HISTOGRAM EQUALIZATION BASED METHODS FOR BRIGHTNESS PRESERVATION AND LOCAL CONTENT EMPHASIS

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HISTOGRAM EQUALIZATION BASED METHODS FOR BRIGHTNESS PRESERVATION AND LOCAL CONTENT EMPHASIS

by

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LIST OF ABBREVIATIONS

1-D	One-Dimensional
2-D	Two-Dimensional
AAMBE	Average Absolute Mean Brightness Error
AHE	Adaptive Histogram Equalization
AMBE	Absolute Mean Brightness Error
BBHE	Brightness Preserving Bi-Histogram Equalization
BERF	Blocking Effect Reduction Filter
BLS	Black Level Stretch
BMHE	Bin Modified Histogram Equalization
BO	Bin Overflow
BOHE	Block Overlapped Histogram Equalization
BPDHE	Brightness Preserving Dynamic Histogram Equalization
BPWCHE	Brightness Preserving Weight Clustering Histogram Equalization
BU	Bin Underflow
BUBOHE	Histogram Equalization with Bin Underflow and Bin Overflow
CDF	Cumulative Density Function
CLAHE	Contrast Limited Adaptive Histogram Equalization
CR	Contextual Region

CSBHE	Conditional Sub-Block Bi-Histogram Equalization
CT	Computed Tomography
DHE	Dynamic Histogram Equalization
DLP	Digital Light Processing
DNA	DeoxyriboNucleic Acid
DSIHE	Dual Sub-Image Histogram Equalization
EGC	Enhancement Gain Control
FFT	Fast Fourier Transform
GHE	Global Histogram Equalization
HE	Histogram Equalization
HP	Histogram Projection
IAHE	Interpolated Adaptive Histogram Equalization
LCD	Liquid Crystal Display
LHE	Local Histogram Equalization
LIHE	Local Information Histogram Equalization
MBPHE	Mean Brightness Preserving Histogram Equalization
MHE	Multi-Histogram Equalization
MLBOHE	Multi-Levels Block Overlapped Histogram Equalization
MMBEBHE	Minimum Mean Brightness Error Bi-Histogram Equalization
MMLSEMHE	Minimum Middle Level Squared Error Multi-Histogram Equalization

MPHE	Multipeak Histogram Equalization
MRI	Magnetic Resonance Imaging
MSSI	Mean Structural Similarity Index
MWCVMHE	Minimum Within-Class Variance Multi-Histogram Equalization
NOBHE	Non-Overlapped Block Histogram Equalization
PDF	Probability Density Function
POSHE	Partially-Overlapped Sub-Block Histogram Equalization
PSNR	Peak Signal-to-Noise Ratio
QBHE	Quantized Bi-Histogram Equalization
RMSHE	Recursive Mean-Separate Histogram Equalization
RSIHE	Recursive Sub-Image Histogram Equalization
RSWHE	Recursive Separated and Weighted Histogram Equalization
SAPHE	Self-Adaptive Plateau Histogram Equalization
STAHE	Spatial-Temporally Adaptive Histogram Equalization
VRAHE	Variable Region Adaptive Histogram Equalization
WAHE	Weighted Adaptive Histogram Equalization
WLS	White Level Stretch
WTHE	Weighted and Thresholded Histogram Equalization

LIST OF SYMBOLS

α	the power index
η	the cluster weight ratio
$\forall$	for all; for each; for any
γ	the amount of emphasis given in frequency
μ	mean
σ	standard deviation
A_{MBE}	AMBE
$\overline{A}_{MBE}$	AAMBE
$\overline{b}$	the normalized of B channel
b'	the processed of $\overline{b}$ channel
B	blue channel
B'	the processed of B channel
$\mathbb{B} \times \mathbb{B}$	the size of a overlapped block
$c(X_k)$	CDF
$\widetilde{c}(X_k)$	the resultant CDF
c_{BU}	the bin underflow threshold
c_{BO}	the bin overflow threshold
$c_{CR}(x)$	the CDF of CR

$c_L(x)$	the CDF for $\{\mathbf{X}\}_L$
$c_U(x)$	the CDF for $\{\mathbf{X}\}_U$
$c_{wt}(x)$	the CDF of p_{wt}
$C(R_{sub})$	the cost function
$C_n(i)$	a unique cluster
$Disc(R_{sub})$	the discrepancy
E	entropy
$\overline{E}$	the average of E
E_{hor}	the horizontal amount of edges
E_{ver}	the vertical amount of edges
$f(x)$	the transform function
$f_{CR}(x)$	the transform function of CR
$f_L(x)$	the transform function of $\{\mathbf{X}\}_L$
$f_U(x)$	the transform function of $\{\mathbf{X}\}_U$
$factor_a$	the factor of the gray level distribution for each subhistogram a
$\widehat{F}$	the cluster width
F_a	the total frequency of gray levels in subhistogram a
$\overline{g}$	the normalized of G channel
g'	the processed of $\overline{g}$ channel
G	green channel

G'	the processed of G channel
$\widehat{G}(X_k)$	1-D Gaussian filter
G_{max}	the pre-set maximum gain of dynamic range
$h(X_k)$	the histogram of X_k
$\widetilde{h}(x)$	the modified histogram
$h_{acc}(x)$	the cumulative histogram
$h_{acc}^{CR}(x)$	the cumulative histogram of CR
$h_{CR}(x)$	the histogram of each CR
$h_{int}(X_k)$	the interpolated histogram
$h_s(X_k)$	the smoothed version of h_{int}
H	hue channel
i	the spatial position at the horizontal axis of an image
I	intensity channel
I'	the processed of I channel
j	the spatial position at the vertical axis of an image
K	discrete levels
L	the discrete gray level
L_{in}	the dynamic range of the input image
L_{out}	the dynamic range of the output image
m	the number of partitions of the histogram

$\mathbb{m} \times \mathbb{n}$	the size of a non-overlapped block
l_a	a-th local minimum in the input image histogram
$\mathbb{M} \times \mathbb{N}$	the size of an image
M_{adj}	the mean adjustment factor
M_{CR}	the mean gray level of each CR
M_i	the mean brightness of the original input image
M_o	the mean brightness of image Z
n	the total number of nonzero bins
$\widehat{n}$	the total number of minimum points
n_k	the number of times that the grat level X_k appears in the image
n_L^k	the number of X_k in $\{\mathbf{X}\}_L$
n_U^k	the number of X_k in $\{\mathbf{X}\}_U$
N	the total number of samples in the image
N_{CR}	the total number pixels of CR
N_L	the total number of samples in $\{\mathbf{X}\}_L$
N_{max}	the number of the local maximum values
N_T	the total number of tested images
N_U	the total number of samples in $\{\mathbf{X}\}_U$
O	the time complexity
$p(X_k)$	PDF

$\widetilde{p}(X_k)$	the resultant PDF
$p_{iU}(X_k)$	the PDF of sub-image r_{iU}
$p_L(X_k)$	the PDF for $\{\mathbf{X}\}_L$
$p_{max}(X_k)$	the highest PDF
$p_{min}(X_k)$	the lowest PDF
$p_n(X_k)$	the new probability value
$p_U(X_k)$	the PDF for $\{\mathbf{X}\}_U$
$p_w(X_k)$	the weighted PDF
$p_{wn}(X_k)$	the resultant weighted and normalized PDF
$p_{wt}(x)$	a weighted thresholded PDF
$P(i,j)$	the processed image
P_l	a lower threshold
P_{max}	the peak value or highest probability value of the original PDF
P_u	an upper threshold
$\overline{P}_{SNR}$	the average of PSNR
Q	the selected channel
Q'	the processed of the selected channel
r	the recursive level
$\overline{r}$	the normalized of R channel
r'	the processed of $\overline{r}$ channel

r_i the i-th sub-block

r_{iL} the lower sub-images of i-th sub-block

r_{iU} the upper sub-images of i-th sub-block

$range_a$ the range of gray levels allocated to the subhistogram a

R red channel

R' the processed of R channel

$\mathbb{R}$ indiscernibility relation

R_{sub} total pieces of subhistograms

R_A the common center region

R_B the border region

R_C the corner center region

s the normalized of S channel

$span_a$ the dynamic gray level range used by subhistogram a

S saturation channel

S' the processed of S channel

t the execution time

$\bar{t}$ the average of t

T threshold level

T_{max} the maximum value of T

U the chroma

v'	the normalized of V' channel
V	the chroma or luminance channel
V'	the processed of V channel
W	the cluster weight
W_{filter}	the filter size
$W_n(i)$	a weight
x_{max}	the upper limits of the transform gray level
x_{min}	the lower limits of the transform gray level
$\mathbf{X}$	the original image
$\overline{\mathbf{X}}$	the input mean
$X(i,j)$	the intensity of the input image at (i,j)
$X_{ctr}(i,j)$	the center intensity level
X_d	the median gray level of the image $\mathbf{X}$
X_{end}^{a}	the highest gray level value in subhistogram a
X_k	k-th gray level or intensity
X_l	the left non-empty bin for the empty region
X_{l_a}	the lowest gray level limits defined by subhistogram a
$\mathbf{X}_L$	the lower portion of the input image $\mathbf{X}$
X_m	the mean of the image $\mathbf{X}$
X_{max}	the highest intensity of the input image

X_{min}	the lowest intensity of the input image
X_M^a	the middle value of input sub-image a
$X_{neigh}(i,j)$	the neighbour intensity level
X_r	the right non-empty bin for the empty region
X_{start}^a	the lowest gray level value in subhistogram a
X_{u_a}	the highest gray level limits defined by subhistogram a
$\mathbf{X}_U$	the upper portion of the input image $\mathbf{X}$
Y	luminance or luma channel
$\mathbf{Y}$	the output image
$\overline{\mathbf{Y}}$	the output mean
Y'	the processed of Y channel
$Y(i,j)$	the intensity of output image
$\mathbf{Z}$	the complete enhanced image
$Z(i,j)$	the intensity of complete enhanced image

KAEDAH-KAEDAH BERASASKAN PENYERAGAMAN HISTOGRAM UNTUK PENGEKALAN KECERAHAN DAN PENEGASAN KANDUNGAN SETEMPAT

ABSTRAK

Penyeragaman Histogram Sejagat (*Global Histogram Equalization* (GHE)) adalah salah satu kaedah penyerlahan imej yang terkenal. GHE masih mempunyai batasan walaupun ia mudah dan dikenali. Oleh kerana itu, terdapat dua kaedah baru yang berdasarkan Penyeragaman Histogram (*Histogram Equalization* (HE)) telah dicadangkan iaitu Penyeragaman Histogram Dinamik Kecerahan Berkekalan (*Brightness Preserving Dynamic Histogram Equalization* (BPDHE)) dan Penyeragaman Histogram Bertindih Blok Berbilang Aras (*Multi-Levels Block Overlapped Histogram Equalization* (MLBOHE)). BPDHE dilengkapi dengan kebolehan mengekalkan kecerahan, iaitu salah satu syarat bagi pelaksanaan di dalam produk elektronik pengguna. BPDHE mengandungi tujuh langkah, iaitu pembentukan histogram, proses penentudalaman, proses pelicinan, proses pengesanan maksimum setempat, proses pemetaan, proses HE, dan proses penormalan kecerahan. Daripada perbandingan di antara BPDHE dengan GHE dan tujuh kaedah lain yang berasaskan Penyeragaman Histogram Berkekalan Min Kecerahan (*Mean Brightness Preserving Histogram Equalization* (MBPHE)), telah ditunjukkan bahawa BPDHE adalah yang terbaik bagi pengekalan kecerahan. Purata Ralat Sebenar bagi Min Kecerahan (*Average Absolute Mean Brightness Error* (AAMBE)) untuk BPDHE adalah hanya 1.06. Tambahan pula, BPDHE dapat menghasilkan penyerlahan yang asli dengan tidak menimbulkan sebarang artifak yang tidak dikehendaki. Satu lagi kaedah, iaitu MLBOHE, direka bagi penggunaan yang memerlukan penyerlahan setempat. MLBOHE terdiri daripada tiga

peringkat, iaitu Penyeragaman Histogram bagi Blok Bertindih (*Block Overlapped Histogram Equalization* (BOHE)), pengurangan aras hingar dan penggabungan imej untuk menghasilkan hasil keluaran terakhir. Secara objektif, MLBOHE memenuhi kesemua syarat rekabentuknya dengan dari segi masa pelaksanaan (iaitu $\bar{t} = 19.2$ saat < 1 minit bagi setiap Mega piksel), entropi (iaitu $\overline{E} = 7.35 >$ input), Nisbah Isyarat Puncak kepada Hingar (*Peak Signal-to-Noise Ratio* (PSNR)) (iaitu $\overline{P}_{SNR} = 54.77$, tertinggi di antara kaedah-kaedah yang diuji), dan Ralat Sebenar bagi Min Kecerahan (*Absolute Mean Brightness Error* (AMBE)) (iaitu $\overline{A}_{MBE} = 8.25$, terendah di antara kaedah-kaedah yang diuji). MLBOHE juga mempunyai prestasi yang lebih baik berbanding dengan GHE dan tiga kaedah Penyeragaman Histogram Setempat (*Local Histogram Equalization* (LHE)) yang lain. Daripada pemeriksaan melalui penglihatan, hasil daripada MLBOHE mempunyai aras hingar yang boleh diterima, dan tidak mengalami masalah ketepuan keamatan dan kesan bongkahan. Di samping itu, MLBOHE dapat mengekalkan bentuk asal histogram supaya tidak mengubah maklumat di dalam imej. Ujikaji bagi tujuh skim pemprosesan warna untuk kedua-dua BPDHE dan MLBOHE juga dijalankan. Daripada keputusan yang didapati, dicadangkan bahawa proses perlu berdasarkan saluran hijau (*Green* (*G*)) daripada sistem warna Merah Hijau Biru (*Red Green Blue* (*RGB*)) kerana kaedah ini adalah yang paling mudah, tetapi memberikan keputusan yang baik.

HISTOGRAM EQUALIZATION BASED METHODS FOR BRIGHTNESS PRESERVATION AND LOCAL CONTENT EMPHASIS

ABSTRACT

Global Histogram Equalization (GHE) is one of the well-known image enhancement methods. Despite of its simplicity and popularity, GHE still has limitations. Therefore, in this work, two novel Histogram Equalization (HE) based methods have been proposed, which are Brightness Preserving Dynamic Histogram Equalization (BPDHE) and Multi-Levels Block Overlapped Histogram Equalization (MLBOHE). BPDHE is equipped with brightness preserving ability, i.e. one of the requirements for the implementation in consumer electronic products. BPDHE consists of seven steps, which are histogram creation, interpolation process, smoothing process, local maximums detection process, mapping process, HE process, and brightness normalization process. From the comparison of BPDHE with GHE and seven other Mean Brightness Preserving Histogram Equalization (MBPHE) based methods, it is shown that BPDHE is the best in terms of brightness preservation. Its Average Absolute Mean Brightness Error (AAMBE) is only 1.06. Furthermore, BPDHE produces natural enhancement, without unwanted artifacts. Another method, which is MLBOHE, is designed for the applications that need local enhancement. MLBOHE consists of three stages, which are Block Overlapped Histogram Equalization (BOHE), noise level reduction, and merging of images to form the final image. Objectively, MLBOHE met all of its design requirements in terms of its average execution time (i.e. $\bar{t} = 19.2$ s < 1 min per Mega pixels), entropy (i.e. $\overline{E} = 7.35$ > input), Peak Signal-to-Noise Ratio (i.e. $\overline{P}_{SNR} = 54.77$, the highest among the methods tested), and Absolute

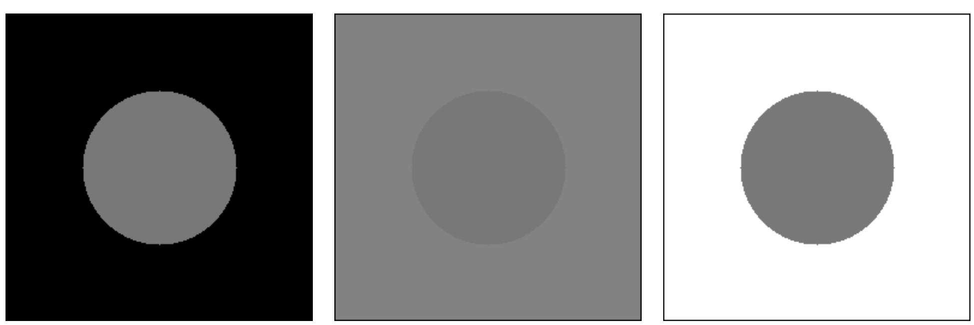
Mean Brightness Error (i.e. $\overline{A}_{MBE} = 8.25$, the lowest among the methods tested). MLBOHE also has a better performance as compared to GHE and three other Local Histogram Equalization (LHE) methods. By vision inspection, the results from MLBOHE have an acceptable level of noise, and do not suffer from intensity saturation problem and blocking effect. Besides, MLBOHE preserves the shape of the original histogram in order to maintain the image's information. Investigation on seven color processing schemes for both BPDHE and MLBOHE also have been carried out. From the results, it is proposed that the process should be based on the Green (G) channel from Red Green Blue (RGB) color space as this method is the simplest, but gives good results.

CHAPTER 1

INTRODUCTION

1.1 Background

Image enhancement, which is also known as contrast enhancement[1], is one of the most interesting and important processes in both human and computer vision field. The main purpose of image enhancement is to bring out details that are hidden in an image, or to improve the quality of the image, so that it will become suitable as an input to some specific automated processing systems [2]. Normally, image enhancement produces an output image which is subjectively looks better than the original image by changing the intensity values of the input image [1, 3, 4]. It stretches up the dynamic range of the image, and enlarges the intensity difference among objects and background. This is based on the assumption that the contrast is proportional to the ratio between the brightest and the darkest pixel intensities contained in an image [5, 6, 7].



(a) A circle with intensity 120 and background of 0

(b) A circle with intensity 120 and background of 130

(c) A circle with intensity 120 and background of 255

Figure 1.1: An example to illustrate the meaning of contrast

[1]The scope of image enhancement is very wide. It includes gray level and contrast manipulation, noise level reduction, edge crispening and sharpening, interpolation and magnification, and pseudocoloring [1]. However, the scope of this thesis is limited to contrast manipulation only. Thus, the terms of image enhancement and contrast enhancement will be used interchangeably.

Simulated images in Figure 1.1 show the role of intensity difference in contrast enhancement. In this figure, a circle with intensity level of 120, is overlaid to different backgrounds. The circle is easier to be seen when the intensity difference between this circle and its background is big, as shown in Figure 1.1(a) and 1.1(c). These images have a better contrast compared to Figure 1.1(b) where the intensity difference between the circle and its background is small. However, the contrast cannot rely on the intensity value alone [2].

Histogram is one of the important features which is very related to image enhancement. The histogram does not only gives us a general overview on some useful image statistics (e.g. mode, median, mean, and dynamic range of an image), but it is also very useful in image processing applications such as image compression and segmentation [2]. In order to define a histogram, first, assume that $\mathbf{X} = \{X(i, j)\}$ is an image that is composed of L discrete gray levels[2] denoted by $\{X_0, X_1, \cdots, X_{L-1}\}$. $X(i, j)$ represents the intensity of the image at spatial location (i, j) with the condition that $X(i, j) \in \{X_0, X_1, \cdots, X_{L-1}\}$. As the intensities are all in discrete values, the histogram of a digital image is a discrete function. Then, the histogram h is defined as:

$$h(X_k) = n_k, \quad \text{for } k = 0, 1, \cdots, L-1 \tag{1.1}$$

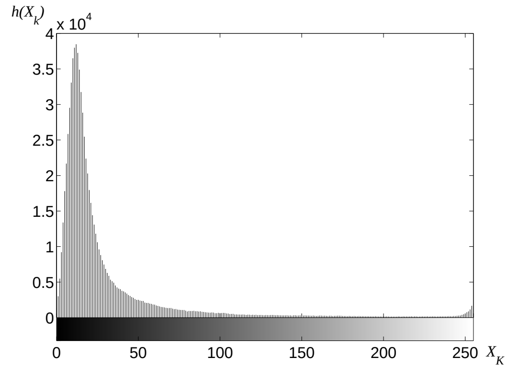
where X_k is the k-th gray level and n_k presents the number of times that the gray level X_k appears in the image. In other words, the histogram is the frequency of occurrence of the gray levels in the image [8]. Alternatively, as used by Wang et al. in [9, 10], the histogram also can be defined as the statistic probability distribution of each gray level in a digital image[3]. Usually, the histogram of an image $\mathbf{X}$ is presented as a graph plots of $h(X_k)$ versus X_k.

Some of the example images and their corresponding histograms are shown in Figure 1.2.

[2]The value of L presents the intensity resolution of an image. Typically, an grayscale image is recorded as an 8-bits depth per pixel image. Thus, for this case, $L = 2^8 = 256$.

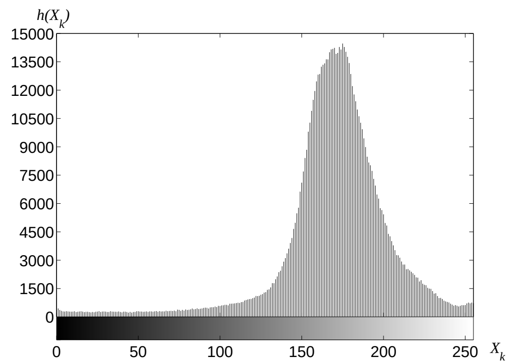
[3]The probability for the occurrence of intensity X_k in an image is defined as equation (1.1) normalized to the total number of pixels contained in an image.

(a) Image of *Akihabara* (Night view)



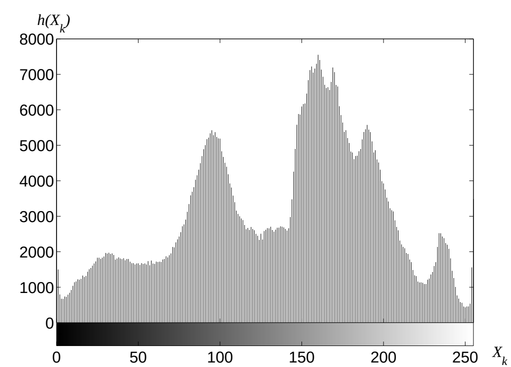
(b) Histogram of *Akihabara*

(c) Image of *Snow*



(d) Histogram of *Snow*

(e) Image of *Griffin*



(f) Histogram of *Griffin*

Figure 1.2: Example images and their corresponding histograms

These figures demonstrate the relationship between the shape of the histogram with the intensity characteristic and appearance of an image. Figure 1.2(a) shows a night view of *Akihabara* city. This image is dominated by low intensity pixels. Hence, the corresponding histogram, as shown in Figure 1.2(b), is more concentrated on the left side (i.e. darker side) of the gray scale axis. In contrast, Figure 1.2(c) shows an image that is dominated by high intensity pixels. As shown by the corresponding histogram in Figure 1.2(d), the graph is biased towards the right side (i.e. brighter side) of the gray scale axis. The image of *Griffin*, as shown in Figure 1.2(e), is an example of image with good contrast. As referred to its histogram in Figure 1.2(f), the components of the histogram occupy almost all the available gray scale range, and do not concentrated only on one side of the gray scale. Hence, this high contrast image exhibits a large variety of gray tones.

There are many images (e.g. medical images, remote sensing images, electron microscopy images and even real-life photographic pictures) suffer from poor contrast. Therefore, it is very necessary to enhance the contrast of such images before further process or analysis can be conducted. Currently, there are many image enhancement techniques that have been proposed and developed. Image enhancement can be carried out in spatial domain or in frequency domain[4] [1, 2, 11]. One of the most popular image enhancement methods in spatial domain is Global Histogram Equalization (GHE), from Histogram Equalization (HE) family. GHE becomes a popular technique for contrast enhancement due to its simple function and its effectiveness.

1.2 Global Histogram Equalization (GHE)

Global Histogram Equlization (GHE)[5] is also known as Tradisional HE [12, 13], Conventional HE [14, 15, 16], Classical HE [8, 17], and Typical HE [18, 19, 20]. GHE technique has been applied in many fields including medical image processing [4, 7, 10, 12, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29], radar image processing [18, 19, 20, 23], sonar image processing [24, 30], satellite image processing [22, 31, 32], near-sensor image processing [33], holography image processing [34], optical image processing [35], watermarking system [36], motion detection [37], speech recognition [12, 26, 38], texture synthesis [12, 26] and texture

[4]For the processing in frequency domain, an image in spatial domain must be transformed first into frequency domain, for example, by using Fast Fourier Transform (FFT). Frequently, these frequency components are processed by multiplying them with a transfer function. The results are then converted back to the spatial domain by using corresponding inverse transform.

[5]Although GHE is normally denoted as HE in most of the literature, this thesis does not use this convention in order to avoid confusion.

classification [39]. GHE also has been widely implemented in many image editing packages such as Adobe Photoshop, National Institutes of Health Image and Lispix [16]. Furthermore, GHE is also useful to be applied for backlight scaling which maximizes backlight dimming while maintaining a pre-specifies image distortion level for Liquid Crystal Display (LCD) [40]. Moreover, GHE is widely applied in preprocessing stage for human face recognition system [24, 41, 42, 43, 44]. This is due to the fact that GHE not only can improve the contrast of the image, but this method is also capable to cope with illumination variations (which is caused by uneven lighting environment during data acquisition process) and thus yields improvement in the recognition performance.

The basic idea of GHE method is to remap the gray levels of an image based on the image's gray levels cumulative density function. GHE uses the information of the whole intensity values inside the image for its transformation function and thus this method is suitable for global enhancement [45]. Its goal is to redistribute the intensity of an image uniformly[6] over the entire range of gray-levels (i.e. to make the image's cumulative histogram to become linear). Thus, GHE is expected to be very effective for enhancing low contrast detail [46] and to maximize the entropy of an image[7] [8, 10]. GHE attempts to "spread out" the intensity levels belongs to an image to cover the entire available intensity range [21]. GHE flattens and stretches the dynamic range of the resultant image histogram and as a consequence, the enhanced image will optimally utilize the available display levels [40]. This then yields an overall contrast improvement.

[6]Refer to [46] in order to obtain more information regarding on how to obtain an approximate uniform distribution of gray levels from the use of a simple gray level transformation.

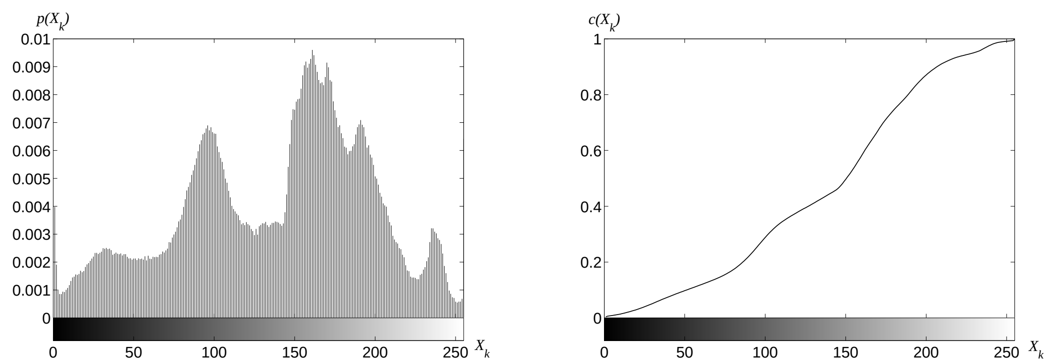
[7]Entropy relates to the information contained in an image. Higher entropy means more information can be extracted from the data. Entropy will be covered in Section 1.2.2.

1.2.1 GHE Algorithm

For a given image **X**, the Probability Density Function (PDF) for intensity X_k, $p(X_k)$, is given by:

$$p(X_k) = \frac{n_k}{N}, \quad \text{for } k = 0, 1, \cdots, L-1 \tag{1.2}$$

where N is the total number of samples in the image. By comparing equation (1.2) with equation (1.1), the PDF is actually a normalized version of the histogram[8]. For example, the PDF of the image of *Griffin* shown in Figure 1.2(e) is presented in Figure 1.3(a). Note that the shape of the graph shown in Figure 1.3(a) is exactly the same as the one shown in Figure 1.2(f), except that the magnitude has been normalized.



(a) PDF (b) CDF

Figure 1.3: PDF and CDF of *Griffin* image

The sum of all components of the normalized histogram or PDF results a Cumulative Density Function (CDF) of an image. Based on the PDF in equation (1.2), the CDF for intensity X_k, $c(X_k)$, is defined by:

$$c(X_k) = \sum_{j=0}^{k} p(X_j), \quad \text{for } k = 0, 1, \cdots, L-1 \tag{1.3}$$

By definition, $c(X_{L-1}) = 1$. Similar to PDF, CDF of an image also can be presented as a plot of

[8] Similar to the histogram, the PDF of an image also can be presented as a graph plot of $p(X_k)$ versus X_k.

$c(X_k)$ versus X_k. An example of CDF is shown in Figure 1.3(b). In GHE, CDF of the histogram is used as the intensity transfer function for intensity value mapping.

GHE is a scheme that maps the input image into the entire dynamic range, $[X_0, X_{L-1}]$, by using CDF as its transformation function. Now, let $x = X_k$. The transform function, $f(x)$, is defined based on the CDF as:

$$f(x) = X_0 + (X_{L-1} - X_0) \cdot c(x) \tag{1.4}$$

From here, the output image produced by GHE, $\mathbf{Y} = \{Y(i, j)\}$, can be expressed as:

$$\mathbf{Y} = f(\mathbf{X}) = \{f(X(i, j)) | \forall X(i, j) \in \mathbf{X}\} \tag{1.5}$$

1.2.2 The Entropy of Message Source

Ideally, GHE will produce a flat histogram and PDF (i.e. a linearly CDF), and thus it can maximize the entropy contained in an image. This subsection will show the relationship between a flat PDF with the maximum entropy value. In information theory, the entropy, E, is defined as the expectation of the uncertainty of a message source and the formula is given as:

$$E(x) = -\sum_{i}^{x} p_i \log_{10} p_i \tag{1.6}$$

where $\sum_{i} p_i = 1$, and $p_i \geqslant 0$. The entropy is the measurement of the average information content that can be obtained from the message source [2, 9, 10].

Consider a digital image $\mathbf{X}$ with its gray intensities are digitized into L levels, with $\{p_0, p_1, \cdots, p_{L-1}\}$, denote the PDF of each gray level respectively. Now, suppose E denotes the entropy of the image information which is the average information content of the image. Then, its formula is given by:

$$E = -\sum_{i=0}^{L-1} p_i \log_{10} p_i \tag{1.7}$$

where $\sum_{i=0}^{L-1} p_i = 1$.

The following equations show the condition to achieve the maximum E value [9]. By considering that $\forall p > 0$, then $\log_{10} p \leqslant (p-1)$. Now, introduce two variables, ω and υ, such that $\forall \omega_i, \upsilon_i > 0$, and

$$\sum_{i=0}^{L-1} \omega_i = \sum_{i=0}^{L-1} \upsilon_i = 1 \tag{1.8}$$

Let $p_i = \frac{\omega_i}{\upsilon_i}$, then

$$-\sum_{i=0}^{L-1} p_i \log_{10} p_i = \sum_{i=0}^{L-1} \frac{\omega_i}{\upsilon_i} \log_{10} \frac{\upsilon_i}{\omega_i} \tag{1.9}$$

From here,

$$\sum_{i=0}^{L-1} \omega_i \log_{10} \frac{\upsilon_i}{\omega_i} \leqslant \left(\sum_{i=0}^{L-1} \omega_i \left(\frac{\upsilon_i}{\omega_i} - 1 \right) = \sum_{i=0}^{L-1} \upsilon_i - \sum_{i=0}^{L-1} \omega_i = 0 \right) \tag{1.10}$$

Thus,

$$\sum_{i=0}^{L-1} \omega_i \log_{10} \upsilon_i \leqslant \sum_{i=0}^{L-1} \omega_i \log_{10} \omega_i \tag{1.11}$$

So,

$$-\sum_{i=0}^{L-1} \omega_i \log_{10} \omega_i \leqslant -\sum_{i=0}^{L-1} \omega_i \log_{10} \upsilon_i \tag{1.12}$$

Especially, let

$$\upsilon_i = \frac{1}{L} \sum_{i=0}^{L-1} \omega_i = \frac{1}{L} \tag{1.13}$$

Then,

$$-\sum_{i=0}^{L-1} \omega_i \log_{10} \omega_i \leqslant \left(-\sum_{i=0}^{L-1} \omega_i \log_{10} \upsilon_i = -\sum_{i=0}^{L-1} \omega_i \log_{10} \frac{1}{L} = \log_{10} L \right) \tag{1.14}$$

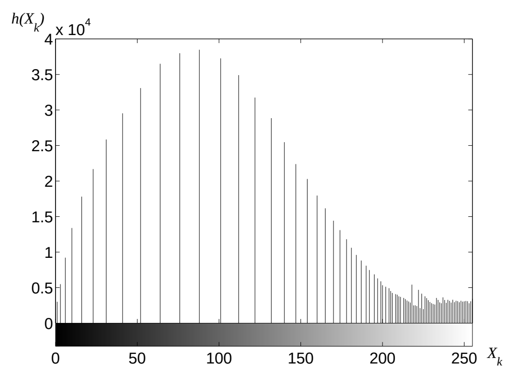
Because $0 < (p_i = \frac{\upsilon_i}{\omega_i}) \leqslant 1$, equation (1.14) is reasonable only when $\omega_0 = \omega_1 = \cdots = \omega_{L-1}$.

Based on equations (1.8) to (1.14), it can be concluded that the entropy of the message source will have its maximum value when the message has been uniformly distributed. Hence, an image will have its maximum entropy value when its histogram or PDF is flat or uniformly distributed. Theoretically, GHE produces images with flat histogram. Thus, this becomes one of the reasons why GHE is used for contrast enhancement.

1.2.3 Limitations of GHE

Although GHE is suitable for an overall contrast enhancement, practically there are some limitations associated with GHE. To ease the discussion, Figure 1.4 shows an image processed using GHE and its corresponding histogram. By comparing histograms in Figure 1.4(b) with its original version in Figure 1.2(b), it is shown that GHE successfully stretches and expands the dynamic range of the image [2]. However, the histogram is far from being flat. The histogram in Figure 1.4(b) has many empty bins[9] because this histogram is actually a shifted version of the histogram shown in Figure 1.2(b) [17]. Thus, the entropy value of the GHE enhanced image is almost similar to the original version, and not maximized as expected in theory.

(a) GHE processed image



(b) Histogram of image (a)

Figure 1.4: GHE processed image of *Akihabara* and its histogram

GHE also usually causes level saturation[10] (clipping) effects in small but visually important

[9] A bin is defined as a series of equal intervals in a dynamic range of an image employed to describe the divisions in a histogram.

[10] Saturation is the decrease in the absorption coefficient of a medium when the power of the incident radiation exceeds a certain value [47].

areas [12]. This happen because GHE extremely pushes the intensities towards the right or the left side of the histogram. If the input image is dominated by dark intensity pixels, the histogram is pushed to the right side, and thus bright saturation effect (as shown in Figure 1.4(a)) will be clearly visible. On the other hand, if the input is dominated by bright intensity pixels, the output normally suffers from dark saturation effect. This saturation effect, not only degrades the appearance of the image, but also leads to information loss [45]. For example, the contents on the billboards in Figure 1.4(a) are completely demolished because these billboards are saturated with bright intensity values.

The damage of the contents on the billboards in Figure 1.4(a) is also because GHE does the enhancement globally, without considering the local contents of the image. GHE method is effective in enhancing the low contrast image when the input image contains only one big single object, or when there is no appearance contrast change between the object and the background in the image [3]. For other images, GHE mapping often results in undesirable effects such as over enhancement for intensity levels with high probabilities, and loss of contrast for levels with low probabilities [3, 5, 12, 45, 48]. Thus, the enhancement might be biased towards the depiction of parts of the image which are unimportant for the viewer such as the background area of the image [49, 50]. As a consequence, GHE algorithm is not applicable to many images, such as infrared image, because this algorithm usually enhances the image's background instead of the object that occupies only a small portion of the image [51].

Furthermore, GHE often causes the shifting on the average (i.e. mean) luminance of the image [9], which is a well-known mean-shift problem [48]. Hence, GHE is rarely employed in consumer electronic products (e.g. video surveillance, digital camera, and digital television) where the brightness preserving characteristic of the enhancement method is crucial [10]. A dark movie scene displayed on television, for example, should be maintained dark in order to keep its artistic value [9].

Besides, the excessive change in brightness level introduce by GHE leads to annoying artifacts and unnatural enhancement, as shown in Figure 1.4(a). The noise in the image is also enhanced or magnified [52]. Thus, although GHE can increase the brightness level in the image, this technique might significantly degrade the quality of the image.

1.3 Objectives of Study

Based on the limitations of GHE stated in Subsection 1.2.3, the objectives of this research are to develop two independent HE based methods, where the aims of these methods are:

1. To develop an extension of HE with brightness preserving ability.

2. To develop an extension of HE that can emphasis local contents.

In addition to grayscale image processing, an extension to color image processing for both methods will be also developed.

1.4 Organization of Thesis

The structure of this thesis is as follows. As a first step in this work, Chapter 1 gives an introduction to the basic concepts of contrast enhancement, histogram and the GHE method. Then, Chapter 2 will briefly review the previous works on the extensions to HE method which are Mean Brightness Preserving HE, Bin Modified HE, and Local HE. Next, the novel method which is based on Mean Brightness Preserving HE, Brightness Preserving Dynamic Histogram Equalization (BPDHE) is presented in Chapter 3. Chapter 4 will discuss another novel method, Multi-Levels Block Overlapped Histogram Equalization (MLBOHE), which is a method based on Local HE. Besides, an extension of BDPHE and also MLBOHE to color image is presented in Chapter 5. Some experimental results from the application of these proposed methods are presented in Chapters 3, 4, and 5. Finally, a conclusion to summarize this entire work is drawn in Chapter 6. Some suggestions for future work are also provided in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

2.1 Extensions of HE

The limitations of GHE as mentioned in Subsection 1.2.3, have encourage many researchers to actively develop various extensions to HE method. Generally, these variations of HE can be classified into four groups as shown in Figure 2.1. In addition to GHE, they are Mean Brightness Preserving HE, Bin Modified HE, and Local HE.

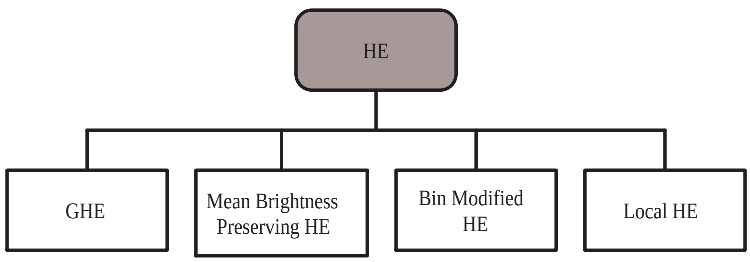


Figure 2.1: Block diagram of HE's extensions

Hence, this chapter provides a literature review on some of the extensions of HE. Section 2.2 will discuss about the Mean Brightness Preserving HE methods. Then, the Bin Modified HE will be presented in Section 2.3. After that, Section 2.4 will give in details about the Local HE. Finally, the last section will summarize all the methods discussed in this chapter.

2.2 Mean Brightness Preserving Histogram Equalization (MBPHE)

Mean Brightness Preserving Histogram Equalization (MBPHE) is a novel extension to HE. This type of enhancement method is specially developed for the use in consumer electronic products such as digital television, digital camera and camcorder. The idea of keeping the mean brightness of an image for consumer electronic products was first introduced by Kim [18]. By preserving the mean brightness, this not only can maintain the artistic value of the image, but it is also proven that this methodology can reduce the saturation effect, and able to avoid unnatural enhancement and annoying artifacts on the output image.

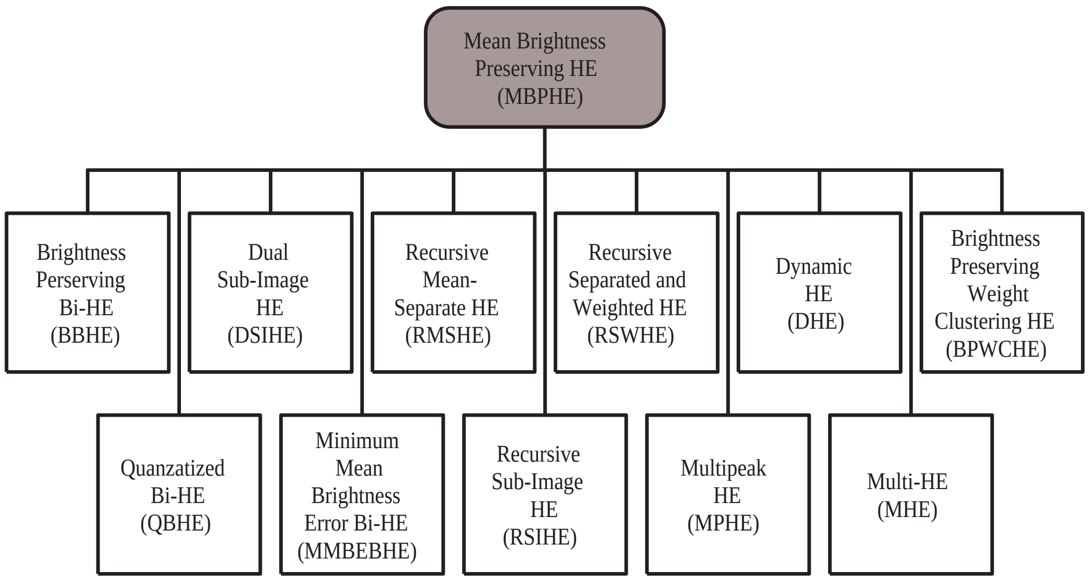


Figure 2.2: Block diagram of MBPHE's extensions

Commonly, MBPHE decomposes the input image into two or more sub-images, and then it equalizes the histograms of these sub-images independently. The major difference among the MBPHE methods is the criteria used to decompose the input image. As shown by Figure 2.2, MBPHE can be divided into 11 methods, which consists of Brightness Preserving Bi-Histogram Equalization (BBHE), Quantized Bi-Histogram Equalization (QBHE), Dual Sub-Image Histogram Equalization (DSIHE), Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE), Recursive Mean-Separate Histogram Equalization (RMSHE), Recursive Sub-Image Histogram Equalization (RSIHE), Recursive Separated and Weighted Histogram Equalization (RSWHE), Multipeak Histogram Equalization (MPHE), Dynamic Histogram Equalization (DHE), Multi-Histogram Equalization (MHE), and Brightness Preserving

Weight Clustering Histogram Equalization (BPWCHE). The details of each MBPHE methods are discussed in the following subsections.

2.2.1 Brightness Preserving Bi-Histogram Equalization (BBHE)

Brightness Preserving Bi-Histogram Equalization (BBHE) is a novel method invented by Kim [18]. First, BBHE finds the value of X_m which is the mean of the image $\mathbf{X}$, where $X_m \in \{X_0, X_1, \cdots, X_{L-1}\}$. Then, BBHE decomposes the input image into two sub-images $\mathbf{X}_L$ and $\mathbf{X}_U$ based on the mean (i.e. X_m) as given in equations (2.1) to (2.3).

$$\mathbf{X} = \mathbf{X}_L \cup \mathbf{X}_U \tag{2.1}$$

where

$$\mathbf{X}_L = \{X(i,j) | X(i,j) \leqslant X_m, \forall X(i,j) \in \mathbf{X}\} \tag{2.2}$$

and

$$\mathbf{X}_U = \{X(i,j) | X(i,j) > X_m, \forall X(i,j) \in \mathbf{X}\} \tag{2.3}$$

Note that the sub-image $\mathbf{X}_L$ is composed of $\{X_0, X_1, \cdots, X_m\}$ and the another sub-image $\mathbf{X}_U$ is composed of $\{X_{m+1}, X_{m+2}, \cdots, X_{L-1}\}$. This is shown in Figure 2.3.

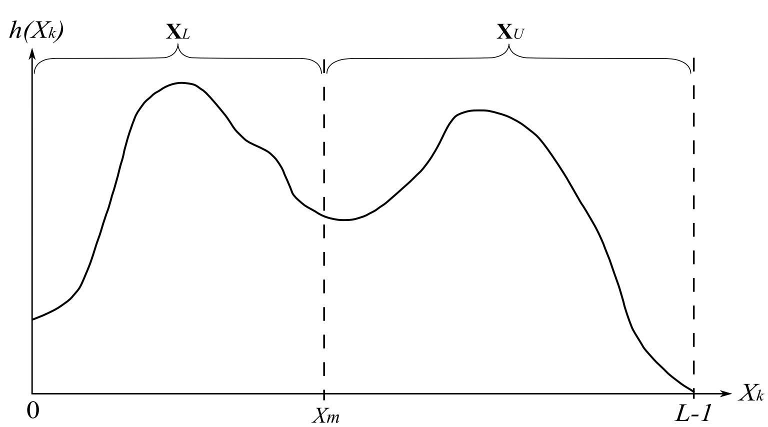


Figure 2.3: An input histogram of BBHE is divided based on its mean

Then, the respective PDFs of the sub-images $\mathbf{X}_L$ and $\mathbf{X}_U$ are given by:

$$p_L(X_k) = \frac{n_L^k}{N_L}, \quad \text{for } k = 0, 1, \cdots, m \tag{2.4}$$

and

$$p_U(X_k) = \frac{n_U^k}{N_U}, \quad \text{for } k = m+1, m+2, \cdots, L-1 \tag{2.5}$$

in which n_L^k and n_U^k represent the respective numbers of X_k in $\{\mathbf{X}\}_L$ and $\{\mathbf{X}\}_U$, and N_L and N_U are the total numbers of samples in $\{\mathbf{X}\}_L$ and $\{\mathbf{X}\}_U$, respectively. Note that $N_L = \sum_{k=0}^{m} n_L^k$, $N_U = \sum_{k=m+1}^{L-1} n_U^k$ and $N = (N_L + N_U)$.

Next, the respective CDFs for $\{\mathbf{X}\}_L$ and $\{\mathbf{X}\}_U$ are then defined by:

$$c_L(x) = \sum_{j=0}^{m} p_L(X_j) \tag{2.6}$$

and

$$c_U(x) = \sum_{j=m+1}^{L-1} p_U(X_j) \tag{2.7}$$

where $x = X_k$. By definition, $c_L(X_m) = 1$ and $c_U(X_{L-1}) = 1$.

Similar to the case of GHE, CDFs are used as the transform functions to assign the new intensity values to the input image. The transform functions for BBHE are defined as:

$$f_L(x) = X_0 + (X_m - X_0) \cdot c_L(x) \tag{2.8}$$

and

$$f_U(x) = X_{m+1} + (X_{L-1} - X_{m+1}) \cdot c_U(x) \tag{2.9}$$

The decomposed sub-images are equalized independently based on their transform functions. Finally, the output image of BBHE, $\mathbf{Y}$, is expressed by equations (2.10) to (2.12).

$$\mathbf{Y} = \{Y(i,j)\} = f_L(\mathbf{X}_L) \cup f_U(\mathbf{X}_U) \tag{2.10}$$

where

$$f_L(\mathbf{X}_L) = \{f_L(X(i,j)) | \forall X(i,j) \in \mathbf{X}_L\} \tag{2.11}$$

and

$$f_U(\mathbf{X}_U) = \{f_U(X(i,j)) | \forall X(i,j) \in \mathbf{X}_U\} \tag{2.12}$$

From equations (2.11) and (2.12), it can be said that $f_L(\mathbf{X}_L)$ equalizes the sub-images $\mathbf{X}_L$ over the range $[X_0, X_m]$ while $f_U(\mathbf{X}_U)$ equalizes the sub-images $\mathbf{X}_U$ over the range $[X_{m+1}, X_{L-1}]$. As a consequence, the input image $\mathbf{X}$ is equalized over the entire dynamic range $[X_0, X_{L-1}]$ with the constraint that the samples less than the input mean are mapped to $[X_0, X_m]$ and the samples greater than the mean are mapped to $[X_{m+1}, X_{L-1}]$.

Unlike GHE that always produce the output mean intensity at the middle gray level regardless of the input mean brightness [18, 20], BBHE is able to preserve the mean brightness quite well [16, 18, 20, 23, 24]. If the intensity distribution of the input is symmetry around its mean, it proves that the average intensity of BBHE output will be at the middle of the input mean and the middle gray level [8]. Thus, BBHE normally gives results with more natural enhancement compared with GHE.

Yet, as the mean-separation[1] of BBHE method is done only once, BBHE only can preserve the mean brightness to a certain extent. However, some cases do require higher degree of preservation to avoid unpleasant artifacts [20]. Furthermore, BBHE can only preserve the orig-

[1] Mean-separation refers to the separation of an input image into sub-images based on the mean of an input image. In other words, mean-separation separates the histogram into two based on the mean of the input image's histogram.

inal brightness if and only if the input histogram has a quasi-symmetrical distribution around its mean [10]. However, most of the input histograms do not have this property. This condition leads to the failure of BBHE in preserving the mean intensity in real life applications.

2.2.2 Quantized Bi-Histogram Equalization (QBHE)

Quantized Bi-Histogram Equalization (QBHE), as proposed by Kim [19], is a modification to BBHE. QBHE follows the same procedures as BBHE. However, QBHE provides much simple hardware structure than BBHE since it utilizes the CDF of a quantized image, which require less number of components, such as comparators, counters, and dividers. Thus, the realization of BBHE in real consumer electronic applications is more feasible. The goal of QBHE is to preserve the mean brightness of a given image effectively with less hardware complexity while enhancing the contrast of a given image [19].

The first step of QBHE is the quantization[2] process, where the L discrete gray levels are quantized into K discrete levels (i.e. $K \leqslant L$). Next, similar to BBHE, the input image is then decomposed into two sub-images based on its mean. After that, quantized CDF is defined for each of the sub-images. A linear interpolation is then used to evaluate the function value at every input gray level in order to perform equalization properly. Finally, the decomposed sub-images are equalized independently based on the transfer functions obtained.

The brightness preservation by QBHE is comparable to BBHE (regardless the level of K). But, the degree of contrast enhancement by QBHE decreases as K discrete levels decreases. Furthermore, quantization is also not so recommended when an image is needed to be transformed from one color space to another [53].

[2] In image processing, quantization is a process in which each pixel in an image is assigned one of a finite set of gray levels [47].

2.2.3 Dual Sub-Image Histogram Equalization (DSIHE)

Dual Sub-Image Histogram Equalization (DSIHE) that is developed by Wang et al. [9], is an extension to BBHE. DSIHE outperforms BBHE in terms of preserving the image's brightness and also in image content or entropy. By following the fact from BBHE, which is the brightness can only be preserved well when the input histogram is symmetry around its separating point, DSIHE method decomposes the input image into two equal area sub-images based on its gray level PDF. In other words, DSIHE separates the histogram using threshold level with CDF equal to 0.5. Then, the two sub-images are equalized independently. Finally, the processed sub-images are composed back into one image to obtain the output image.

The implementation of BBHE and DSIHE are almost similar except that DSIHE separates the histogram based on the gray level with CDF equal to 0.5 (i.e. the median value) instead of using the mean[3]. Theoretically, if the separation of the histogram is done based on the median of the input image's brightness (i.e. median-separation[4]), the maximum Shannon's entropy can be obtained after the two equal areas, which corresponds to dark and bright areas, are equalized independently.

Unlike BBHE, DSIHE changes the brightness to the middle level between the median level and the middle of the input image [10]. Thus, DSIHE can enhance image information effectively and also keep the original image luminance well enough [9]. However, although DSIHE can overcome the aforementioned problems of GHE, DSIHE fails to preserve the original brightness of an image when the higher degree of preservation is needed.

[3]For the implementation of DSIHE, X_m in Figure 2.3 presents the median value.

[4]Median-separation is similar to mean-separation except it is using median value instead of mean value.

2.2.4 Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE)

Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) is introduced by Chen and Ramli [23, 24], to overcome the limitation of GHE, BBHE, and DSIHE in preserving the image's original brightness [10, 24]. The ultimate goal of this method is to allow maximum level of brightness preservation, which indirectly can avoid the output from unpleasant artifacts and unnatural enhancement due to excessive equalization.

Unlike BBHE and DSIHE, MMBEBHE will first test all possible values of the separating intensity from 0 to $(L-1)$ in order to find the optimal threshold level that can produce the smallest Absolute Mean Brightness Error (AMBE) [23]. AMBE is defined as the absolute difference between input and output mean, A_{MBE}, as given by:

$$A_{MBE} = |\overline{\mathbf{X}} - \overline{\mathbf{Y}}| \tag{2.13}$$

where $\overline{\mathbf{X}}$ is the input mean and $\overline{\mathbf{Y}}$ is the output mean. From equation (2.13), lower AMBE indicates that the brightness is better preserved.

The following are the procedures of MMBEBHE method. First, the AMBE is calculated for each of the possible threshold levels. Then, the algorithm finds the threshold level that yield minimum AMBE and consider it as the separating point. Lastly, the input is decomposed into two based on this separating point (as refer to Figure 2.3, X_m is the separating point that can produce the minimum AMBE value for the given input image). These two sub-images are then equalized independently.

From the procedures described in the previous paragraph, the process of calculating the AMBE for each of the possible threshold levels, especially when the number of gray level is large, requires considerable amount of computations [23]. As the process of selecting the separating point is based on enumeration method, this could become a major drawback of MMBEBHE for a real time implementation.

2.2.5 Recursive Mean-Separate Histogram Equalization (RMSHE)

Recursive Mean-Separate Histogram Equalization (RMSHE), which is introduced by Chen and Ramli, is a generalization of BBHE method [20, 24]. Similar to BBHE, RMSHE uses the mean values to decompose the input image into several sub-images. The only difference between BBHE and RMSHE is the mean-separation in BBHE is done only once while the mean-separation in RMSHE is done recursively. RMSHE further separates each of the new subhistograms based on their respective means. In other words, RMSHE uses BBHE repeatedly [10].

The first step of RMSHE is the same with BBHE, which is to separate the input histogram into two pieces based on its mean. Then, recursive separations are applied many times, using the mean value of the sub-images, depending on the scale r (i.e. the recursive factor that is set by the user) to generate $R_{sub} = 2^r$-pieces of subhistograms. An example of RMSHE is presented in Figure 2.4.

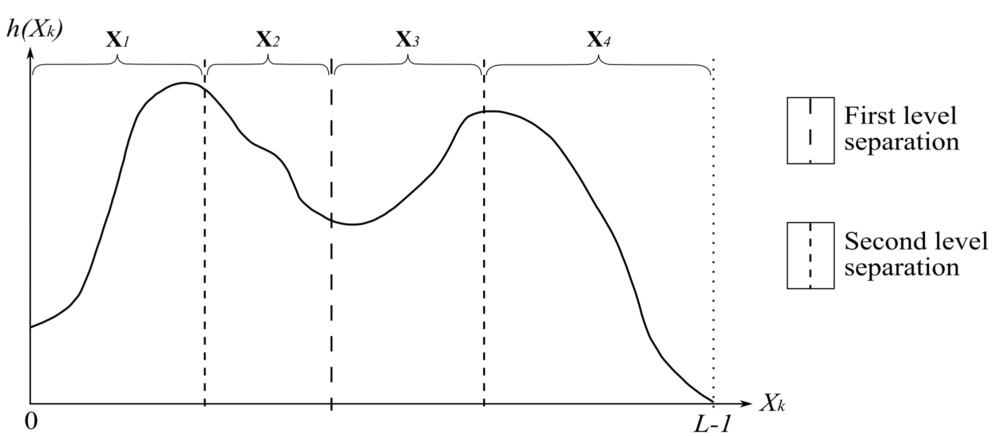


Figure 2.4: An input histogram of RMSHE is divided based on its mean recursively (i.e. $r = 2$)

Finally, each subhistogram a, with CDF defined as $c_a(x)$, is equalized independently by using transform function, $f_a(x)$, as:

$$f_a(x) = X_{l_a} + (X_{u_a} - X_{l_a}) \cdot c_a(x), \quad \text{for } a = 1, 2, \cdots, R_{sub} \tag{2.14}$$

where X_{l_a} and X_{u_a} are the lowest and the highest gray level limits defined by subhistogram a, respectively. Lastly, the output image of RMSHE, $\mathbf{Y}$, is expressed as:

$$\mathbf{Y} = \{Y(i,j)\} = \bigcup_{a=1}^{R_{sub}} f_a(\mathbf{X}_a) \tag{2.15}$$

where

$$f_a(\mathbf{X}_a) = \{f_a(X(i,j)) | \forall X(i,j) \in \mathbf{X}_a\} \tag{2.16}$$

The ultimate goal of RMSHE is to allow higher level of brightness preservation to avoid unpleasant and unnatural enhancement due to excessive equalization while enhancing the contrast of a given image as much as possible. RMSHE provides better and also scalable brightness preservation [20]. The performance of RMSHE is better in some cases where GHE and BBHE fail in their applications [16].

RMSHE with recursive level, $r = 0$ is similar to GHE because there is no mean-separation performed. As a result, the mean brightness cannot be preserved. RMSHE with $r = 1$ is similar to BBHE because the mean-separation is done only once. Hence, the original brightness can be preserved to a certain extent. For RMSHE with r is greater or equal to 2, if more mean-separation is done recursively (i.e. larger r value), then a better brightness preservation can be achieved. Thus, in general, RMSHE can preserve an image's original brightness in a scalable manner, which allows scalable degree of brightness preservation range from 0% ($r = 0$, output of GHE) to 100% ($r = \infty$, original input image) [20]. Theoretically, the output mean converges to the input mean when r grows larger, and thus yields a good brightness preservation. However, when r grows to infinite, the output histogram is exactly the same with input histogram, and thus the output image has no enhancement at all [10].

2.2.6 Recursive Sub-Image Histogram Equalization (RSIHE)

Recursive Sub-Image Histogram Equalization (RSIHE) that is developed by Sim et al. [16], subdivides the input image into 2^r sub-images. RSIHE is a generalization of DSIHE method. RSIHE shares the same characteristics (i.e. recursive framework) with RMSHE in generating the sub-images, except RSIHE chooses to separate the histogram based on the median value (i.e. CDF $\cong 0.5$) rather than the mean-separation approach.

RSIHE first determines median of the input image and separates the histogram into two equal areas. Then, depending to the level r set by the user, RSIHE further recursively subdivides these subhistograms by using the median value of the corresponding histogram sections. The separation process halts once 2^r sub-images are generated[5]. Then, the equalization process is carried out to these subhistograms, independently using equations (2.14) to (2.16).

Similar to RMSHE, the output from RSIHE with recursive level, $r = 0$ is the same as the one produced by GHE because there is no median-separation performed. Therefore, there is no brightness preservation can be obtained when $r = 0$. RSIHE with $r = 1$ is similar to DSIHE as the histogram separation, which is based on the median value is done only once. The original brightness can be preserved to a certain extent. For RSIHE with r greater or equal to 2, the separation is done recursively based on median to further preserve the original brightness [16].

RSIHE provides better brightness preservation and also the high structure similarity. Besides, RSIHE can preserve the quality of image while producing a more natural enhancement. Hence, the features of RSIHE are energy preservation, better contrast, and better image with

[5]Here, 2^r must be less than L in order to allow enhancement to happen.

high Peak Signal-to-Noise Ratio (PSNR)[6] and Mean Structural Similarity Index (MSSI)[7]. Similar to RMSHE, parameter r allows the brightness preserving ability posses by RSIHE to be controlled [16]. However, when $r > 3$ (i.e. total sub-images greater than 8), the resultant image produced by RSIHE will result in ineffective image enhancement and also consumes more computation time. Furthermore, RSIHE also shares the same problem with RMSHE where the image cannot be enhanced if the value of r is set to a very large value.

2.2.7 Recursively Separated and Weighted Histogram Equalization (RSWHE)

Recursively Separated and Weighted Histogram Equalization (RSWHE) is proposed by Kim and Chung [48]. RSWHE method is similar to both RMSHE and RSIHE in terms of recursively decompose the input histogram into 2^r subhistograms. However, unlike RMSHE and RSIHE, the weighting function is applied to RSWHE method. The main difference between the previous MBPHE methods and RSWHE is that all the previous methods, which are discussed in subsections before, do not modify the shape of the input histogram. RSWHE, on the other hand, changes the input histogram before it performs HE process.

Generally, RSWHE method consists of three modules which are histogram segmentation module, histogram weighting module, and HE module. These RSWHE modules are discussed as follow:

1. Histogram Segmentation Module

   First, the image is decomposed into 2^r subhistograms based on some specified recursive level, r. There are two types of segmentation process which is based on mean and median of the subhistogram, respectively. The mean-based histogram segmentation of RSWHE is defined as RSWHE-M, which the procedures of segmentation is the same as RMSHE.

[6]PSNR is a common measure used to indicate the strength of the signal toward its surrounding noise.
[7]MSSI measures the structural similarity between two images.