NUMERICAL MODEL FOR TSUNAMI PROPAGATION DUE TO A TIME DEPENDENT SOURCE ALONG PENINSULAR MALAYSIA AND SOUTHERN THAILAND

by

UMI HAZIRAH BINTI ISHAK

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IN THE NAME OF ALLAH S.W.T. (THE ALMIGHTY) THE GRACIOUS AND MOST MERCIFUL

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MODEL BERANGKA BAGI PERAMBATAN TSUNAMI BERDASARKAN SUMBER BERASASKAN MASA SEPANJANG SEMENANJUNG MALAYSIA DAN SELATAN THAILAND

ABSTRAK

Kejadian Gempa Bumi Sumatra-Andaman yang berlaku pada 26 Disember 2004 menghasilkan kesan yang amat dahsyat memandangkan ia menghasilkan tsunami yang tersebar luas ke Negara-negara yang berada di dalam lingkungan Lautan India. Kawasan persisran pantai Semenanjung Malaysia dan Selatan Thailand juga tidak terlepas dari terkena dari ancaman ombak tsunami tersebut lalu membuktikan dua kawasan yang terjejas ini turut berada di dalam kedudukan yang tidak selamat terhadap ancaman tsunami pada masa hadapan berdasarkan sumber tetap yang terdapat di Sumatra, Indonesia.

Sejak kejadian yang dahsyat itu, banyak pembelajaran dan kajian telah dilakukan oleh pengkaji-pengkaji dari serata dunia untuk lebih memahami fenomena tersebut seperti Kowalik et al. (2005) dan Roy dan Ismail (2005). Pengakaji-pengkaji ini menimbangkan dasar laut terangkat secara serentak sepanjang zon zumber yang mana ia disebabkan oleh pergeseran Kerak India/Burma secara tiba-tiba dan ia dijadikan sebagai syarat awal. Tetapi, pergerakan dasar laut sepanjang garis patahan itu dianggarkan sepanjang 1300km selama kira-kira 8 minit (Ishii et al., 2005). Ammon et al. (2005) juga menyatakan bahawa pergerakan itu melitupi hampir 1200km dari kawasan pergerakan garis patán selama kira-kira 500 saat. Jadi, daripada fakta-fakta berkenaan, pergerakan

permukaan air sepanjang zon sumber dikatakan bersandarkan masa. Jadi, menggunakan model Roy dan Ismail (2005), kejadian Tsunami Lautan India 2004 yang melanda persistan pantai Pulau Phuket di selatan Thailand dan Pulau Pinang di Semenanjung Malaysia dikaji dengan menggunakan sumber bersandarkan masa. Sumber bersandarkan masa ini dilaksanakan dengan membahagikan tempat pergerakan dasar laut yang berukuran 1200km panjang dan 300km lebar itu kepada 42 bagi 4 bahagian supaya ia diaktifkan secara beransur-ansur dari selatan ke utara dan barat ke timur pada 45 selang masa yang berlainan di mana selang masa ditetapkan pada 10 saat. Waktu ketibaan ombak tsunami adalah masing-masing kira-kira 90 minit dan 240 minit bagi Phuket dan Pulau Pinang. Paras air maksimum didapati tercatat di utara Phuket dan barat laut Pulau Pinang dan ini membuktikan kedua-dua lokasi persisiran ini lebih terdedah pada bahaya tsunami di masa akan datang.

Juga, dengan mempertimbangkan sempadan pulau dan sempadan persisiran pantai di Thailand dan Semenanjung Malaysia berada yang tidak lurus secara semulajadi, dengan menggunakan model Karim et al. (2007), sumber bersandarkan masa telah diaplikasikan ke atas model Air Cetek untuk mengkaji beberapa aspek Tsunami Lautan Indian 2004 dengan menggunakan sistem sempadan- penuh gris berlengkung berbanding model asal yang menggunakan sumber serentak. Di dalam model yang sama itu juga, model berjaring telah dilaksanakan berdekatan persistan pantai selatan Thailand dan Semenanjung Malaysia di mana kedalaman air laut mula berubah secara drastik lalu mengakibatkan perubahan halaju air laut yang seterusnya akan meningkatakan keberkesanan model yang baru yang terhasil. Keputusan yang terhasil daripada kedua-ua

model ini didapati hampir sama dengan data sebenar yang terdapat di laman web United Status Geological Survey (USGS). Kaedah beza terhingga (FTCS) yang digunakan di dalam kajian ini juga didapati amat berkesan bagi menyelesaikan Persamaan Air Cetek tidak linear terbabit.

Kata Kunci: Sumber bersandarkan masa, Model Air Cetek Tidak Linear, Tsunami Lautan India 2004, sempadan-penuh grid berlengkung, model berangka berjaring

NUMERICAL MODEL FOR TSUNAMI PROPAGATION DUE TO A TIME DEPENDENT SOURCE ALONG PENINSULAR MALAYSIA AND SOUTHERN THAILAND

ABSTRACT

The event of the 26th December 2004 Sumatra-Andaman Earthquake was very profound in its impact since it generated a widespread tsunami throughout the Indian Ocean rim countries. The west coast of Peninsular Malaysia and the coast of southern Thailand were also attacked by the massive tsunami waves thus proving that these two affected regions are also in a vulnerable position for tsunami surge due to a permanent source in Sumatra, Indonesia.

Ever since the devastating event, lots of studies and researches had been done by various researchers from all over the world in order to better understand the phenomenon. Kowalik et al. (2005) and Roy and Ismail (2005). These researchers considered an instantaneous sea surface uplift and subsidence along the whole source zone which was caused by an abrupt slip at the India / Burma plate interface and that was assigned as the initial condition. But, the rupture of the sea surface elevation along the fault line spreads over 1300km long and it propagated for about 8 minutes (Ishii et al., 2005). Ammon et al. (2005) also indicated that the rupture propagated approximately 1200km of the ruptured fault length in about 500 seconds. Thus, by considering these facts, the initial disturbance of the sea surface along the whole source zone is time dependent. Using the model of Roy and Ismail (2005), the event of 2004 Indian Ocean Tsunami along the coastal belts of Phuket Island in southern Thailand and Penang Island in Peninsular Malaysia has been investigated with response to a time dependent source. A time dependent source is applied by dividing the 1200km long by 300km width of the rupture into 42 by 4 segments which then will be activated gradually from south to north and west to east in 45 different time steps in which the time step is taken to be at 10 seconds. It is found that it took approximately 90 minutes and 240 minutes for the tsunami waves to reach both Phuket and Penang Island respectively. Also, maximum water level were computed at the north and north-west coast of Phuket and Penang respectively thus proving that these two coastal locations are more vulnerable for tsunami attack in the future.

By considering the fact that the coastal and island boundaries of Southern Thailand and Peninsular Malaysia are curvilinear in nature, using the model of Karim et al. (2007), a time dependent source is applied to the Shallow Water model in order to compute several aspects of the 2004 Indian Ocean Tsunami using the boundary-fitted curvilinear grids systems instead of the instantaneous source used in the original model. Also, in the same model, a nested numerical model is developed near the coastal belts of Phuket and Penang as the water depth changes rapidly which leads to huge velocity gradient thus increasing the accuracy of our new developed model. The results obtained from both models are found to be in reasonable agreement with the observed data available in USGS website. The finite difference method (or Forward in Time and Central in Space, FTCS, method in particular) used in this study was found to be a relevant method in discretizing the Non-Linear Shallow Water Equations.

Key words: Time dependent source, Non-Linear Shallow Water model, 2004 Indian Ocean Tsunami, boundary-fitted curvilinear grid, nested numerical model.

CHAPTER 1 - INTRODUCTION

1.0 Introduction

Tsunami is a natural disaster with a high impact. The latest big tsunami that occurred in this century was the 26th December 2004 Indian Ocean tsunami. The earthquake generates a widespread tsunami around the Indian Ocean rim countries where the total of death toll is estimated to be as high as 310,000 people. From the extensive number of loss of lives and damages on properties due to tsunami, we can see how dangerous a tsunami can be. In this study the event of 2004 Indian Ocean Tsunami that hit the coastal belts of Phuket, Thailand and Penang Island located in the west coast of Peninsular Malaysia is simulated using mathematical and numerical models.

Tsunami modeling is very important in helping researchers to understand the aspects involved in tsunami. Tsunami modeling can help people to better understand how tsunami waves are generated, how it propagates from the open sea towards the land and how much damage can it caused to the environment and society. Tsunami modeling is not only beneficial for academic research. It is also useful in developing tsunami early warning system that is capable of reducing the impact of the disastrous hazard. After December 26, 2004 Indonesian Tsunami, significant developments have been achieved in tsunami modeling for serving not only academic research but also operational demands such as tsunami early warning systems. Some of these studies were done by Kowalik et al. (2005) who developed a global model that shows the propagation of the 2004 Indian

Ocean Tsunami over the whole world, Roy and Ismail (2005) who developed a shallow water model to compute several aspects of the 2004 Indian Ocean Tsunami in Thailand and Peninsular Malaysia and many others. All of these work done by these researchers assumed that the tsunami source generation (initial condition) is generated by an instantaneous sea surface elevation due to sea floor uplift and subsidence. Also, in Meah, (2008) thesis, he considered the initial disturbance of the sea surface of 2004 Indian Ocean Tsunami to be instantaneously uplifted.

But, the sea surface disturbance does not happen instantaneously as what was used as the initial condition of Roy and Ismail (2005) study. It took approximately 8 minutes for the rupture to propagate northwards which took place along the 1300km long source zone (Ishii et al., 2005). Moreover, the seismic inversion model developed by Ammon et al. (2005) indicated that the rupture of the 2004 Sumatra-Andaman Earthquake propagated northwards from its epicenter, parallel to trenches at a shear wave speed of 2-3km/s thus covering approximately 1200km of ruptured fault length in about 550 seconds. Thus, the initial disturbance of the sea surface elevation is dependent on time and that makes the initial condition of our work to be time dependent. Furthermore, the results obtained from our model is found to be consistent with the observed data and is in good agreement with the data found in the United States Geological Survey (USGS) website. Thus, this makes our work to be meaningful in attaining a better approximation in studying several aspects of the tsunami.

In this thesis, several effects of the 2004 Indian Ocean Tsunami in Phuket and Penang will be studied. The aspects that will be studied in this research are the propagation of tsunami waves, arrival time at a certain coastal location and the maximum water level in every particular place. Proper numerical models will be developed in order for us to get the results. The model used in this dissertation is the Non-Linear Shallow Water Model in Cartesian Coordinates. The finite difference method is applied to the system of the non-linear Shallow Water Equations in order to discretize the spatial variables included in the equations.

The coastal boundaries of Southern Thailand and west Peninsular Malaysia together with the island boundaries of Phuket Island and Penang Island are curvilinear in nature. Moreover, the bending is especially high along the coast of Southern Thailand. Due to this matter, the boundary-fitted curvilinear method is applied whereas the coastal boundaries of Peninsular Malaysia and Southern Thailand together with the western open sea boundary will be presented by two functions. Also, the northern and southern open sea boundaries will be considered as straight lines. By applying the boundary-fitted curvilinear grid approach, the boundaries mentioned above can be matched accurately since the model grids will fit well with the coastline and bathymetry. Then, necessary transformation of independent coordinates will be applied so that the curvilinear physical domain will be transformed to a rectangular computational domain. Only then, the finite difference method can be applied to this new system.

Also, rapid change of sea water velocity is expected near the coast of Southern Thailand and Peninsular Malaysia as the whole model domain also covers the eastern part of the Indian Ocean. The water in this area varies from 5m to 3000m. Since the water velocity depends on ocean depth, large gradient of velocities are expected near shallow regions. Rapid changes of velocities are expected since the model domain covers both shallow water and deep sea. Due to this matter, the nested numerical method is also applied since it can maximize the accuracy of our developed model with less grid numbers. We will develop a one-way nested numerical model in this study. In one-way nested numerical model, a fine mesh (inner model) with smaller grid size is used in covering the coast of Southern Thailand and Peninsular Malaysia and the Phuket Island and Penang Island of longitude5°10’N-5°35’N and 7°6’N-8°N and latitude 100°E-100°30’E and 98°E-98°4’E. The fine mesh (inner model) will be developed in the coarse mesh (outer model) that covers the open sea up to the coastal belts of Southern Thailand and Peninsular Malaysia of latitude 91°E-100.5°E and longitude 2°N-14°N which also includes the 2004 Sumatra-Andaman earthquake source in North Sumatra, Indonesia. In the one-way nested numerical model, the values of velocities and water elevations computed in the coarse mesh will enter and affect the fine mesh through the interface between then in each time step of the solution process. The disturbances from the fine mesh will not feed back into the coarse mesh and hence making the coarse mesh to be entirely independent of the fine mesh. Thus, by applying the nested numerical grid approach, more accurate results can be obtained. These different approaches will be used in computing several aspects of 2004 Indian Ocean Tsunami that hit the coasts of Phuket Island in Southern Thailand and Penang Island in Peninsular Malaysia.

1.1. Background

Tsunami which originated from the Japanese phrase for ‘harbor waves’ is defined as a series of ocean waves with an extremely long wavelength compared to its water depth and long period that is generated due to an impulsive disturbance that then will vertically displace the water from its equilibrium position. The word tsunami comes from a Japanese word consisting of two syllables, “tsu”, meaning “harbor” and “nami” which means “wave” and is used to describe the event because tsunamis are common throughout Japanese history.

Tsunami is generated due to an impulsive disturbance that will vertically displace the water volume from its equilibrium position. These disturbances may be due to several factors such as underwater earthquakes, mass movements above or below sea water, volcanic eruptions and other underwater explosions, landslides, earthquakes and several others but mostly generated due to submarine earthquakes. A volcano’s slope failure can generate tsunami where the sea water will be displaced as the volcano erupts. Submarine landslides which usually triggered by an earthquake can generate tsunami as the slide material and sediments that were redistributed across the sea floor affect the amplitude of the sea water causing tsunami to occur. Testing of nuclear weapons has also been known to generate tsunami just like the tsunami that occurred at the Marshall Islands in 1940 (Yalciner et al.,2005).

Submarine earthquake has generated most of the tsunami that ever occurred during the past history. Submarine earthquake generates tsunami when the sea floor is abruptly deforms due to the released of strain accumulated over years that then will displace the water from its equilibrium position. The process starts by the friction of two the subducting plate and the overriding plate that is caused due to the oceanic plate being forced down to the mantle via plate tectonic forces. Further friction between these two plates then caused them to “stuck”. As the stuck plates continue to descend, energy will be accumulated there and its amount is very similar to the amount of energy stored in a compressed spring. The energy will be accumulated over a long time of period and once its amount exceeds the frictional forces between the two colliding plates, the overriding plate will snap back into an unrestrained position and causing tsunami to occur due to the enormous shove of the overlying water. At the same time, inland areas of the overriding plate are suddenly lowered. Then, the resulting tsunami will spread. Some of the wave will travel to the deep ocean as a distant tsunami and some will travel to the nearby coast as a local tsunami. The impact of the local tsunami can be very devastating due to the immense volumes of water and the amount of energy involved which is very large in magnitude.

Tsunami is characterized as shallow water waves since the ratio of its water depth compared to its wave length is very small. Shallow water waves are different compared to wind-generated waves that we usually see on the beach. The period of a tsunami is extremely long as it is in the range of ten minutes up to two hours and its length is in excess of 500 kilometers while the wind-generated waves usually have period of five to twenty seconds and wavelength of about 100 to 200 meters. Since the wavelength of a tsunami is very large (compared to its depth), it will lose some of its energy as it propagates. Hence, in very deep water, a tsunami will travel at a high speed over a great transoceanic distance with limited energy loss. As it travel towards the nearby coastal belt where the water depth there decrease (shallow water), the velocity also decreases but the amount of the energy remains constant. The period between the successive waves which also remain constant then will force the water between the waves causing the height of water to increase. The speed of tsunami is equal to the square root of the product of the acceleration of gravity (given by 9.81m/s2) and its corresponding water depth (Yalciner et al., 2005**)**.

1.2 2004 Indian Ocean Tsunami

On 26 December 2004, a devastating earthquake generated tsunami occurred in the Indian Ocean. The earthquake occurred at 00:58:53 UTC (07:58:53 local time). The main event is then followed by an intense aftershock activity that then spread out over a length of about 1200km and a width about 200km between 3-15°N latitude along the Andaman-Nicobar island arc region. The intense earthquake then triggered a series of tsunamis causing fatalities in various Indian Ocean rim countries such as Indonesia, India, Malaysia, Sri Lanka, Maldives, Thailand and many others. More surprisingly, the tsunami waves also reach the east coast of Africa which is about 4500km (2800 miles) away from the epicenter of the earthquake.

Figure 1.1: Tsunami earthquake map provided by USGS

(Source: http://geology.com/articles/tsunami-map.shtml)

Figure 1.1 depicts the map of the 2004 Indian Ocean Tsunami map associated with the Sumatra-Andaman Earthquake. This 26 December 2004 earthquake measuring at 9.0 magnitudes on Richter scale was the fourth largest earthquake that ever occurred during the century where the largest earthquake occurred happened to be the 1960 Great Chilean Earthquake where the earthquake was at magnitude 9.5 on Richter scale. After the main earthquake, several aftershock activities are reported to occur in the Andaman Islands, Nicobar Islands and at the other places in the region of the epicenter of the earthquake. The hypocenter of the earthquake took place at 3.316°N, 95.854°E which is about 160km away from the west of Sumatra, Indonesia. The depth was estimated to be at about 30km below mean sea level. It was observed that the center where the earthquake took place was exactly at the extreme western end of the Ring of Fire where almost 81% of world's largest earthquakes occurred.

The 2004 Indian Ocean Tsunami occurred as result of the subduction of two tectonic plates, that is the India Plate and Burma Plat located in the Sunda Trench. The 2004 Indian Ocean Tsunami is generated due to the India Plate that dived under the Burma Plate as a result of the main earthquake where the fault line is estimated to be at about 1200km (750 miles) long at about 15m (45 feet) along the subduction zone. The slip took place in two phases where the first phase involved a rupture at about 400km long and 100km wide. It proceeded for about 100 seconds at a speed of about 2km/s. After a pause for about 100 seconds, the rupture continued towards the Andaman and Nicobar Islands.

1.3 Numerical Modeling

Mathematical modeling is the formulation of a problem in terms of mathematical equations. Often, simplifying assumptions and approximations are required to enable the governing mathematical equation to be solved in order to yield useful information. Numerical modeling refers to a situation in mathematical modeling where the use of computers and techniques from numerical analysis is necessary. Numerical models can be used to study the effects of tsunami. Various parameters can be changed or neglected in order to study the effect of the tsunami. When the effects are known, effectiveness mitigation measures can be undertaken. There are three well known numerical methods. The methods are finite element method, finite difference method and finite volume method.

The finite difference method (FDM) is the method that will be used in this study since it is conceptually simple and popular for modeling tsunami. Thus, we can compare our results with other researchers. Finite difference method is a method that numerically solves certain partial differential equations by approximating its solutions via approximating the derivatives of the finite difference equations. The solutions will be replaced by an equivalent different quotient. The famous Taylor’s theorem will be used in approximating the quotient which then will results in the forward difference equation, backward difference equation and central difference equation. Some examples of finite difference method are the Forward in Time and Central in Space method (FTCS) used for the heat equation, Central in Time and Central in Space method (CTCS) used for the wave equation and many others**.**

1.4 Problem Statement

Tsunamis rarely occur in the Indian Ocean where we live right now since there are less seismic activities occurred here compared to the Pacific Ocean. But, on 26th December 2004, a shocking tsunami occurred in Indian Ocean as a result of a mega earthquake that took place in Sumatra, Indonesia region. The earthquake was very devastating as it generated a widespread tsunami that hit the coastal belts of various Indian Ocean rim countries including Malaysia, Sumatra and Thailand. Thus, it is safe to say that this region now is in a vulnerable position of a tsunami attack. In fact, we cannot prevent this hazard from occurring but we can reduce its impact via proper planning. Hence, this is why this research is conducted where we will study certain behavior of the Indian Ocean tsunami that attacked the coast of Phuket (Thailand) and Penang (Malaysia). The aspects that we are interested in studying is the arrival time of the tsunami wave at a certain location, how does the tsunami wave propagates as it reaches a certain point as well as the maximum water level at a particular location. In particular, we are interested to study models based on a time dependent source as previous studies on Phuket and Penang had assumed an instantaneous source and sea level rise.

1.5 Objectives and Methodology

There are two objectives in this study. The objectives are

(i) To develop a numerical model that shows the propagation of the 2004 Indian Ocean Tsunami, the arrival time and maximum water levels computed in the coastal belts of Penang and Phuket using the staggered grid scheme in Cartesian Coordinates Model based on a time dependent source.

(ii) To develop a numerical model that shows the propagation of the 2004 Indian Ocean Tsunami, the arrival time and maximum water levels computed in the coastal belts of Penang and Phuket using the boundary fitted curvilinear and nested-numerical approach based on a time dependent source.

To get the results of our research, we will be using the Non-Linear Shallow Water Equations as our governing equations for this problem. These equations will be discretized using a finite difference method or Forward in Time and Central in Space (FTCS) method in particular. Also, the boundary fitted nested model will be used in computing several aspects of tsunami that hit the coastal belt of Phuket and Penang since the coastal belt is curvilinear in nature. A boundary fitted nested numerical model is also built to calculate the maximum water levels, propagation of tsunami waves and the arrival time of tsunami at Penang's coast. These two models will give us a more accurate approximation. Then, the results of all models used will be compared with observed data and the results obtained from other models.

1.6 Outline

This thesis contains six chapters. In the first chapter, the introduction of this topic will be discussed briefly such as the background of the 2004 Indian Ocean Tsunami, the characteristics of tsunami and the factors that can generate tsunami. The objective of this research will also be discussed in this chapter. In chapter 2, the basic concepts, theory and method used for this research work will be discussed in detail. Literature survey used in helping this research work will be discussed in the third chapter of this thesis where several papers have been used as a reference. Also, in chapter 4, the 2004 Indian Ocean tsunami source generation will be discussed in detail. The numerical simulation of the 2004 Indian Ocean Tsunami due to a time-dependent source using a staggered grid scheme will be discussed in chapter 5 whereas numerical simulation of the 2004 Indian Ocean Tsunami using the boundary fitted curvilinear and nested numerical model approach is discussed in the sixth chapter of this thesis. Last but not least, in chapter 7, the conclusions of the results and suggestions for further research work will be presented.

CHAPTER 2 - BASIC CONCEPTS, THEORY AND METHODS

2.0 Introduction to numerical methods

Numerical method is an area in computer science and mathematics that creates, analyzes, and implements algorithm to solve problems of continuous mathematics numerically. It is a mathematical technique used to generate approximate solutions to problems that arise in our daily life. Such problems may occur throughout natural sciences, social sciences, engineering, business and medicine. It is very suitable in solving problems that cannot be solved analytically. Fast growth in power and capabilities of digital computers has led in increasing use of realistic mathematical models in science and engineering.

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There are several examples of numerical methods used in computations. They are finite difference method, finite element method and several others. For this research, the finite difference method will be used to compute the maximum water levels and the arrival time of a tsunami wave at a certain location of our interest. In particular, the Forward in Time and Central in Space method (FTCS) will be used. The explanation to this method will be discussed later in this chapter.

2.1 Finite Difference Method and Approximations

In this research, a rectangular grid system is generated to represent the coastal area of Phuket Island in Southern Thailand and Penang Island in Peninsular Malaysia. In order for us to do that, some knowledge on finite difference method is needed where the grid system that we generated to represent the analysis area is a finite difference grid. The finite grid is quite accurate in giving us the solutions for this problem.

Finite difference method is a numerical method used to approximate solutions to differential equations using a finite difference representation to approximate derivatives. Finite difference is useful in obtaining the numerical solutions to the governing equations of flow and relevant to tsunami.

Suppose that $u = u(x, y)$. The first quadrant of the xy-plane is divided into several uniform rectangles by grid lines parallel to the x-axis (uniform length Δy) and grid lines parallel to the y-axis (uniform length Δx). Figure 2.1 depicts the finite difference grid used in this study.

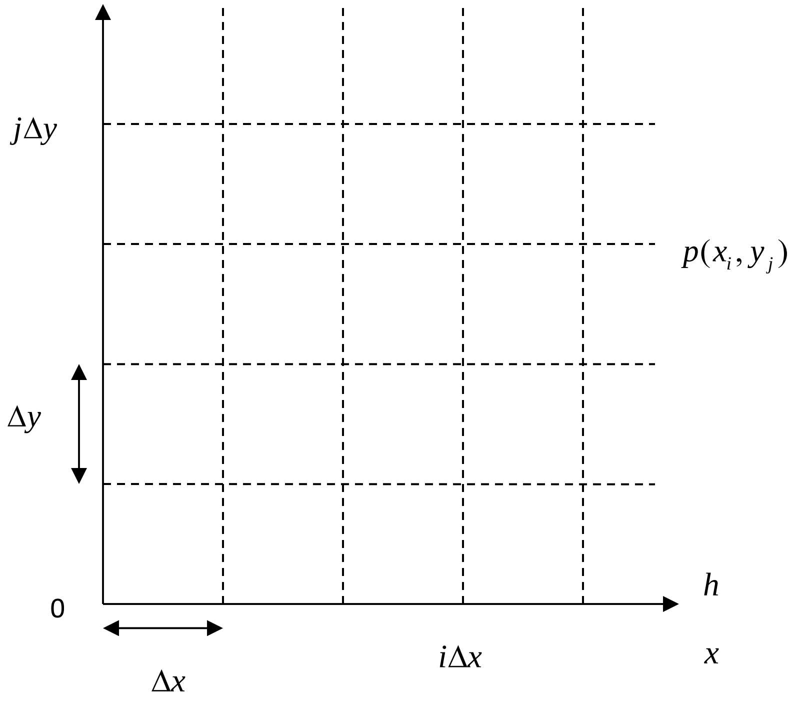

Figure 2.1: Finite Difference Grid

The function whose derivatives are to be approximated is assumed to be properly-behaved whereas it must have only one numerical value at any point in the space, the function itself together with its first and second derivatives must be finite and continuous and it must have a finite integral over all space. Taylor's Theorem gives a sequence of approximations of a differentiable equation around a given point by polynomials whose coefficients depend only on the derivatives of the function at that point. The theorem says that suppose that f and its first n derivatives are continuous on $[a,b]$, it is guaranteed that its $n+1$ derivatives on $[a,b]$ also exists. Then, for any x in $[a,b]$, there is a $c(x)$ between x and x_0 with

$$f(x) = P_n(x) + R_n(x)$$

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

$$R_n(x) = \sum_{k=0}^{n} \frac{f^{(k+1)}(c(x))}{(k+1)!}(x - x_0)^{k+1}$$

Or, the Taylor's Theorem is also written as

$$u(x+h) = u(x) + \frac{h}{1!}u'(x) + \frac{h^2}{2!}u''(x) + \frac{h^3}{3!}u'''(x) + \cdots$$

The above form can be truncated in order for us to give a representation of u in a form of polynomial such as follows:-

$$u(x+h) = u(x) + hu'(x) + \frac{h^2}{2}u''(\xi) + \cdots$$

where $x < \xi < x + h$. If the series is truncated after one term one gets the Mean Value Theorem of elementary calculus. We can write the above equation in a Big Oh Form.

$$u(x+h) = u(x) + hu'(x) + O(h^2)$$

which means that the error due to the truncation of the series is some expressions that goes to 0 as fast as h^2 as $h^2 \to 0$.

We applied the above Taylor series expansion for $u(x_i + h, y_j)$ about (x_i, y_j), and thus we get :-

$$u(x_i + h, y_j) = u(x_i, y_j) + \frac{h}{1!}u_x(x_i, y_j) + \frac{h^2}{2!}u_{xx}(x_i, y_j) + \frac{h^3}{3!}u_{xxx}(x_i, y_j) + \cdots \tag{2.1}$$

Suppose we decide to truncate the series on the RHS beginning with the third term. If h is sufficiently small, the fourth and higher terms are much smaller than the third term. Hence,

$$\begin{aligned}
u(x_i+h,y_j) &= u(x_i,y_j)+\frac{h}{1!}u_x(x_i,y_j)+O(h^2) \\
&= u(x_i,y_j)+hu_x(x_i,y_j)+O(h^2) && (2.2) \\
(\div h) \Rightarrow \frac{u(x_i+h,y_j)}{h} &= \frac{u(x_i,y_j)+hu_x(x_i,y_j)+O(h^2)}{h} \\
\frac{u(x_i+h,y_j)}{h} &= \frac{u(x_i,y_j)}{h}+u_x(x_i,y_j)+O(h) \\
\frac{u(x_i+h,y_j)-u(x_i,y_j)}{h} &- O(h) = u_x(x_i,y_j) \\
\frac{\partial u}{\partial x}\left(x_i,y_j\right) = u_x(x_i,y_j) &= \frac{u(x_i+h,y_j)-u(x_i,y_j)}{h}-O(h) && (2.3)
\end{aligned}$$

$\frac{u(x_i+h,y_j)-u(x_i,y_j)}{h}$ is said to be the 'forward difference approximation' to $\frac{\partial u}{\partial x}$ at (x_i,y_j) and is said to be first order accurate or $O(h)$ accurate.

The Taylor series expansion for $u(x_i-h,y_j)$ about (x_i,y_j) is given by:-

$$u(x_i-h,y_j)=u(x_i,y_j)-\frac{h}{1!}u_x(x_i,y_j)+\frac{h^2}{2!}u_{xx}(x_i,y_j)-\frac{h^3}{3!}u_{xxx}(x_i,y_j)+\frac{h^4}{4!}u_{xxxx}(x_i,y_j)+\cdots \quad (2.4)$$

Again, by truncating the RHS of the above equation beginning with the third term and if h is sufficiently small, the fourth and fifth term will be smaller than the third term. Hence,

$$u(x_i - h, y_j) = u(x_i, y_j) - hu_x(x_i, y_j) + O(h^2) \tag{2.5}$$

$$(\div h)\frac{u(x_i - h, y_j)}{h} = \frac{u(x_i, y_j) - hu_x(x_i, y_j) + O(h^2)}{h}$$

$$\frac{u(x_i - h, y_j)}{h} = \frac{u(x_i, y_j)}{h} - u_x(x_i, y_j) + O(h)$$

$$\frac{\partial u}{\partial x}(x_i, y_j) = u_x(x_i, y_j) = \frac{u(x_i, y_j) - u(x_i - h, y_j)}{h} + O(h) \tag{2.6}$$

$u_x(x_i, y_j) = \dfrac{u(x_i, y_j) - u(x_i - \Delta x, y_j)}{\Delta x}$ is said to be the ‘backward difference approximation’ to u_x at (x_i, y_j) and said to be first order accurate.

When we subtract equation (2.4) from (2.1), we get :-

$$\begin{aligned} u(x_i + h, y_j) - u(x_i - h, y_j) &= \left[u(x_i, y_j) + \frac{h}{1!}u_x(x_i, y_j) + O\left(h^2\right)\right] \\ &\quad - \left[u(x_i, y_j) - \frac{h}{1!}u_x(x_i, y_j) + O\left(h^2\right)\right] \\ &= 2hu_x(x_i, y_j) + O\left(h^3\right) \end{aligned} \tag{2.7}$$

$$(2.7) \div 2h \;\Rightarrow\; \frac{u(x_i + h, y_j) - u(x_i - h, y_j)}{2h} = u_x(x_i, y_j) + O(h^2)$$

$$\Rightarrow \frac{\partial u}{\partial x}(x_i, y_j) = u_x(x_i, y_j) = \frac{u(x_i + h, y_j) - u(x_i - h, y_j)}{2h} - O(h^2) \tag{2.8}$$

$\dfrac{u(x_i + h, y_j) - u(x_i - h, y_j)}{2\Delta x}$ is said to be the ‘central difference approximation’ to u_x at (x_i, y_j) and is said to be second order accurate or $O(h^2)$. Because the approximation to the derivative is more accurate, central difference approximation should be used as much as possible (provided that it does not cause problem).

$(2.1)+(2.4):-$

$$u(x_i+h,y_j)+u(x_i-h,y_j)=[u(x_i,y_j)+hu_x(x_i,y_j)+\frac{h^2}{2}u_{xx}(x_i,y_j)+O(h^3)]$$

$$+[u(x_i,y_j)-hu_x(x_i,y_j)+\frac{h^2}{2}u_{xx}(x_i,y_j)+...]$$

$$u(x_i+h,y_j)+u(x_i-h,y_j)=2u(x_i,y_j)+h^2u_{xx}(x_i,y_j)+O(h^4) \qquad (2.9)$$

$$(2.9)\div h^2 \Rightarrow \frac{u(x_i+h,y_j)-u(x_i-h,y_j)}{h^2}=\frac{2u(x_i,y_j)+h^2u_{xx}(x_i,y_j)+O(h^4)}{h^2}$$

$$\frac{u(x_i+h,y_j)-u(x_i-h,y_j)}{h^2}=\frac{2u(x_i,y_j)+O\left(h^4\right)}{h^2}+u_{xx}(x_i,y_j)$$

$$\frac{\partial^2 u}{\partial x^2}(x_i,y_j)=u_{xx}(x_i,y_j)=\frac{u(x_i+h,y_j)-2u(x_i,y_j)+u(x_i-h,y_j)}{h^2}-O(h^2) \qquad (2.10)$$

$\frac{u(x_i+h,y_j)-2u(x_i,y_j)+u(x_i-h,y_j)}{h^2}$ is said to be the ‘central difference approximation’ to u_{xx} and second order accurate or $O(h^2)$ accurate.

Using similar arguments:-

$$\frac{\partial u}{\partial y}(x_i,y_j)\approx\frac{u(x_i,y_j+h)-u(x_i,y_j)}{h} \text{ (forward difference to } u_y\text{)}$$

$$\frac{\partial u}{\partial y}(x_i,y_j)\approx\frac{u(x_i,y_j)-u(x_i,y_j-h)}{h} \text{ (backward difference to } u_y\text{)}$$

$$\frac{\partial u}{\partial y}(x_i,y_j)\approx\frac{u(x_i,y_j+h)-u(x_i,y_j-h)}{2h} \text{ (central difference to } u_y\text{)}$$

$$\frac{\partial^2 u}{\partial y^2}(x_i,y_j)\approx\frac{u(x_i,y_j+h)-2u(x_i,y_j)+u(x_i,y_j-h)}{h^2} \text{ (central difference to } u_{yy}\text{)}$$

The points schematics used in finite difference scheme is presented in Figure 2.2. As seen in the figure, the i points are located in the x-axis while the j points are located in the y-axis.

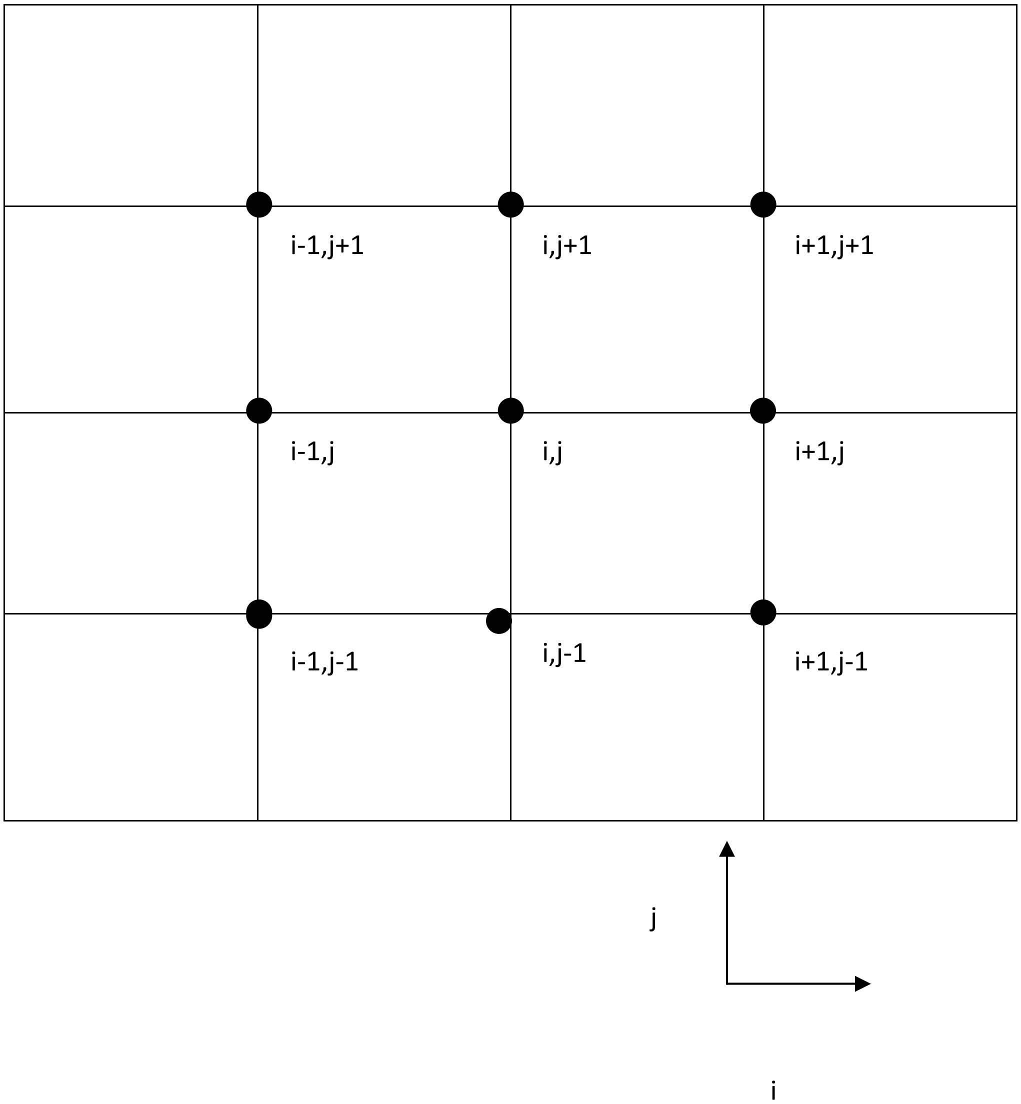

Figure 2.2: Schematics points in finite difference scheme

In this particular research, we will be using the FTCS method (Forward in Time and Central in Space), the same method as used in Roy and Ismail in 2005. This means that, the Forward Difference formula will be used in dicretrizing the partial variables with respect to time (t) while the Central Difference formula will be used in the discretization of partial variables with respect to space (x,y,r,θ,etc) **.** Figure 2.3 depicts the illustration of forward, backward and central difference approximation.

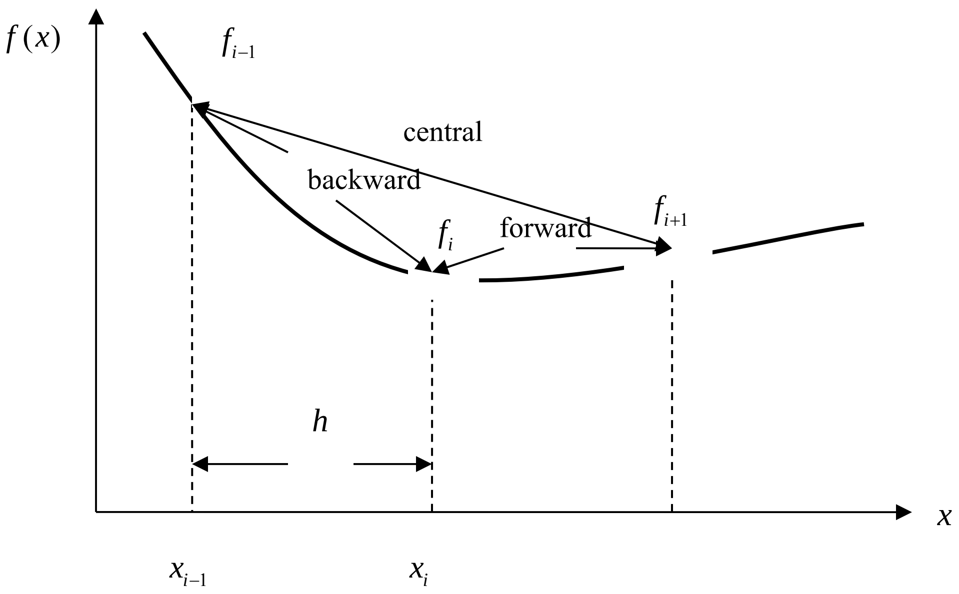

Figure 2.3: Forward, backward and central difference approximation

2.2 Governing Equations – Non-Linear Shallow Water Equations

Shallow water equations describe the motion of an incompressible fluid (a type of fluid where the density is constant) in response to gravitational and rotational accelerations (Randall, 2006). They are a set of hyperbolic partial differential equations and often used for tsunami modeling.

Tsunami wave is known to have a very large wavelength compared to its depth. Hence, due to its very small ratio of water depth over its wavelength which is typically about 0.05, tsunami is characterized as shallow water. The equations can be derived from the Navier-Stokes equations which are used to describe the motion of a fluid. Essentially, the equations describe the conservation of momentum while the other equation involved is the continuity equation or also known as the conservation of mass equation.

The continuity and Navier Stokes equations are as follows (Chlorin, 1968):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.11}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+E_x \tag{2.12}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+E_y \tag{2.13}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+E_z \tag{2.14}$$

where

$u, v, w =$ instantaneous velocities of flow particle in x, y and z directions respectively

$\rho =$ fluid density

$p =$ pressure

$E_x, E_y, E_z =$ External forces per unit mass in x, y and z directions respectively

$\mu =$ kinematic viscosity $\mu = \left(\dfrac{\nu}{\rho} \right)$

The equation (2.11) is the continuity equation while the equations (2.12), (2.13) and (2.14) are the momentum equations with respect to the x, y and z directions respectively. From these equations, several assumptions can be made to derive the shallow water equations. The assumptions are:-

(i) Vertical acceleration of the fluid particles can be neglected, i.e. vertical velocity of fluid particles change very slowly and the vertical magnitude is very small compared to the horizontal magnitude

(ii) Only gravitational forces act in the vertical direction

(iii) Amplitude of oscillations (waves) are very small compared to water depth

After such assumptions are made, the momentum equations in the z-direction can be approximated by the hydrostatic equation $\dfrac{dp}{dz} = -\rho g$. This means that the pressure only varies in the z-direction, not in the x and y directions. The negative sign on the front indicates that the pressure is inversely proportional to the z-direction, i.e. the pressure increases as we move down and decreases as we move up.

After neglecting the molecular viscosity and averaging the turbulent, the continuity equation and momentum equations turn out to be as follows (given by Roy,1998) :-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.15}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_x}{\partial z} \tag{2.16}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_y}{\partial z} \tag{2.17}$$

where

$\tau_x, \tau_y =$ eddy stress

$f = 2\Omega \sin\varphi =$ Coriolis parameter (a parameter due to the rotation of the earth) where Ω is the angular speed of the earth while φ is the latitude of the location

2.3 Depth Averaged Shallow Water Equations in Cartesian Coordinates

Figure 2.4 shows the displaced position of the free surface (or usually known as water elevation) to be taken as $z=\zeta(x,y,t)$ while the water depth from the mean sea level (MSL) to the sea floor is taken to be $z=-h(x,y)$. Hence, the total depth is equal to $D=\zeta+h$.