

**INVERSION OF SURFACE WAVE PHASE
VELOCITY USING NEW GENETIC
ALGORITHM TECHNIQUE FOR
GEOTECHNICAL SITE INVESTIGATION**

LA HAMIMU

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GEOTECHNICAL SITE INVESTIGATION**

by

LA HAMIMU

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LIST OF SYMBOLS

$A(t)$	A perturbation amplitude
c	Phase velocity
C_{ijkl}	The constants of proportionality (<i>elastic moduli</i>)
D_{hs}	The depth to the half space
E_H	Thickness error
E_J	Error values of joint inversion
E_L	Love dispersion curve error
E_R	Rayleigh dispersion curve error
E_S	Shear wave velocity error
E_{SH}	The total error of shear wave velocity and thickness
f	Frequency
f_{cn}	The cut-off frequency of the n th higher mode
f_i	The body force in i th component
f_j	The fitness function
$f-k$	Frequency-wave number
F_L	The characteristic equation of Love wave
$f-p$	Frequency-slowness
F_R	The characteristic equation of Rayleigh wave
H	The thickness of layer
H_i^{inv}	The inverted thicknesses of the i th layer
H_i^{true}	The true thicknesses of the i th layer
H^{\min}	The lower search limits of thickness
H^{\max}	The upper search limits of thickness
Im	Imaginary number
k	Wave number
l	Wavelength
m	Model parameters
M	The number of shear wave velocities
n	Number of layer
N	Number of the observed data
N_4	Four-layer model
N_6	Six-layer model
N_{yq}	Nyquist
P	Primary
p_c	Crossover probability
p_m	Mutation probability
Q	The number of model in the population
Re	Real number
RH_i	The relative thickness error for i th layer
RSH_i	The total error of relative shear wave velocity and thickness
RS_i	The relative shear wave velocity error for i th layer
S	Secondary
t	Time parameter
U	A vector form of (u_1, u_2, u_3)
u_i	The displacement of the particle
v	Velocity

V_i^{inv}	The inverted shear wave velocities of the i th layer
V_i^{true}	The true shear wave velocities of the i th layer
$V_{R,i}^{inv}$	Rayleigh wave dispersion curve obtained from the inverted profiles
$V_{L,i}^{inv}$	Love wave dispersion curve obtained from the inverted profiles
$V_{R,i}^{true}$	Rayleigh wave dispersion curve obtained from the true profiles
$V_{L,i}^{true}$	Love wave dispersion curve obtained from the true profiles
V_s^{\min}	The lower search limits of shear wave velocity
V_s^{\max}	The upper search limits of shear wave velocity
\bar{V}_s	An average of shear wave velocities
V_p	The primary wave velocity
V_s	The shear wave velocity
V_{SH}	the inverted horizontal shear wave velocity
V_{SV}	The inverted vertical shear wave velocity
x_j	The Cartesian coordinate axes
α	The P-wave velocity
β	The S-wave velocity
ω	Angular frequency
ν	The Poisson's ratio
δ	The anisotropy percentage
σ_{ij}	The stress component
ρ	The density of medium
λ, μ	Two independent moduli (called Lamé constants)
θ	The cubic dilatation
ε_{ij}	The strain components
ϕ_j	The misfit function
∇	Divergence operator
Ψ	Vector potential
Φ	Scalar potential

LIST OF ABBREVIATIONS

<i>BC</i>	Boundary conditions
CP	Child population
CPU	Central processing unit
FFT	Fourier fast transform
fGA	Forking genetic algorithms
GA	Genetic algorithm
g-fGA	Genotypic forking genetic algorithm
HVL	High velocity layer
JRL	Joint inversion of Rayleigh and Love
L-M	Levenber-Marquardt
LSQR	Least square regression
LVL	Low velocity layer
MASW	Multichannel analysis of surface waves
MV	Mood volcanos
N6	Model-N6
NA	Neighbourhood algorithm
NIPGA	Novel inversion procedure of genetic algorithm
P1	Profile 1
P2	Profile 2
p-fGA	Phenotypic forking genetic algorithm
PP	Parent population
P-SV	Primary-shear vertical
RMS	Root mean square
S/N	Signal/noise
SA	Simulated annealing
SASW	Spectral analysis of surface wave
SH	Shear horizontal
SIL	Single inversion of Love
SIPGA	Standard inversion procedure of genetic algorithm
SIR	Single inversion of Rayleigh
SS	Salient schema
SVD	Singular value decomposition
SWM	Surface wave method

SONGSANGAN HALAJU FASA GELOMBANG PERMUKAAN MENGUNAKAN KAEDAH ALGORITMA GENETIK BARU UNTUK PENYELIDIKAN KAWASAN GEOTEKNIK

ABSTRAK

Prosedur songsangan halaju fasa gelombang permukaan adalah satu masalah yang rumit dalam pendekatan songsangan linear disebabkan oleh sifatnya yang banyak model dan sangat tak linear. Oleh itu, kegunaan kaedah pengoptimuman algoritma genetik (GA) yang merupakan salah satu daripada kaedah-kaedah pengoptimuman tak linear adalah pilihan yang sesuai untuk menyelesaikan masalah songsangan gelombang permukaan yang mempunyai ketaklinearan yang tinggi dan model yang banyak. Satu prosedur songsangan GA (NIPGA) yang novel yang diunjur dalam kajian ini dibentangkan sebagai kaedah songsangan halaju fasa gelombang permukaan yang efektif. Prosedur ini menerima pakai prosedur songsangan GA yang piawai (SIPGA) dengan memasukkan skim gelung baru untuk mengemas kini had-had pencarian daripada parameter model yakni halaju gelombang shear (V_s) dan ketebalan (H). Kod songsangan NIPGA yang dibangunkan di bawah Visual Compact FORTRAN Version 6.6 telah berjaya diaplikasikan kepada songsangan halaju fasa sintetik dan lapangan.

Dua profil sintetik yang berlainan mewaliki lapisan berhalaju tinggi (HVL) dan lapisan berhalaju rendah (LVL) telah digunakan. Profil-profil ini dianggap mewakili situasi yang biasa untuk kajian di kawasan geoteknik. Halaju fasa efektif gelombang Rayleigh dan halaju fasa multimod gelombang Love disintesis daripada profil-profil ini menggunakan bentuk gelombang pantulan P-SV yang lengkap dan bentuk gelombang pantulan SH yang lengkap masing-masing. Halaju fasa kemudian disongsangkan menggunakan kod NIPGA melalui tiga pendekatan yang berbeza, iaitu songsangan tunggal halaju fasa efektif gelombang Rayleigh, songsangan

tunggal halaju fasa multimod gelombang Love dan gabungan songsangan keduanya.

Untuk menguji ketepatan pendekatan tiap-tiap songsangan, perbezaan profil antara halaju gelombang shear yang betul dan songsang dikira berdasarkan kepada error halaju gelombang shear, E_S . Model numerikal menunjukkan ralat pendekatan gabungan halaju gelombang shear berdasarkan kod NIPGA adalah kecil jika dibandingkan dengan pendekatan songsangan tunggal. Ralat-ralat ini menunjukkan bahawa ketepatan halaju gelombang shear terbalik boleh ditingkatkan dengan gabungan songsangan halaju fasa gelombang Rayleigh dan Love.

Implementasi NIPGA menggunakan contoh data lapangan menghasilkan persetujuan yang baik antara halaju fasa yang dihitung dan halaju fasa daripada data lapangan dan dapat mengesan target LVL. Hasil daripada songsangan data lapangan yang dilakukan melalui pendekatan songsangan tunggal dan gabungan menyamai data lubang gerudi yang meyakinkan keefektifan secara praktikal aplikasi kod NIPGA. Perbezaan antara halaju fasa eksperimen dan halaju fasa yang dihitung boleh dibaiki dengan menggunakan lengkung-lengkuk gabungan songsangan halaju fasa efektif gelombang Rayleigh dan multimod Love. Keputusan ini juga menjusfikan bahawa kod-kod songsangan gabungan lengkung (curves) halaju fasa efektif gelombang Rayleigh dan multimod Love berjaya diaplikasi untuk menyongsang data lapangan. Ini akan meningkatkan ketepatan struktur subpermukaan yang mempunyai halaju reversal (target LVL). Walaubagaimanapun, satu halaju reversal yang terdiri daripada lapisan lembik di atas lapisan keras (kes LVL) adalah biasa dan sangat penting untuk kajian kawasan geoteknik. Secara keseluruhan, keputusan songsangan daripada dua ujian numerikal dan satu contoh data lapangan menjuruskan kod-kod NIPGA boleh meningkatkan penumpuan, keberkesanan, kestabilan dan ketepatan songsangan halaju fasa gelombang jika dibandingkan dengan prosedur songsangan GA yang piawai (SIPGA).

INVERSION OF SURFACE WAVE PHASE VELOCITY USING NEW GENETIC ALGORITHM TECHNIQUE FOR GEOTECHNICAL SITE INVESTIGATION

ABSTRACT

The inverse procedure of surface wave phase velocity is a complicated problem for linear inversion approach because of its multi-model and highly non-linear nature. Therefore the use of genetic algorithm (GA) optimization technique which is one of nonlinear optimization methods is an appropriate choice to solve surface wave inversion problem having high nonlinearity and multimodality. A novel inversion procedure of GA (NIPGA) suggested in this study was presented an effective technique for surface wave phase velocity inversion. This procedure adopted standard inversion procedure of GA (SIPGA) inserting a new loop scheme for updating the search limits of model parameters i.e., shear wave velocity (V_s) and thickness (H). Inversion codes of NIPGA developed under Visual Compact FORTRAN Version 6.6 were successfully applied to the synthetic and field phase velocity inversions.

Two different synthetic profiles representing the high velocity layer (HVL) and low velocity layer (LVL) cases were used. These profiles are considered to simulate situation commonly encountered in geotechnical site investigation. Effective Rayleigh wave and multimode Love wave phase velocities are synthesized from these profiles using full P-SV waveform reflectivity and full SH waveform reflectivity, respectively. The phase velocities are then inverted using codes of NIPGA with three different approaches, namely single inversion of the effective Rayleigh wave phase velocity, single inversion of multimode Love wave phase velocities and joint inversion of them.

To assess the accuracy of each inversion approach, differences between the true and inverted shear wave velocity profile are quantified in terms of shear wave velocity error, E_s . Our numerical modeling showed that the shear wave velocity errors of the joint inversion approach based on NIPGA codes are relatively smaller than the single inversion approach. These errors indicate that the accuracy of shear wave velocity reversal can be improved by jointly inverting of Rayleigh wave and Love wave phase velocities.

Implementation of NIPGA to a field data example provided excellent agreement between the calculated and field phase velocities and can well detect LVL targets. Field data inversion performed that results from single and joint inversion approaches were comparable to drilling data, supporting the effectiveness and practical applicability of NIPGA codes. Manifestly, misfit between experimental and calculated phase velocities would be improved by using joint inversion effective Rayleigh and multimode Love wave phase velocity curves. This result also justify that joint inversion codes of effective Rayleigh wave and multimode Love wave phase velocity curves are successfully applied to invert a field data example. This can significantly improve the accurate assessment of a subsurface structure having a velocity reversal (LVL targets). However, a velocity reversal comprising a soft layer below a stiff one (LVL case) is common and very important for geotechnical site investigation. Overall, inversion results from two numerical tests and one field data example indicated that the NIPGA codes could greatly improve the convergence, efficiency, stability and accuracy for surface wave phase velocity inversion, compared to the standard inversion procedure of GA (SIPGA).

CHAPTER 1

INTRODUCTION

1.3 Overview

Surface waves are seismic waves propagating parallel to the surface of the earth. Their amplitude distribution over depth is stationary with horizontal position and decreases exponentially with the depth (Aki and Richard, 2002). The most essential characteristic in the surface wave analysis is the calculation of dispersive phase velocities. Surface wave methods are based upon the inversion of dispersive phase velocity which is obtained by the processing of full waveform data. Inversion of surface wave phase velocity is a challenging problem for linear inversion procedures because of its multi-model and highly non-linear nature (Dal Moro et al., 2007).

In the last few decades, many papers have been extensively published on various methods to invert surface wave phase velocity. Many authors inverted surface wave phase velocity using linear inversion approach such as the damped least square regression (LSQR) method, Levenberg–Marquardt (L-M) method, singular value decomposition (SVD) as well as Occam’s algorithm. A brief description of linear inversions which are frequently used by various authors for inversion of surface wave phase velocity is described as follow.

Least Square Regression Method: An azimuthally anisotropic 3D shear wave velocity model of the Australian upper mantle obtained from the dispersion of fundamental and higher modes of Rayleigh waves can be estimated by employing least square method (Simons et al., 2002). Ju and Ni (2007) developed a least squares method to find the Rayleigh damping parameters for modeling the surface

wave propagation in soils. A simple method in the time-space domain to detect near-surface features based on a travel time equation of Rayleigh wave diffraction can be determined to accurately calculate layer parameters by using least square method (Xia et al., 2007). Furthermore, Xia et al. (2008) demonstrated how the data resolution matrix can reveal the intrinsic properties of surface wave inversion using a damped least square method.

Levenberg-Marquardt Method: Estimation of near-surface shear-wave velocity by inversion of Rayleigh waves can be efficiently calculated by selecting the appropriate damping factor in the L-M method (Xia, et al., 1999). Construction of 2D vertical shear-wave velocity field by the multichannel analysis of surface wave technique can be obtained by inverting Rayleigh wave phase velocities using Levenberg-Marquardt (L-M) method (Xia et al., 2000). This method was also applied for inversion of high frequency surface waves with fundamental and higher modes (Xia et al., 2003). Moreover, the use of the damping factor in the L-M method can be then developed for utilization of high frequency Rayleigh waves in near surface geophysics (Xia et al., 2004). The use of multiple-mode dispersion data in surface wave inversion to derive shear-wave velocity can be presented for reducing the error associated with the nonuniqueness in inversion by using L-M method (Supranata et al., 2007).

Singular Value Decomposition (SVD) Method: The SVD combined with the damped least square method can significantly contribute to the joint inversion for apparent velocities of Rayleigh and Love waves (Joh et al., 2006). An iterative solution technique to a weighted damping equation using the SVD techniques and Levenber-Marquardt (L-M) method are very efficient in determining the elastic moduli of the Gibson half space and recovering shear wave velocity by the inversion

of Rayleigh wave phase velocities (Xia et al., 2006). Luo et al. (2007) showed that the SVD technique combined with the damped least-square method in inverting surface-wave data can be applied for joint inversion of high frequency surface waves with fundamental mode and higher modes. A new scheme to generate a pseudo-2D shear-wave velocity section can be presented by inverting a series of 1D dispersion curves using the SVD technique and the damped least-square method (Luo et al., 2008).

Occam's Algorithm: O'Neill et al. (2003) used a linear optimization based on Occam's algorithm to invert surface wave dispersion curve which is calculated using full waveform P-SV reflectivity and applied for shallow engineering. Multiple-mode Love wave dispersion curves generated from full SH-wavefield modeling by the reflectivity method can be well inverted to provide a layered shear-wave velocity variation with depth by using a linearised optimization known as Occam's inversion (Safari et al., 2006). Estimation of shallow subsurface shear-wave velocity by inverting fundamental and higher-mode Rayleigh waves can be appraised employing Occam's algorithm (Song et al., 2007).

For the linear approach, most of the authors require a priori information about a subsurface structure (Yamanaka and Ishida, 1996). A priori information incorporated in a linear inversion is to hasten to find an optimal solution that is the global minimum of a misfit function (defined as the sum of differences between calculated and observed data). The linear approach can be considered as an acceptable and computational effective solution only when some robust a priori information is available and good starting model can thus be established that is close to the real solution. In fact, the main problem of the traditional approach is that the final solution intrinsically depends on the starting model and that poor or missing a

priori information makes the final solution particularly weak (Dal Moro et al., 2007). However, if a priori information is either scant or unavailable, the inversion may find local optimal solution. Linearized methods can only lead to a good answer if one has good a priori information on the model parameters (Sen et al., 1993). Therefore, to reduce these limitations, in this study the Genetic Algorithm (GA) as one of the global optimization technique has been suggested by the researcher to improve the inversion results of surface wave phase velocity without using a priori information.

Generally, a phase velocity is a nonlinear function of shear (and/or compressional) wave velocities, densities, and thicknesses for each layer (Yamanaka and Ishida, 1996). Therefore the use of nonlinear optimization method or global optimization method is a natural choice to solve surface wave inversion problem having high nonlinearity and multimodality. This method, however, requires computation of the forward modeling problem many more times than that required by linearized inversion techniques, although it does not require computation of any partial derivatives or the inversion of matrices (Sen et al., 1993). Several global optimization methods used frequently to invert surface wave phase velocity are simulated annealing (SA) method, genetic algorithm (GA) optimization technique, neighborhood algorithm (NA), and pattern search algorithm. A brief description of global optimization methods which are commonly used by various authors for inversion of surface wave phase velocity can be described as follow.

Simulated Annealing (SA) Method: From 2000 to the present, a number of papers using the various SA methods for inverting the surface wave phase velocity have been extensively published. Martinez et al. (2000) used SA optimization method to invert Rayleigh wave phase and group velocities. Application of SA method to invert multimode Rayleigh wave dispersion curves for geological

structures was presented by Beaty et al. (2002). Pei et al. (2005) studied on a 1-D inversion of shallow surface wave dispersion curves using simulated annealing. Ryden and Park (2006) used the fast simulated annealing inversion of surface wave on pavement to minimize the difference between the measured phase velocity spectrum and that calculated from theoretical layer model including the set up geometry. Luke and Macias (2007) showed that the SA inversion can be used to seismic surface wave data for resolve complex profiles.

Genetic Algorithm (GA) Optimization Technique: During the last decades, application of GA for surface wave inversion have been extensively developed as the more reasonable global optimization technique to improve the accurate assessment of shear wave velocity profiles. Yamanaka and Ishida (1996) developed successfully GA as a global optimization method to an inversion of surface wave dispersion data. Iglesias et al. (2001) inverted dispersion curves of group velocity of fundamental mode Rayleigh waves using simultaneous genetic and simulated annealing algorithms. In addition, Yamanaka (2004) compared the inversion results of surface wave phase velocity from microtremor exploration using GA optimization technique with those using SA method. Chang et al. (2004) applied the GA optimization technique for Rayleigh-wave phase velocity measurements combined with teleseismic *P*-wave receiver function data. At the same time, Lawrence and Wiens (2004) combined inversion of body wave receiver function and surface wave phase velocity inversion using GA to yield a regional model of crustal structure in Chilean Patagonia. A GA applied to the inversion of fundamental mode Rayleigh wave dispersion over a 3-layered and shallow vertical fault model was successfully utilized by Nagai et al. (2005).

Pezeshk and Zarrabi (2005) introduced a new inversion procedure for spectral analysis of surface wave (SASW) using a GA. An inversion technique based on a GA was applied to calculate layer parameters for soil classification at seismic site effect evaluation in Dinar region SW Turkey by Kanli et al. (2006). Lu et al. (2007) estimated shear wave velocity profiles using GA for multimode Rayleigh waves in the presence of a low velocity layer based on the experimental dispersion curves of fundamental and/or higher modes. Rayleigh wave dispersion curves resulted from seismological and engineering geotechnical methods were inverted for estimating the local shear wave velocity depth profiles using GA (Richwalski et al., 2007). Song and Gu (2007) implemented a multimode Rayleigh wave dispersion curve inversion scheme for characterizing roadbed structure based on the modified genetic algorithms as developed by Yamanaka and Ishida (1996). A combined inversion with a genetic algorithm and a strategy for selecting possible model parameters are examined by Fah et al. (2008) to analyze phase velocities of fundamental and higher mode Rayleigh and Love waves, and particle motion (ellipticity) retrieved from H/V spectral ratios.

Neighbourhood Algorithm (NA) Method: The NA and pattern search algorithm are also the alternative global optimization methods which can be applied for inversion of surface wave phase velocity. Inversion analysis on surface wave dispersion curve and H/V spectra by NA were tested to acquire shear wave velocity structure by Nagashima and Maeda (2005). Tokeshi et al. (2008) used a NA for estimating the shear wave velocity profiles using its natural frequency and Rayleigh wave dispersion characteristic. Yao et al. (2008) applied also a NA to determine the 3-D shear wave speed variations in the crust and upper mantle in the southeastern borderland of the Tibetan Plateau, SW China using Rayleigh wave dispersion curves.

A new code using NA was successfully developed and then tested for inverting dispersion curve of surface wave and its application to ambient vibration measurements by Whatelet et al. (2004). Song, et al. (2008) implemented and tested a Rayleigh wave dispersion curve inversion using pattern search algorithms to nonlinear inversion of high frequency surface wave data.

Iglesias et al. (2001) have tested the efficiency of the GA and SA inversion methods. They find that GA saves 30% of computation time as compares to SA. On the other hand, GA needs more memory as compared to SA. In problems involving a large number of parameters, a small computer might be insufficient for GA. A further advantage of the SA inversion is that it converges with a lesser misfit as compared to the GA inversion. SA requires smaller memory and shows better convergence during the final iterations. Yamanaka (2004) compared performances of heuristic optimization technique in phase velocity inversion. The GA and SA show the rapid convergence of the misfits. However, the smallest misfits can be achieved by using SA method.

To reduce the some difficulties in the GA method, in this study a novel scheme in GA will be introduced to invert the surface wave phase velocity. Basic ideas of this scheme are adopted from Sen et al. (1993). They used simulated annealing (SA) method for inverting the resistivity sounding data. To invert the observed apparent resistivity acquired from field measurements, they carried out the inversion in two steps. First, they chose large search limits with coarse search intervals and then based on the results from this step they manually redefined their search limits with finer search intervals for their final inversion run. Basic ideas in redefining the search limits for the specified run of generation are further developed in this study. Here, a novel inversion scheme is realized with aim of decreasing the

capacity of computational memory and lowering the computational time. More detailed explanation of a novel inversion scheme for surface wave phase velocity based on GA optimization technique can be found in Chapter 3. This scheme is not only to reduce a huge computational memory in GA method but also guarantee that in the final run of generation the smallest misfit can be achieved. Finally, the use of the proposed inversion scheme can be then applied for accurate assessment of a shear wave velocity reversal profile.

However, accurate assessment of a shear wave velocity profile derived from inverting of the surface wave phase velocity is one of important keys for geotechnical characterization. In the following some of the applications of surface wave inversion that can be found in the various literatures are summarized. Rix et al. (2001) reported that surface wave methods can also be used to determine the material damping ratio of soils in situ by measuring the spatial attenuation of Rayleigh wave amplitudes. Acquisition and inversion of Love wave data can be used to measure the lateral variability of geo-acoustic properties of marine sediments (Winsborrow et al., 2003). Dal Moro et al. (2003) used determination of Rayleigh wave dispersion curves for near surface applications in unconsolidated sediments.

Multimode Rayleigh phase velocity inversion can be employed to seismic pavement testing for evaluating the stiffness properties and the thickness of the top pavement layer (Ryden et al., 2004). The dispersion analysis of the Rayleigh-type surface waves can be successful applied to characterize stiffness distribution of water-bottom sediments by using spectral analysis of surface waves (Park et al., 2005). Song and Gu (2007) showed that an inversion of Rayleigh wave dispersion curves can be implemented to reconstruct the fine structure of the tested roadbed subsurface. Shear wave velocity of Rayleigh wave propagating along the ground

surface can be measured by using the continuous surface wave (CSW) test for determining ground stiffness (Heymann, 2007).

As reported by O'Neill et al. (2003), the shear wave velocity in the shallow surface wave is an important parameter in geotechnical engineering. In many typical geotechnical sites, a shear wave velocity reversal profile comprising a soft layer below a stiff one is common and very important in geophysical, environmental, and geotechnical site investigation. In civil engineering, low velocity layers (LVL) are of special importance because such soft horizons can give rise to various problems, such as settlement or liquefaction during an earthquake (e.g. Feng et al., 2005). Therefore, the researcher will focus his attention on the estimation of a shear wave velocity profile having a velocity reversal (HVL and LVL cases).

1.4 Problem statement

The surface wave method can be conventionally divided into three steps: field data acquisition, picking of phase velocities, and inversion of phase velocities (O'Neill et al., 2003; Xia et al., 2004). Inversion of surface wave phase velocities is the most important step in the surface wave interpretation. Final results of this step can certainly be used to assess a shallow subsurface shear wave velocity profile (Lai et al., 2002).

However, inversion of surface wave phase velocities, as with most other geophysical optimization problems, is typically a nonlinear, multi parameter, and multimodal inversion problem. As a consequence, the use of linear inversion approaches or local optimization methods for surface wave inversion, such as least square, SVD, conjugate gradients, L-M method, Occam's algorithm, can be practically trapped by local minimal or incorrect solution (Song et al., 2008). These methods can be considered as an acceptable and computationally effective solution

only when some robust a priori information about the subsurface structure is available and a good starting model can thus be established that is sufficiently close to the real solution (Yamanaka and Ishida, 1996; Dal Moro et al., 2007).

Genetic algorithm (GA) as one of the global optimization methods that can overcome these limitations are particularly attractive for surface wave analysis (Beatty et al., 2002; Chang et al., 2004; Nagai et al., 2005; Song et al., 2007; Fah et al., 2008). In the technical computing implementation, GA needs the big capacity of computational memory as compared to SA. Therefore, in problems involving a large number of parameters, a small computer might be insufficient for GA. The GA inversion shows also the rapid convergence of the misfits with a greater misfit as compared to the SA method (Iglesias et al., 2001; Yamanaka, 2004).

Shear wave velocity as a function of depth can be derived from inverting the phase velocity of the surface (Rayleigh and or Love) wave (Xia et al., 2003). Accurate assessment of shear wave velocity profile is very important for geotechnical site investigation and has become common practice recently. In many typical geotechnical sites, large velocity reversal and or contrast cause the layered system to vibrate anomalously and one or more higher mode to propagate with more energy than fundamental. Dominant higher modes of surface waves are known to be generated at sites with large stiffness contrast and/or reversal between layer (O'Neill and Matsuoka, 2005). Most of the engineering studies employ Rayleigh wave, while Love wave method are not as widely used. However, Love waves can contribute valuable information and theoretically have a more efficient modeling algorithm than Rayleigh waves (e.g. Safani, 2007).

Joint inversion analysis for apparent phase velocities of Rayleigh and Love waves using the damped least square method with singular value decomposition for

the inverse matrix calculation can be used to reduce non-uniqueness inherent in surface-wave methods and increase the accuracy of a resulting shear-wave velocity (Joh et al., 2006). In addition, Zeng et al. (2007) have also studied a comparative analysis of sensitivities of Rayleigh and Love waves. Their result indicates that joint inversion of multimode Love waves and Rayleigh waves would obtain a more accurate shear wave velocity profile. Accurate assessment of shear wave velocity in a earth subsurface with a velocity reversal, e.g., high velocity layer (HVL) or low velocity layer (LVL), can be estimated using joint inversion Rayleigh and Love waves.

1.3 Research objectives

The primary objectives of the study are to:

- Develop inversion procedure of surface wave phase velocities with the target of finding the best inversion scheme to assess a more accurate and reasonable estimate of the shear wave velocity profile as the important parameter in geotechnical site investigation. A novel inversion scheme for surface wave phase velocities based on the GA optimization technique will be introduced in this study to reduce the huge memory in the computation processes, to less time consuming as well as to achieve the smallest misfit that defined as the sum of differences between experimental/observed and theoretical/calculated phase velocities.
- Improve the accurate assessment of a shear wave velocity reversal profile including HVL and LVL cases. These cases are considered to simulate situation commonly encountered in geotechnical site investigation. In this study joint inversion of Rayleigh wave and Love wave phase velocities based on a novel

inversion scheme in GA will be developed to map subsurface geological structure having a velocity reversal (HVL and LVL targets).

1.4 Organization of the thesis

This thesis comprises five chapters which reflect the work in chronological sequence and can be briefly described as follows:

Chapter 1 includes an overview of this study, statement of the problem, the research objective and organization of the thesis. Overview is a historical background concerning the surface wave analysis and the inversion methods that are common used to derive the shear wave velocity profile. Problem statement exposes several limitations and difficulties which are frequently appeared in inverting surface wave phase velocities. The main point of this chapter is the research objective which can explain how to overcome the limitations and difficulties that are previously described in statement of problem and why it is necessary.

Chapter 2 is a literature review divided into two major parts that is surface wave analysis and GA description employed in this study. The most important discussions concerning to surface wave analysis will be briefly cited in this part such as surface wave properties, basic steps of surface wave (including steps of acquisition, processing and inversion). Deriving the wave equation comprising fundamental equation of motion in the medium, Hooke's law for linear elasticity and equation of motion in three-dimensional is further reviewed. A brief description of P-SV and SH waves is presented by employing several fundamental equations of surface waves propagating in the layered medium. General solutions of characteristic equation for Love and Rayleigh waves are also reported in brief. The last part is dedicated to the description of GA inversion employed to invert the surface wave

phase velocity. A standard GA description involves parameter representation (e.g., decoding and search space), genetic operators (including selection, crossover and mutation). In addition, a forking GA (fGA) which is recommended by Tsutsui et al. (1997) constitutes the most important reference to be discussed once more.

Chapter 3 is a methodology sketch applied in this study. This chapter is devoted to describe the techniques in generating the phase velocity of surface waves and the inversion procedures using GA optimization techniques. A step by step of computational methods of phase velocity including Rayleigh and Love waves is in detail presented. This chapter provides also a brief description of basic operators of GA and parameter representation in GA. Standard inversion procedures of GA implemented under FORTRAN computer program is described by employing a number of subroutine codes. Last section of this chapter introduces a novel inversion procedure of GA. Moreover, a step by step algorithm is performed by using flowchart diagram.

Chapter 4 performs detailed computation results and discussion. This chapter consists of several parts. The first section discusses the importance of surface wave phase velocity inversion. The section explains the significance of inversion of the effective Rayleigh wave and multimode Love wave phase velocities. The use of GA inversion technique for inverting surface wave phase velocity presented by major research groups is also summarized at the last part of the first section. The second section performs synthetic data from the numerical test including steps of numerical test, generating experimental phase velocity, full-waveform and dispersion image, inversion results of synthetic phase velocities and summary of synthetic inversion results. A comparison of inversion results obtained from single inversion Rayleigh/Love waves and joint inversion of Rayleigh and Love is also presented

with aim of evaluating the improvement of joint inversion in resolving shear wave velocity. The inversion procedure of GA proposed in this thesis is further applied to invert a field data. More detailed explanation of field data inversion is presented in the third section. This section consists of overview of the field data, setting the initial model and discussing of inversion results of field example. In the last section, a comparison of inversion results obtained from this study and the previous works is also presented with aim of evaluating a new contribution of this research.

Chapter 5 is a last chapter containing several general conclusions that may be drawn from the overall work together with indications for further development of the research. This chapter involves also a number of suggestions for the future works.

CHAPTER 2

LITERATURE REVIEW

2.1 Surface wave methods

2.1.1 Basic properties of surface waves

In addition to body waves (P-waves and S-waves) penetrating deeply into the earth, a seismic source such as an explosion or earthquake can also generate waves that travel along the surface of the earth and often called as surface waves (in seismic exploration known as *ground roll*). Main differences between seismic body waves and surface waves, as observed on seismograms of distant earthquakes, can be summarized as follows (Novotny, 1999):

1. Records of a seismic event begin with longitudinal waves, followed by transverse waves, and finally by surface waves,
2. Surface waves usually have larger amplitudes and longer periods, and
3. Surface waves display a characteristic dispersion and polarisation.

The most commonly observed surface waves are Rayleigh waves, after Lord Rayleigh who predicted their existence in 1887. A Rayleigh wave is frequently dominant events in seismograms, because of their greater energy and because of a geometrical spreading that is lower than that of body waves spreading energy in all directions (spherical spreading in a homogeneous medium). This wave travels along the surface of the earth and involves an interference of the P-wave and SV-wave. The particle motion is confined to the vertical plane that includes the direction of propagation of the wave. The amplitude of the Rayleigh wave motion decreases exponentially with depth.

The second type of surface waves is called Love waves, after Augustus Edward Hough Love, who developed a theory for their existence early in the twentieth century. A Love wave is formed through the constructive interference of multiply reflected S-waves. The particle motion is horizontal (S_H) and in the direction of SH-wave. The amplitude of a Love wave motion decreases also exponentially with depth.

In each case, the depth to which the waves penetrate is a function of their frequency. Higher frequencies have shorter wavelengths, because of the relationship $f=v/l$ (where f = frequency, v =velocity, and l = wavelength), and penetrate shallower into the earth. Due to velocity commonly increases with depth in the layered earth, the higher frequencies pass through material with lower average velocity. The existence of vertical medium-velocity gradients in the real earth caused velocity of surface waves varies with frequency (i.e., velocity is a function of frequency), which is that they are dispersed. The propagation of surface waves in a vertically heterogeneous medium shows a dispersive phase velocity (dispersion curve). *Dispersion* means that different frequencies have different phase velocities.

On seismograms, surface waves appear as long wave trains in which the frequency slowly increases, and they are not compact wavelets like body waves. Surface wave is employed to estimate a general picture of subsurface earth structure through the analysis of their phase velocity. Due to different zones of the earth have different distributions of phase velocity with depth each zone is characterized by different dispersion curves (plots of the variation of phase velocity with frequency).

2.1.2 Basic steps of surface wave method

The surface wave method (SWM) is a seismic characterization method based on the analysis of the surface wave phase velocity. The dispersive nature of surface

waves in a layered medium is utilized to estimate a shear wave velocity (i.e. stiffness) profile of the test site. The complete testing procedure can be divided into three basic steps (Ryden and Park, 2004): (1) generation and measurement of surface wave in the field (*acquisition step*); (2) data processing and extraction of an experimental phase velocity (*processing step*); (3) inversion of the experimental phase velocity to obtain an estimated shear wave velocity with depth profile (*inversion step*). Figure 2.1 illustrates a schematic of three basic steps in surface wave methods.

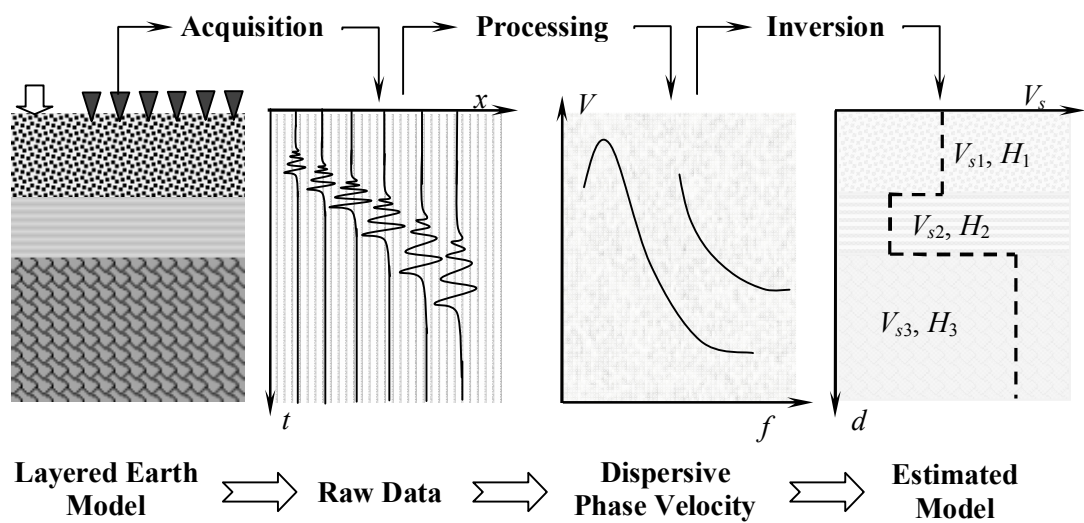


Figure 2.1: A schematic of three basic steps of surface wave methods

The acquisition comprises the gathering raw data of the seismic wave propagation containing surface waves in a wide frequency band. From these data the processing extracts the information about the phase velocity of surface waves, which is then used by the inversion to assess the model parameters, namely the mechanical properties of the subsurface structure. The inversion procedure is certainly based on the forward modeling that is able to compute the Rayleigh wave propagation for a known model (Socco and Strobia, 2004).

2.1.2.1 Step of acquisition

The main task of the acquisition is to measure surface waves and thus produce information about the dispersive phase velocity. Many acquisition techniques have been employed in surveying of surface wave, depending on the type of application, the depth of investigation and the scale acquisition. For geotechnical site investigation the scale is smaller namely from centimeters up to some tens meters. The sources are low explosions, small vibrations, sledgehammers and weight drops, even noise can often be used. The receivers are a number of geophones. These equipments are to be light, portable, and cheap. The effects of the acquisition on the data lead to an experimental/observed phase velocity, which is the consequence of the difficulties in separating the energy associated with different modes. The effect is particularly relevant for the small scale involved in geotechnical problem. A brief description for a better understanding of the surface wave testing procedure and for a best practice of acquisition can be summarized as follow.

Sampling of space

The basic aspects to be considered are related to the spatial sampling of surface waves (the total array length and the receiver spacing), while time sampling is less critical (Socco and Strobina, 2004). The array length affects the number of resolution Δk and therefore the possibility of mode separation. However, long arrays should be preferred because they can improve the modal separation and because they can reduce the data uncertainties. In contrast, short arrays are less sensitive to lateral variation, produce a better S/N ratio, are less affected by high frequency attenuation and produce less severe spatial aliasing. For receiver spacing (Δx), as stated by the Nyquist sampling theorem, the maximum receiver spacing that can be identified

depends on spatial sampling rate: $\Delta x = \frac{1}{2} (2\pi/k_{Nyq}) = \pi/k_{Nyq}$

The number of receivers, obviously related to array length and receiver spacing, affects the propagation of the uncertainties over the data. The uncertainty in the estimated wave number (and hence in the phase velocity) depends on the uncertainty in the phase of each frequency component, but also on the number and position of receivers. For a given array length, increasing the number of receivers reduces the amplification of the uncertainty. For example, a 24-receiver array reduces the uncertainty by a factor of four with respect to a two-receiver array, and enables a solution for the trade-off length-spacing to be found.

For source offset, two main aspects have to be considered in planning an optimum source offset. At small distances, the near-field effects contaminate the signal at low frequencies, while the attenuation reduces the S/N of traces at large distances, especially in the high frequency band. These two phenomena are strongly dependent on the site and the experimental conditions, and in general cannot be predicted to determine the best source-offset. Alternative solutions are the acquisition with different source-offsets to recognize the near-field, or the use of a small offset and the filtering the near-field during processing.

Sampling of time

The time-sampling parameters have a minor effect with respect to spatial sampling. The sampling rate is chosen depending on the highest frequency that will be acquired according to Nyquist sampling theorem. The time-window has to be long enough to record the whole surface wave on all traces: with long arrays at low velocity sites, several seconds can be needed. A long window with a pre-trigger can be used to evaluate the signal level during the acquisition and to improve the spectral resolution.

2.1.2.2 Step of processing

The aim of the processing is to derive from full waveform records all information about the propagation of surface wave. The processing consists of extracting from the raw data the dispersive phase velocity as a function of the frequency. A number of techniques that have been developed and employed for processing surface wave can be summarized as follow (Socco and Strobria, 2004).

1. Multiple-filter analysis has been used for determination of the group velocity as a function of the frequency from a dispersed wavefront. In addition, multiple-filter analysis can be employed to estimate the spectral amplitudes of various models.
2. Cross-power spectrum of two-station data has been adopted by many authors for working with the spectral analysis of surface wave (SASW) approach.
3. The frequency-wavenumber ($f-k$) plane-wave transform has been presented for an unambiguous investigation of higher modes.
4. The frequency-slowness ($f-p$) plane-wave transform obtained from a slant stack has been applied for imaging of dispersive phase velocity.
5. Frequency-time analysis is an alternative approach which can be used for the processing of surface wave data.

Plane-wave transforms are widely used to perform the analysis in domains where surface wave are easily identified and their properties are estimated. The frequency-wavenumber ($f-k$) transform has the advantage of being a natural approach to the analysis of the seismic event. Other plane-wave transform is the frequency-slowness ($f-p$) transform obtained from the slant stack. This transform can be completely equivalent and applied for the first step of the processing.

2.1.2.3 Step of inversion

Inversion is the last and most important step of surface wave methods in estimation of shear wave velocity profiles. Inversion techniques aim to minimize an objective function, which comprises the RMS error between the observed and forward modeled data. In geophysics the term inversion means the estimation of the parameters of a postulated earth model from a set of observation. In the case of the surface wave method, the inversion supplies the estimated velocity from surface wave phase velocity. It is important to stress that the surface wave method inverse problem is nonlinear and mix-determined; in addition, the object is usually interpreted (dispersive phase velocity) is often continuous and therefore, automatic inversion procedures can be successfully applied only in the case in which branch of modal curves are selected within a proper frequency range (Socco and Strobria, 2004).

Several inversion techniques have been proposed for inverting the surface wave phase velocity. Historically, the more widely used approach is the linear inversion techniques such as the linearised iterative least square method, Levenberg-Marquardt (L-M) method, singular value decomposition (SVD) as well as Occam's algorithm. These methods has been used by many authors, with some differences in the data concerned, the model parameters, the computation of the partial derivatives, the inversion strategies, the use of smoothness constrain, etc. (e.g. Xia et al., 1999; Xia et al., 2000; Simons et al., 2002; Xia et al., 2003; O'Neill et al., 2003; Xia et al., 2004; Joh et al., 2006; Safani et al., 2006; Xia et al., 2006; Ju and Ni 2007; Luo et al., 2007; Xia et al., 2007; Supranata et al., 2007; Song et al., 2007; Xia et al., 2008; Luo et al., 2008, among others).

Like inversion of other geophysical data, the objective functions in surface wave inversion are nonlinear. Therefore the use of linear approaches or local search methods can be made to account for this by iteratively jumping or creeping through model space. As a consequence, nonlinear optimization or global search methods are a natural choice to solve surface wave inversion problem. Some global optimization methods which are common employed for surface wave phase velocity inversion are the simulated annealing (SA) method, the genetic algorithm (GA) method, neighbourhood algorithm (NA) and pattern search algorithm. Yamanaka and Ishida (1996), Martinez et al. (2000), Iglesias et al. (2001), Beaty et al. (2002), Chang et al. (2004), Lawrence and Wiens (2004), Yamanaka (2004), Whathelet et al. (2004), Pei et al. (2005), Nagai et al. (2005), Nagashima and Maeda (2005), Pezeshk and Zarrabi (2005), Ryden and Park (2006), Kanli et al. (2006), Luke and Macias (2007), Lu et al. (2007), Song et al. (2007), Tokeshi et al. (2008), Yao et al. (2008), Fah et al. (2008), and Song et al. (2008) are among of authors who used global optimization methods in inverting the surface wave phase velocity.

2.1.2.4 Limits and advantages of the SWM

The comparison with other geophysical techniques shows advantages and limitations of the SWM. The main limitation of SWM is that the assumed model is 1D, and so the result is one-dimensional: the 2D information present in data can be used only to give warnings on the inversion. Nevertheless with short arrays the lateral variations are often not critic. On the other hand, the SWM presents many advantages. The SWM overcome some intrinsic limitations of the refraction technique (hidden layer, velocity inversion, gradual variations), affecting both P-wave and S-wave refraction. It can increase the reliability of the results, and it can be

applied in situations where the refraction does not work, when a stiff top layer due to a pavement is present.

The SWM shares with the S-waves surveys the advantages on the P-wave techniques: in saturated materials the sensitivity of P-waves to the mechanical properties of the solid skeleton can sometimes be very low, and P-wave surveys do not provide useful information. In such situations, the techniques that investigate the shear properties have to be used. The acquisition of P-wave refraction and SWM data is very similar, and can even be performed simultaneously: some synergies between the two techniques can be found (Strobia, 2002).

2.1.3 Deriving the wave equation

2.1.3.1 Fundamental equation of motion in the medium

For finite motions, the exact equation for motion in the continuum can be derived from the theory of elastic wave that provides mathematical relationships between stresses and strains in the medium (Aki and Richards 2002; Lay and Wallace, 1995). Applying Newton's second law to the medium gives:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = f_i + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2.1)$$

Equation (2.1) is called the *equation of motion* in the medium, where u_i is the displacement in i th component, ρ is the density of medium, σ_{ij} would be the stress component and f_i represents the body force in i th component. For motion in three-dimensional case, stress tensor σ_{ij} is defined as

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad (2.2)$$

The terms of diagonal in equation (2.2) are often called normal stresses and the off-diagonal terms are called shear stresses.

In the case in which sources or body forces such as gravity are not being considered ($f_i=0$), u_i which is the displacement of the particle can be written as:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2.3)$$

where x_j ($j=1,2,3$) are the Cartesian coordinate axes.

Like stress tensor, there are generally relationships between nine strain components (ε_{ij}) and three displacement components (u_i). The first type of strain is also called the normal strains which can be defined as

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \quad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}, \quad \varepsilon_{33} = \frac{\partial u_3}{\partial x_3} \quad (2.4)$$

The second type of strain is shear strains which are defined as

$$\varepsilon_{12} = \frac{1}{2} \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right), \quad \varepsilon_{13} = \frac{1}{2} \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) \quad (2.5a)$$

$$\varepsilon_{21} = \frac{1}{2} \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} \right), \quad \varepsilon_{23} = \frac{1}{2} \left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) \quad (2.5b)$$

$$\varepsilon_{31} = \frac{1}{2} \left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \right), \quad \varepsilon_{32} = \frac{1}{2} \left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3} \right) \quad (2.5c)$$

By using compact indicial notation, nine strain terms which constitute the infinitesimal strain tensor with six independent quantities can be expressed as

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2.6)$$

The trace of the strain tensor called the cubic dilatation (θ) is defined as

$$\theta = \varepsilon_{ii} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \nabla \cdot u \quad (2.7)$$

2.1.3.2 Hooke's law for linear elasticity

Relationship between stress and strain and hence stress and displacement gradients can be derived from constitutive laws. In any given material, a complex relationship exists between stress and deformation, depending on parameters such as pressure, temperature, stress rate, strain history, and stress magnitude. However, for the small-magnitude, short-duration stresses of interest in seismology, almost all earth materials display a linear proportionality between stress and strain (Lay and Wallace, 1995). The most general form of a constitutive law for linear elasticity is *Hooke's law*

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \quad (2.8)$$

The constants of proportionality (C_{ijkl}) are known as *elastic moduli* and define the material properties of the medium. In its general form, C_{ijkl} is a third-order tensor with 81 terms relating the nine elements of the strain tensor to nine elements of the stress tensor by linear sum. In the case of an isotropic elastic substance, where the elastic properties for many materials and material composites in the earth are independent of direction or orientation of the sample, the constants of proportionality have only two independent moduli, called Lamé constants, λ and μ . These are related to C_{ijkl} by

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (2.9)$$

where the Kronecker delta function is used. Inserting this into (2.8) yields

$$\sigma_{ij} = [\lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})] \varepsilon_{kl} \quad (2.10)$$

which reduces (e.g., $\delta_{kl} \varepsilon_{kl} = \varepsilon_{kk}$) to

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \quad (2.11)$$

Using the equation (2.7), equation (2.11) can be written as