transfer stiffness is figured out.

1.5 Thesis outlines

This thesis contained five chapters which include the introduction, literature review, methodology, results and discussion and also the conclusions from the research. The first chapter presented an overview of the thesis by giving brief introduction, motivations, objectives and also the contributions of the research. In Chapter Two, the review of engine rubber mount model and the impotence of damping in the vibration isolation system are presented. The review of recent development of the study of dynamic characterization and some existing damping measurement method are also summarized. Chapter Three listed down the methodology of the experimental and analytical procedures to achieve the thesis objectives. Subsequently, Chapter Four presented the results from the study with some discussions. Finally, the Chapter Five concluded the outcome from the research work.

CHAPTER 2
LITERATURE REVIEW

2.0 Overview

This chapter of literature review presented four main areas which are listed as below:

- Model of engine rubber mount
- Damping properties in vibration isolation system
- Dynamic characterization of engine rubber mount
- Damping measurement method

2.1 Model of engine rubber mount

Rubber is an elastomer which combined both solid and liquid properties (Gent, 2001). The response from these neither elastic solid nor ideal liquids material is called viscoelastic behaviour. Traditionally, this viscoelastic behaviour is demonstrated by employing combinations of elastic and viscous elements such as Maxwell and Voigt models elements. Maxwell element is defined as the dashpot coefficient and the stiffness of element is added in series. On the other hand, the Voigt element is defined as the sum of two effective stiffnesses (dashpot coefficient and stiffness of element) in parallel. The standard model which combined the Maxwell and Voigt elements is also been widely used (Jones, 2001). However, most material exhibits more complexity in material properties in the real case.

Mundo et al. (2005) used Standard Voigt model to represent the viscoelastic model in dynamic parameter identification for automotive rubber connection. Finite element modelling approach was used as a modelling tool to evaluate dynamic complex stiffness of this rubber connection. An experimental method to investigate the dynamic properties was also proposed. The results showed that the stiffness varied by linearly within 0 - 300 Hz range.

Zhang et al. (2006) studied modal analysis of a single mass elastomeric isolation system by Maxwell-Voigt model. Three types of Maxwell elements are defined by different time constant. Figure 2.1 showed the existence of the Maxwell element in the Maxwell- Voigt model will caused the response in natural frequency greater than a Voigt model which has equivalent Voigt elements as the Maxwell-Voigt model. It means that Voigt and Maxwell- Voigt model may have different frequency response function although their natural frequency is the same.

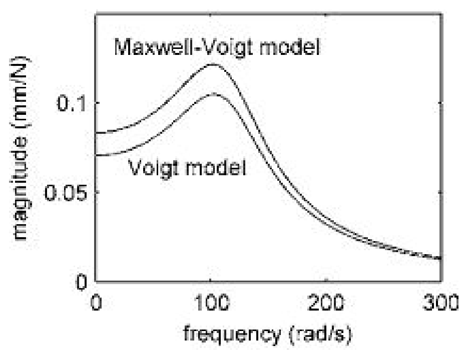


Figure 2.1: Frequency response spectrum for Maxwell-Voigt and Voigt model (Zhang et al., 2006)

Hysteresis damping is one of the damping models used to study energy dissipation mechanisms. The hysteresis damping for single degree of freedom system with free vibration in time domain was solved by Chen et al. (1994) by using the concept of phase plane. The study by Chen et al. was focused on linear time varying system. Crandall (1995) commented this model for not able to fully represent the hysteresis damping model in frequency domain, later Chen et al. (1999) solved time domain governing equation for harmonic loading by applying the direct iteration technique. The hysteresis loop was also generated with the time-domain approach. The results of Chen et al. (1999) improved the hysteresis model and this proved by the results from the governing equation for frequency domain and the time domain approach having close agreement.

Symen et al. (2002) characterized the dynamic behaviour of hysteretic elements for a mechanical system by considering the single degree freedom system of a mass supported on a non linear hysteretic spring. Different researchers (Kucherskii, 2005; Gounaris, 2007; Li, 2007) have extended and applied the hysteresis damping model to different applications or study such as carbon black filled rubber and vibration absorber. Most recently, Banks (2008) gave a summary of survey of the development in hysteresis for viscoelastic materials.

Although elastomeric components are commonly used in vibration isolation system, the finite element model analysis of elastomeric components in automotive industry is not yet common. This is because of the lack of material properties at higher frequencies. For this purpose, Lu (2006) proposed a fractional derivative viscoelastic model which can be used to characterize frequency-dependent complex modulus of automotive elastomers at high frequencies. This model can accurately predict complex modulus of elastomers at high frequency.

2.2 Damping properties in vibration isolation system

Damping helps to remove energy away from system and reduce the vibration levels. Effect of damping on natural frequency of linear dynamic system was first studied by Caughey et al. (1961) and provided some fundamental knowledge of damping in vibration theory where damped natural frequency of higher modes are always less than or equal to the undamped frequency. However, this is not true for the lower modes. At the lower modes, the damped natural frequency may either be greater or less. Different type of damping matrix used will produce different results.

In order to present damping and complex stiffness in correct expression, the concept of complex stiffness for viscous and hysteretic damping was proposed by Neumark (1962) for a single degree freedom system where different case for damping i.e harmonic oscillations and decaying oscillations are reexamined and compared. The importance of each different kind of oscillations related to each type of damping was pointed out and related to its theoretical formulation. Crandall (1970) highlighted the mechanism of damping in the determination of dynamic behavior for a vibration system. The author defined damping as energy removed from vibration system which is either transmitted from system or dissipated within system. Damping was then defined as the ratio of the energy dissipated in a cycle of single frequency vibration to the total potential energy stored in the system during the cycle. Although the damping force is small compared to the elastic and the inertial force, it has a great effect for certain condition for example on the stability of the system.

Damping is an important materials property in vibration isolation. There are many types of damping related to different materials and also the mode of excitation. Some definitions and theoretical knowledge of damping are published (Nashief, 1984; Newland, 1989) as a reference so that the applications of damping in some mechanism are made readily available. Damping properties for engine rubber mount system help to remove energy from engine vibration. The proper understanding of these properties can help to improve the performance of the engine mount. The study and understanding for stiffness and damping are important not only for engine mount but for all others isolator and elastomer.

2.3 Dynamic characterization of engine rubber mount

Abdulhadi (1985) carried out an analytical and experimental study to estimate stiffness and damping coefficients a solid circular cylinder rubber pad. Electrodynamic shaker was used as the vibration exciter. From the measurement result, a mathematical model was developed and used to evaluate the stiffness and damping coefficients for the rubber pad. This study assumed that damping force is equivalent and proportional to first power of the rate of displacement. The authors found that stiffness and equivalent viscous damping coefficient were influenced by the preload mass. In the same study, the authors also solved a heat equation which described the temperature filed in the rubber by calculating the energy dissipated.

In order to get better mathematical model which can represent more complicated structures, Chen et al. (1996) proposed a method which can extract the mass, stiffness and damping matrix from the frequency response function. The authors reported that the existing method does not produce accurate damping matrix and as a result the dynamic behaviour of the system cannot be predicted accurately. The proposed idea by Chen et al. was novel since the estimation of damping matrix is independent of the mass and stiffness matrices. Significant improvement in the result proved the viability of the approach.

Soula et al. (1997) presented a method to evaluate both Young's and shear modulus for viscoelastic materials by using the transmission function. The possibility to measure both moduli of the same sample by using the same experimental setup for large frequency range was discussed. Two electromagnetic shakers are used as a source of excitation to the sample. Dickens (2000) proposed a static compression model to investigate a dynamic model of a vibration isolator under static load.

An improved method and new theoretical procedure for the damping matrix identification directly from measured frequency response function for general dynamic system is developed by Lee at al. (2001). A thin beam with clamped end is used in the study. In the same year, the authors developed another new method with simpler algorithm for more effective damping matrix identification. This new method used the dynamic stiffness matrix to produce fewer numerical analysis steps compared to the previous method. The new method also increased the accuracy in the damping identification from the measured frequency response function.

In 2003, Kari modelled and experimentally measured the preload dependent dynamic stiffness of vibration isolators in the audible frequency range and the results are shown in figure 2.2. In this figure, the dynamic driving point stiffness and dynamic transfer stiffness of an isolator were presented under different pre-compression displacement (preload). The results showed that the effect of the pre-compression displacement on the magnitude of dynamic transfer stiffness was only significant for the frequencies above 500 Hz but the effect on the dynamic driving point stiffness was significant for whole frequency domain. The direction of arrows showed the increases of pre- compression displacement from 0 to 12 mm with the increasing step of 1mm.

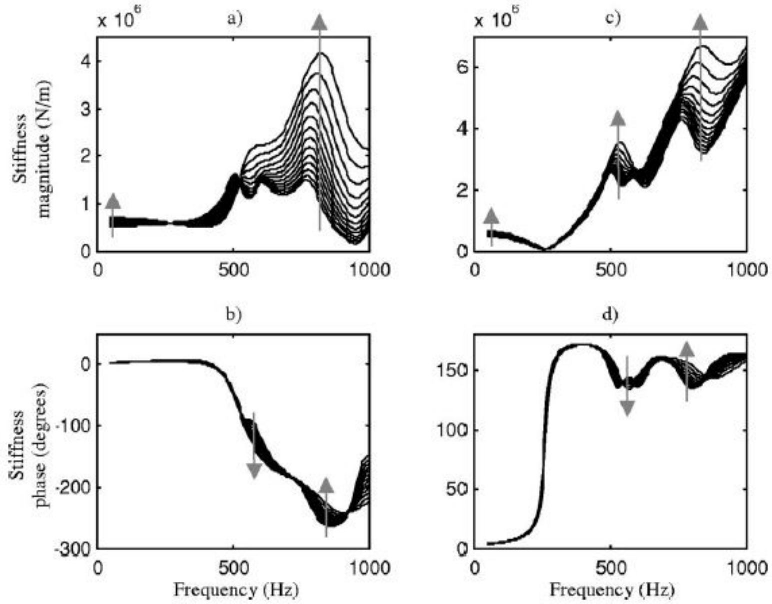


Figure 2.2: The calculated (a) and (b) transfer stiffness; (c) and (d) driving point stiffness under different pre- compression displacement (Kari, 2003)

Phani et al. (2007) analyzed parametric study of viscous damping identification method for linear vibration. Two general groups of the method which are matrix method and modal method are summarized. The authors introduced a new group called enhanced method which was an improvement of the matrix method. The authors also commented on the methods of Chen et al. (1996) and Lee et al. (2001) by reporting that Chen's method was a good fit to the damping matrix but Lee's method was only good for system with high damping but not for low damping. The authors also reported that their enhanced method can help to improve the performance of Chen's method at high noise level but unable to improve the performance of Lee's method.

Lang et al. (2009) performed theoretical analysis for single degree of freedom (SDOF) systems while considering the effects of nonlinear viscous damping on vibration isolation. The results showed that ideal vibration isolation can be performed by nonlinear viscous damping with resonant region modified by damping and non-resonant regions remain unaffected. It also implies that the vibration isolation level which can be performed by using active vibration isolator can be also achieved by the vibration isolator with non- linear viscous damping. This contributed to the design of viscously damped vibration isolators for a wide range of applications.

2.4 Damping measurement method

2.4.1 Hysteresis loops method

The experimental apparatus used to determine the hysteresis of rubber for forced non- resonant vibration at low frequency was introduced by Fletcher in 1951. In that setup, photoelectric pickups are used to measure the force and displacement of the vibrating rubber. The results were used to evaluate the dynamic properties of the rubber in terms of dynamic shear modulus and loss angle. Dynamic hysteretic loops method was applied and studied by Chernyshev in 1969 to increase the sensitivity of the method so that it can be used to measure the energy dissipation for most materials with either low or high damping capacity. An electronically controlled materials testing system (MTS 481.01, produced in the USA) was used to analyze and record the deformation of materials and automatically generate the dynamic hysteresis loops. The results showed that the used of strain gages as the measurement device was better in this kind of measurement compared to tensometer.

Gibson et al. (1977) used electromagnetic shaker as an exciter to measure the material damping for fibre reinforced composite materials under force vibration. The specimen used is the double cantilever beam. The test was run under discrete frequency. The concept used to define loss factor is similar to hysteretic damping which means it is defined as ratio of input energy to strain energy stored in specimen under steady state conditions. The equation used to estimate loss factor is shown as below:

$$\eta = \frac{D}{2\pi U} \tag{2.1}$$

where D is the energy dissipated per cycle and U is strain energy in beam at maximum displacement.

2.4.2 Direct and indirect methods for determination of dynamic stiffness

Complex stiffness is referred as the hysteresis damping and the stiffness are combined into one complex term (Neumark, 1962). Complex stiffness method is commonly used for damping and loss factor determination for resilient elements. Some researchers (Maly et al., 2000; Rao et al., 2001; Bloss et al., 2002) adopt this method in their study to further characterize the resilient element. The measurement of dynamic stiffness is generally divided into two groups which are direct and indirect methods. ISO 10846 provided definition of different dynamic stiffness and the corresponding measurement set up. There are five parts included in BS ISO 10846. The first part is on principles and guideline for measurement. The second part explained the direct method and third part explained the indirect method. Dynamic stiffness of elements other than resilient supports for translatory motion is included in part four and the final part is the explanation for driving point stiffness measurement. Shaker or hydraulic actuator is used as the vibration exciter in BS ISO 10846.

Gade et al. (1994) presented the experimental work on complex modulus and damping measurement on vibration isolator by introducing resonant and non-resonant methods. Shaker is used as the exciter for the resonant and non- resonant methods. The dynamic driving point stiffness is measured where the input force and acceleration are measured at same point. The complex modulus is obtained by rescaling the measured stiffness with a factor l/A which is the ratio of the length, l and cross sectional area, A of the isolator and reported as the function of frequency.

Thompson (1998) applied the indirect method from ISO 10846 for dynamic stiffness measurement with some refinements. Resilient rail pad was used as the test specimen in the study. The author pointed out the dynamic behaviour of the measurement apparatus in the final discussion. The indirect measurement was defined as a method where the output force is not measured directly by using the test jig but derived using Newton's second law. The advantage of the indirect method for the dynamic stiffness measurement was its better accuracy in high frequency performance which proved in the results. The stiffness and loss factor of rail pad were discussed by Fenander (1997) with the application of indirect method. The results were reported in function of frequency under different preload.

Nadeau et al. (2000) applied direct complex stiffness measurement techniques to engine mount to evaluate the loss factor of viscoelastic materials. Two examples of axial and lateral direction measurement setup were presented. The shaker was used as the vibration exciter. However, the equation of motion related to each definition was not included. Blocked transfer stiffness was measured and used to estimate the loss factor. The results showed that the stiffness and loss factor were frequency dependent for the engine rubber mount.

Huang et al. (2007) proposed a new approach to extract modal damping ratios from a linear structure with viscous damping. Single degree of freedom system with free vibration response was used in the study. This method was then compared to traditional logarithmic-decrement method. The author claimed that the approach has advantages such as stronger anti-noise ability, higher precision, better stability and convenience.

2.4.3 Dynamic driving point stiffness and dynamic transfer stiffness

The definition and the difference between dynamic driving point stiffness and dynamic transfer stiffness are listed in BS ISO 10846. Clemen et al. (2003) have modelled cylinder isolator using Abaqus software to look into the dynamic driving point stiffness and dynamic transfer stiffness. Nonlinear analysis was performed for different pre-deformations conditions and superimposed harmonic analysis on the pre-deformed isolator. The observation by the authors indicated that pre-deformation of rubber mount will only affected longitudinal transfer stiffness. The rubber behaves nearly incompressible and high damping properties in this case. The authors also found that the dynamic stiffness is general minimal at the resonant frequency.

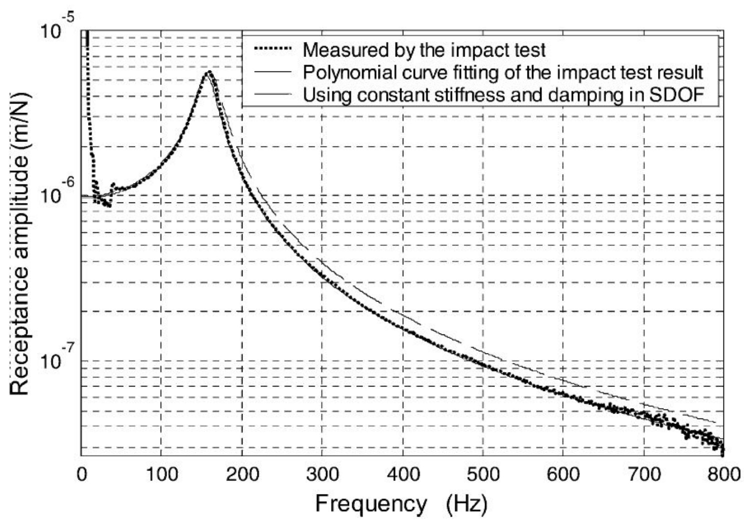


Figure 2.3: Comparison between measured and predicted receptance of rubber- mass system. (Lin et al., 2005)

Lin et al. (2005) applied dynamic driving point stiffness approach to estimate frequency dependent stiffness and loss factor by using impact test. Figure 2.3 shows the comparison of receptance functions which are measured from impact test, estimated from polynomial curve fitting and also predicted using the constant damping and stiffness. The result showed that constant stiffness and damping 3% over predicted the response for frequency range below natural frequency and 20% over predicted the response for frequency range above natural frequency. However, the frequency dependent stiffness and damping can closely reproduce the response for whole frequency range.

2.5 Discussion

From the review, it is clear that the study of the dynamic characteristics of the engine rubber mount is important for engine vibration isolation. Damping and stiffness are important parameter in the dynamic analysis of engine rubber mount. The frequency and amplitude dependent behaviour of stiffness and damping is an important study since it can predict the response of the system more accurate compared to constant stiffness and damping coefficient (Kari, 2003; Lin et al., 2005).

Shaker or hydraulic actuator is commonly used in the experimental determination of complex stiffness and damping (Gibson et al, 1977; Abdulhadi, 1985; Soula et al., 1997; Nadeau et al., 2000; Rao et al, 2001). Rao et al (2001) had compare between the experimental work with shaker setup and with hydraulic excitation method. The authors' opinion was that the shaker was less time consuming and inexpensive compared to hydraulic excitation method. However, hydraulic excitation method is suitable for large displacements and large preload requirement. The selection of the suitable measurement method depended on the requirements and the purposes of the elastomer. Impact test is a new development in damping and stiffness measurement technique (Lin et al., 2005). This method is faster and less cumbersome compared to shaker because it can allow researcher to do the test in real application condition. This is necessary to further develop this technique so that it can be used to measure other dynamic characteristics of the engine mount, namely the transfer stiffness, k_{21} and driving point stiffness, k_{11} as these are the most commonly measured parameters. The effect of the preload mass was not discussed by Lin et al (2005).

The existing technique for determination of the transfer stiffness and driving point stiffness require two different and separated set ups individually due to the need to measure the applied force and the transmitted force of the blocked side of the mount (BS ISO 10846). It is also clear from literature review (Lin et al., 2005) that the impact technique has not been widely adopted or reported. There is a need to test the effectiveness and accuracy of the impact technique which will enable wider adoption in the industry.

The experimental determination of loss factor of the rubber mount to be used in modelling work must be treated cautiously. The loss factor measured using different technique cannot be used generally in applications that are not supported by the technique. For example, the loss factor calculated using the dynamic driving point stiffness is in general used to represent the loss factor of the system and the loss factor calculated using the dynamic transfer stiffness is in general used to represent the loss factor of the resilient element itself.

CHAPTER 3
METHODOLOGY

3.0 Overview

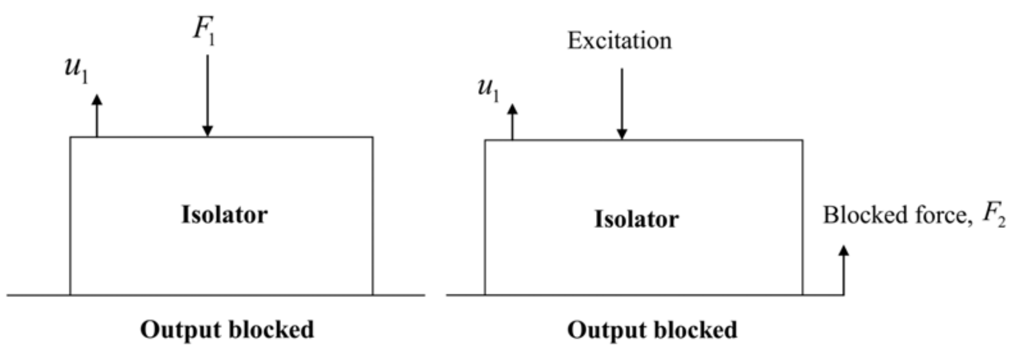
In this chapter, the analytical and experimental measurements of dynamic driving point stiffness and dynamic transfer stiffness are presented. Firstly, the frequency response function according to each approach and the mathematical formulation are derived. The parameters to be measured are frequency response function for each dynamic driving point stiffness and dynamic transfer stiffness. In order to measure these parameters, the necessary experimental set up is proposed and discussed in this chapter. The main topics discussed in this chapter are:

- Analytical of dynamic characterization of engine mount
- Experimental measurement and procedures
- Validation
- Reproduction of dynamic driving point stiffness and dynamic transfer stiffness

3.1 Analytical investigation of dynamic stiffness of engine rubber mounts

In order to study the dynamic properties of engine rubber mounts, proper understanding of analytical investigation related to each measurement method is very important. In this section, the comparison of the theoretical analysis for dynamic driving point stiffness with the dynamic transfer stiffness is carried out. Mathematical modelling for dynamic driving point stiffness and dynamic transfer stiffness related to the SDOF system and resilient element are derived to clearly show the effect of the different damping measurement method. The formulation is the same for both the impact test and sine swept testing. The difference is in the experimental application and the related procedures.

There are two types of dynamic stiffness under the direct complex stiffness method; the dynamic driving point stiffness (k_{11}) and dynamic transfer stiffness (k_{21}). Each type of the dynamic stiffness has their corresponding formulation so that the analytical results and the experimental measurements are compatible. Basically, the dynamic driving point stiffness can be used to represent dynamic transfer stiffness at low frequency range and cannot be applied to the high frequency range. The overall concept of the dynamic driving point stiffness and the dynamic transfer stiffness are shown in figure 3.1 as below:



(a) (b)

Figure 3.1.: The concept of (a) dynamic driving point stiffness and (b) dynamic transfer stiffness for isolator

According to BS ISO 10846-5: 2008, dynamic driving point stiffness (k_{11}) is the frequency dependent ratio of force on the input side of vibration isolation to the displacement on the input side with the output side blocked. Based on figure 3.1 (a), dynamic driving point stiffness is defined as $k_{11}= F_1/u_1$; where subscript 1 indicated that the force and displacement are measured on the input side of the vibration isolator. The isolator in figure 3.1 (a) is subjected to the input force, F_1 on the top and the response, u_1 is measured also at the input side (on the top). The dynamic driving point stiffness, k_{11} is mainly determined by elastic and dissipative force.

The dynamic transfer stiffness (k_{21}) is frequency dependent ratio of the blocked force on the output side of the vibration isolator to the displacement of the input side. Based on figure 3.1 (b), dynamic transfer stiffness is defined as $k_{21}= F_2/u_1$; where subscripts 1 and 2 denote the input and output respectively. The figure 3.1 (b) showed the condition where excitation is at the input and the transmitted force is measured at the blocked output, F_2. In the case of dynamic driving point stiffness, the response is measured at the input side.

3.1.1 Dynamic driving point stiffness (k_{11})

The dynamic driving point stiffness is an important dynamic property which can be used to represent the dynamic behaviour of a single degree of freedom (SDOF) system. A SDOF system which assumed hysteretic damping is used as the element model and shown in figure 3.2. The sinusoidal varying force is applied on the top of the preload to the specimen. This will result in a steady state response of the preload plate itself. Another end of the rubber is blocked and fixed. The rubber element is assumed to have hysteretic damping and represented by complex stiffness (Maly et al., 2000).

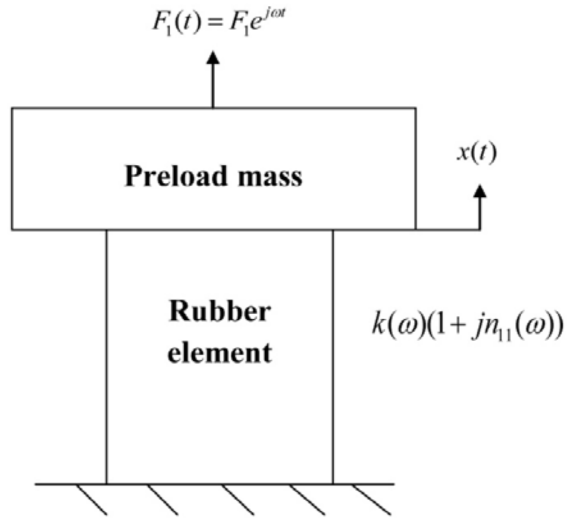


Figure 3.2: Single degree of freedom system with hysteretic damping for rubber mass system

For SDOF system with hysteretic damping, the equation of motion for this rubber mass system can be written as below:

$$m\ddot{x}(t) + k^* x(t) = F_1(t) \tag{3.1}$$

where $k^* = k(\omega)(1 + j\eta_{11}(\omega))$ is the complex stiffness for the rubber accounting for both stiffness and damping behaviour. By inserting this complex stiffness into the equation of motion of the SDOF system,

$$m\ddot{x}(t) + k(\omega)\big(1 + j\eta_{11}(\omega)\big)x(t) = F_1(t) \tag{3.2}$$

where m is the mass of the system, $k(\omega)$ is the frequency dependent stiffness and $\eta_{11}(\omega)$ is the loss factor. When the system is subjected to the applied force where

$$F_1(t) = F_1 e^{j\omega t}$$

and the response of the system is

$$x(t) = H_{11}(j\omega) F_1 e^{j\omega t}$$

where $H_{11}(j\omega)$ is the complex receptance function which is defined as the ratio of the output displacement to the input force of the system as a function of frequency and can be divided into real ($Re\{H(j\omega)\}$) and imaginary ($Im\{H(j\omega)\}$) part.

Assuming the applied force is complex,

$$H_{11}(j\omega) = \frac{1}{-m\omega^2 + k(\omega)(1 + j\eta_{11}(\omega))} \tag{3.3}$$

The dynamic driving point stiffness is the reciprocal of the receptance function where

$$k_{11} = -m\omega^2 + k(\omega)(1 + j\eta_{11}(\omega)) \tag{3.4}$$

Phase angle, θ obtained in this measurement is the phase lag between the output (displacement) and the input (force) where

$$\tan\theta = -\frac{Im\{H_{11}(j\omega)\}}{Re\{H_{11}(j\omega)\}} \tag{3.5}$$

The phase angle (θ) is related to the loss angle (φ) by the equation below:

$$\eta_{11} = \tan\varphi = \tan\theta\,(1 - \frac{\omega^2}{{\omega_n}^2}) \tag{3.6}$$

In here, ω_n is the natural frequency of the system and η_{11} is the loss factor which is obtained from the experimental measurement.

3.1.2 Dynamic transfer stiffness (k_{21})

Figure 3.3 presented rubber element which is placed under a preload mass for dynamic transfer stiffness. The excitation is on the top of the preload mass plate and the response from the plate itself is measured. The transmitted force from the bottom of rubber mount is also measured. The dynamic transfer stiffness is normally used in the forced vibration study of resilient element.

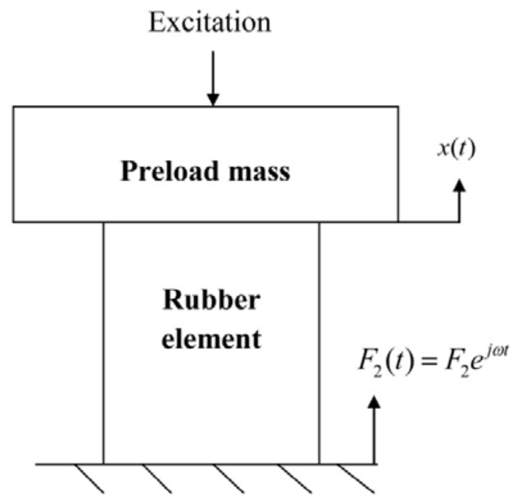


Figure 3.3: Simplified model for rubber element with the response under impact hammer excitation

The equation of motion for the resilient element is as below:

$$c\frac{dx(t)}{dt} + kx(t) = F_2(t) \qquad (3.7)$$

where c is damping coefficient and k is stiffness. The k can be frequency dependent or not. The element is then excited by using an impact hammer and the transmitted force through the rubber mount is $F_2(t) = F_2 e^{j\omega t}$. The response of the system which is measured at the input point become

$x(t) = H_{21}(j\omega)Fe^{j\omega t}$

The receptance function is given by $H_{21}(j\omega) = \frac{1}{c(j\omega)+k}$

Since $\tan\varphi = \frac{c\omega}{k}$, then $H_{21}(j\omega) = \frac{1}{k(1+j\tan\varphi)}$

The spectrum of the force and displacement can be measured independently and used to determine the dynamic transfer stiffness which is the ratio of force to displacement and can be written as:

$$k_{21} = k(1 + j\tan\varphi) \qquad (3.8)$$