THE DEVELOPMENT OF PLANAR CURVES WITH HIGH AESTHETIC VALUE

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by

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LIST OF ABBREVIATIONS

Abbreviation	Description
BLINCE	Bilinear Curvature Element
CAAD	Computer Aided Aesthetic Design
CAD	Computer Aided Design
CADCAM	Computer Aided Design & Computer Aided Manufacturing
CAGD	Computer Aided Geometric Design
CAM	Computer Aided Manufacturing
CAS	Computer Aided Styling
CG	Computer Graphics
CNC	Computer Numerical Control
DMU	Digital Mockup
FIORES	Formalization and Integration of an Optimized Reverse Engineering Styling Workflow
GCS	Generalized Cornu Spiral
GLAC	Generalized Log-Aesthetic Curve
IFCA	Involute From Circular Arc
IFSL	Involute From Straight Line
LAC	Log-Aesthetic Curve
LCG	Logarithmic Curvature Graph
LDDC	Logarithmic Distribution Diagram of Curvature
LDGC	Logarithmic Distribution Graph of Curvature
LINCE	Linear Curvature Element
MCV	Minimum Curvature Variation
MVC	Minimum Variation Curves
NAA	North American Aviation
OGH	Optimized Geometric Hermite Curves
SCP	Scientific Computation Program
STL	Standard Tessellation Language
TLS	Total Least Square

PEMBANGUNAN LENGKUNG SATAH DENGAN NILAI ESTETIK YANG TINGGI

ABSTRAK

Penyelidikan terhadap lengkung satah bagi menghasilkan suatu produk yang cantik dan pengubahsuaian lengkung bagi bidang tertentu telah dibangunkan sejak tahun 70-an. Pola penyelidikan dalam bidang ini boleh dibahagikan kepada lima cabang utama iaitu pembangungan algoritma adil, pembangunan lengkung satah melalui kaedah sintesis lengkung, pembangunan algoritma baru bagi mengubahsuai lingkaran asli untuk kegunaan reka bentuk, pengubahsuaian lengkung fleksibel (Bézier dan NURBS) supaya profil kelengkungan adalah monoton dan akhirnya, pembangungan algoritma dalam proses pemyuaian dan penghampiran terhadap lingkaran asli menerusi lengkung fleksibel. Bulatan involut adalah lengkung satah yang mempunyai nilai estetik tinggi dan terkenal untuk reka bentuk gerigi gear. Sumbangan pertama adalah pemgubahsuian bulatan involut agar dapat disesuaikan dalam bidang reka bentuk. Proses evolut-involut digunakan bagi menjana dua jenis splin; involut dari garis lurus (IFSL) dan involut dari lengkok bulatan (IFCA). Sumbangan kedua penyelidikan ini adalah pegenalpastian dan penambahbaikan Graf Kelengkungan Logaritma (LCG) untuk menghasilkan lengkung yang mempunyai nilai estetik yang tinggi. Persamaan LCG dan kecerunan LCG diguna-pakai sepanjang kajian yang dijalankan ini sebagai suatu kaedah siatan reka bentuk efektif. Analisis terhadap lingkaran Cornu teritlak (GCS) telah menjelaskan elemen yang tepat bagi mengenalpasti suatu lengkung satah mempunyai nilai estetik. Oleh kerana GCS dikenali sebagai suatu lengkung estetik, penelitian dilakukan terhadap pembentukan lengkung baru yang mempunyai kecerunan LCG sebagai suatu persamaan garis lurus. Kajian ini telah menghasilkan lengkung log estetik teritlak (GLAC). Lengkung ini mempunyai darjah kebebasan tambahan dan terdiri dari pelbagai lengkung seperti Nielsen spiral, Logarithma spiral, clothoid, bulatan involut, segmen Lengkung Log Estetik (LAC) dan GCS. Akhirnya, pengukuran berangka bagi nilai estetik lengkung satah dilakukan dengan mengubahsuai formula Birkhoff. Hasil kajian membuktikan bahawa formula tersebut boleh diguna pakai setelah clothoid mendapat ukuran nilai estetik tertinggi berbanding dengan lengkung satah yang lain.

THE DEVELOPMENT OF PLANAR CURVES WITH HIGH AESTHETIC VALUE

ABSTRACT

The research on developing planar curves to produce visually pleasing products and modifying planar curves for special purposes has been progressing since the 1970s. The pattern of research in this field of study has branched to five major groups; the development of fairing algorithms; the development of planar curves via curve synthesis, the development of algorithms to modify natural spirals to suit design intent, the modification of flexible curves (Bézier and NURBS) so that the curvature profile is strictly monotonic and finally, the development of algorithms to fit natural spirals and approximation via flexible curves. A circle involute is a planar curve with high aesthetic value and it is famous for gear teeth design. The first contribution is the algorithm to construct circle involute to suit curve styling/design environment. Using the evolute-involute process, two types of splines were developed; involute from straight lines (IFSL) and involutes from circular arcs (IFCA). The second contribution of this research is the identification and enhancement of Logarithmic Curvature Graph (LCG) in order to identify and develop aesthetic curves. The LCG and its gradient formula have been used throughout this research as an effective shape interrogation tool. The analysis of GCS yielded an insight into what makes a curve aesthetic, where the ambiguity of a planar curve being aesthetic is elucidated. Since the GCS is identified as a potential aesthetic curve, the conditions for the curve to possess the LCG gradient as a straight line equation are identified. The extension of the investigation led to the Generalized Log Aesthetic Curve segment (GLAC). This planar curve has extra degrees of freedom and it comprises of many curves; Nielsen's spiral, Logarithmic spirals, clothoids, circle involutes, LAC segments and GCS segments. The methods to formulate GLAC segment are detailed. The final contribution is the numerical measurement formula to evaluate the aesthetic value of planar curves via the customization of Birkhoff's formula. The usability of the modified formula is valid since the clothoid scored the highest aesthetic value as compared to other planar curves.

CHAPTER 1

INTRODUCTION

1.1 Preliminaries

Geometric modelling deals with the study of free-form curve and surface design and it is one of the most basic tools in product design environment. The superset of geometric modeling is computational geometry which encompasses computer-based representation, analysis and synthesis of shape information. The latter is now well known as Computer Aided Geometric Design or (CAGD). The mathematical entities of product development involve CAGD functionalities which lead to Computer Aided Design (CAD) systems development. The general phases of product development are the creative, conceptual and engineering phases. CAx (Figure 1.1) is an acronym which represents various IT support systems of all the phases involved in the lifecycle of a product [1].

The definition of CAGD deals with the construction and representation of free-form curves, surfaces and volumes [2]. The abbreviation was first introduced by Barnhill and Riesenfeld at a conference in 1974 held at the University of Utah. The event successfully gathered many researchers from all over the world who were enthusiastic about topics involved in computational geometry used for design and manufacture. The conference is regarded as the founding event of the new discipline; CAGD. Since then, many conferences and workshops have been held, text books and journals have been published around the

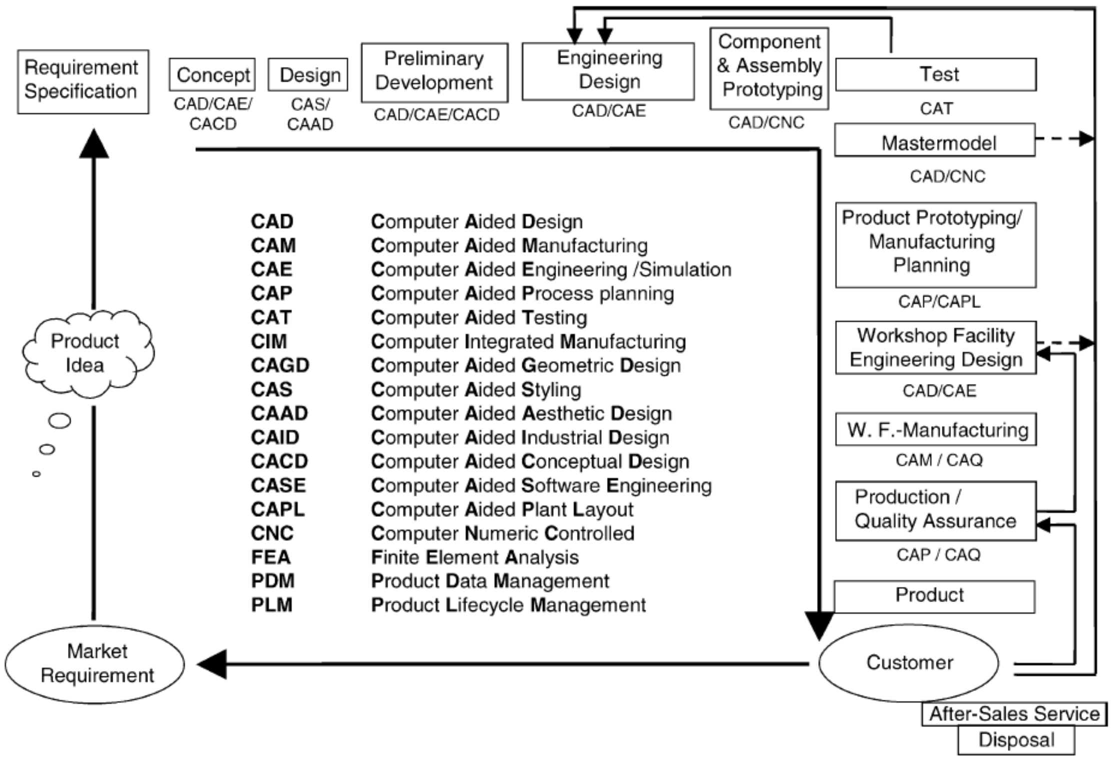


Figure 1.1: General process chain of product development [1]

United States of America and Europe which contributed to the intensification of CAGD.

Even though differential geometry explains in detail the concerns of curves and surfaces, its potential was not known to the Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) environment. CAD models are computer represented geometrical descriptions of a physical object and the description include numerical data as well as algorithms to prescribe the geometry of the object [3]. The exploration and detailed investigation of mathematics for CAD/CAM and computer graphics was the precursor in the CAGD field [2].

The process of extraction of information from a geometrical model is known as geometry processing or shape interrogation [4]. Shape interrogation techniques are vital for product manufacturing to verify whether the designed product meets its functionalities and aesthetic shapes. The most widely used shape interrogation technique in the 2D environment is curvature profiling where the designer reshapes the curve while inspecting the curvature profile or porcupine plot. There are many techniques available for the 3D environment, for example contour lines, lines of curvature, asymptotic lines, isophotes and reflection lines [5, 6]. In the automobile industry, the skin of the car is investigated using reflection lines method [7, 8]. Since the tweaking of shape parameters to reach to a pleasing curve or surface is a painstaking process, researchers use artificial intelligence techniques, such as genetic algorithms to carry out the task [9].

1.2 CAGD and its History

Between about AD 1450 and 1800, ships were built based on simple curves in order to reduce nature's force and to meet hydrodynamic entities; hence it has been an efficient transportation in the water [10]. Ships hulls were first visualized when wooden beams or well known as "spline" were introduced in England back in 1600s. Aeronautics has played a vital role in the enhancement of shape design for manufacturing; for example, the North American Aviation (NAA) under the guidance of R. Liming, developed fighter planes such as P-51 Mustang (Figure 2.4) [11]. In his book entitled "Analytical Geometry with Application to Aircraft", the description of hybridization of classical methods and computational techniques were elaborated for the first time [12]. The prominent idea from this textbook was to translate classical drafting into numerical algorithms intended to circumvent the confusion during blueprint interpretation. In 1950s, many US companies adopted this method. The development of numerical control (NC) machine in the late 1950s is another precursor for shape design in manufacturing. In September 1952, the first ever NC machine was demonstrated to a selected group of US military, aerospace and machine industries.

With the development of powerful microprocessors, computer numerical control (CNC) machines the CADCAM technology became available. In addition, well developed human/machine interfaces enabled NC machine operators to program interactively without considering the calculation for the real tool paths. These machines are not just used for cutting manufactured objects, but they can be used for any processes which involve

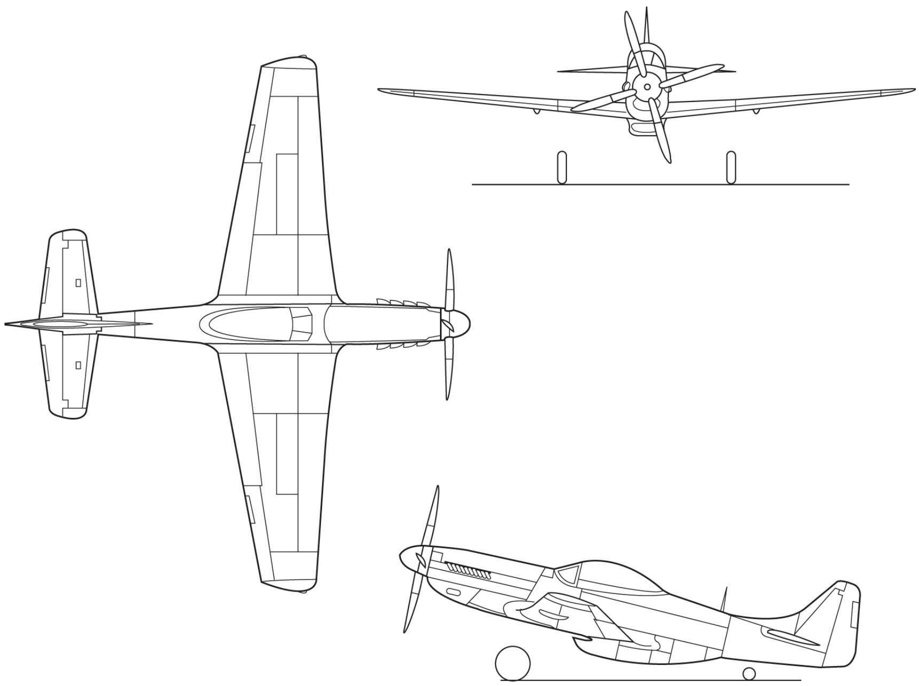


Figure 1.2: P51 Mustang

machine tool motion, for instance, lasing (to produce laser light), welding, friction stir welding, ultrasonic welding, flame cutting, bending, spinning, pinning, gluing, fabric cutting, sewing, tape and fiber placement, routing, picking and placing (PnP: where the robots move from one place to another), sawing etc. CAD systems have become robust and powerful with the embedment of complex algorithms to solve problems that arise in shape design, for example products which need to be visually pleasing. Nowadays, the evolution of the end product clearly indicates the advancement of shape design in the manufacturing industry. The shapes which were assumed impossible to manufacture are manufactured effortlessly, for example a propeller blade.

In the early CAD system, the most widely used planar curves were circular arcs along with straight lines, ellipses, parabola and hyperbola. Since the properties of geometric shapes are different from the stated functions, these functions may not be suitable for free form curve design in the product design environment. For example, a geometric shape is independent of coordinate system where in whatever coordinate system we choose, the entities of the shape (bend, singularities, inflection points, etc.) remains unaffected. However, the stated functions depend on the coordinate system in the xy-plane. Consequently, the equation of a curve changes in accord with the selected coordinate system. To note, piecewise circular arcs or biarcs are still in use since it can be directly processed by NC machines without polygonisation as needed for parametric splines. As a result, researchers looked into ways to develop curves which will be easier to control. The idea is to develop a mathematical representation of a planar curve with the properties of a pen plotter; in other words it must be flexible and convenient to control the shape of a curve. The result was overwhelming, where many types of curves have been proposed. The proposal of the parametric representation and control polygon were made in order to intuitively control the curve. Examples include Bézier, Ball [13, 14], Timmer [15], Overhauser curves or Catmull-Rom spline [16], B-spline and Non Uniform B-Spline (NURBS).

The most prominent curve since the 1960s has been the Bézier curve. The first person to derive the Bézier curve was Paul de Casteljau, who also introduced the de Casteljau algorithm to evaluate Bézier Curves. His findings were kept secret by Citröen, however the Bézier curve was made public by another mathematician who was working independently at Renault; Pierre Bézier [2]. The properties that make Bézier Curves suitable curve for free-form curve design are coordinate system independence, convex hull property, symmetry, linear independence, endpoint interpolation and the variation diminishing property. Convex hull properties means the curve will always be in the control polygon generated by joining the control points, variation diminishing properties means that no straight line intersects the curve more times than it intersects the curve's control polygon and the rest of the properties of Bézier curve are self-explainatory.

In the later years, B-splines have grown in porpolarity in the CADCAM and computer graphics (CG) environment. The term B-spline was first coined by Schoenberg in 1946 when he used B-splines for statistical data smoothing [17]. His paper was regarded as the precursor for modern theory of spline approximation. The important developments of B-spline recurrence in the context of CAGD can be found in [18] and [19]. It was first put into practise by Gordon and Riesenfeld [20]. The most popular curve in CADCAM is NURBS as it provides an exact representation of conic sections and surfaces, for example circular arcs, circles, cylinders, cones, spheres and surface of revolution [21, 22].

Since the foundation of curve development revolves around the flexibility of free-form curve construction, its curvature, which indicates the shape aesthetics, has been overlooked. Hence, controlling the curvature of the stated curves has been a major problem. Curvature controlled curves are not just vital for product design but play an important role in railway design, highway design, robot trajectories, roller coaster design and etc. In the automotive design environment, the feature curves (a curve which makes the outline of a product, e.g., a hood and a door of a car) must have perfect shape in order to produce aesthetically pleasing surface [23].

1.3 Emotion Aspects, Shape Geometry and Styling

Styling is said to be a creative activity and a social construction where the designer's goal is to define a product that evokes some emotion while satisfying imposed ergonomics and engineering constraints [24]. This field of study is called Computer Aided Styling (CAS) and Computer Aided Aesthetic Design (CAAD).

1.3.1 FIORES-I

Since the integration of styling work and engineering (in terms of concepts and terminology) is still vague, an effort has been carried by the European Commission under the project called The European Project FIORES-I and FIORES-II [25]. Formalisation and Integration of an Optimized Reverse Engineering Styling Workflow stage 1 or denoted as FIORES-I deals with the identification and standardization of aesthetic design workflow. The proposed workflow is illustrated in Figure 1.3 [26]:

Manual shapes or models which are formed using materials like clay and hardened foam are expensive and time consuming. However, it creates a direct sensory connection between stylist and the model. Currently, designers use virtual models (DMU) to

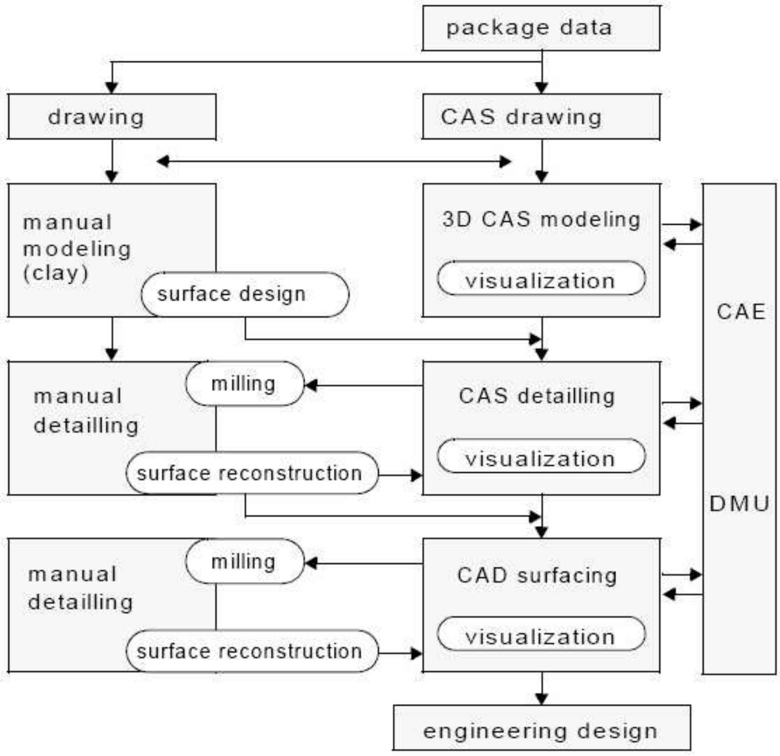


Figure 1.3: Generalized aesthetic design workflow as proposed by FIORES-I

interrogate the shape to reach to the final stage. This process is carried out with the help of 3D Screen projections or virtual reality models. Since computer generated models can only be visualized and not touched, the impression that the models are artificial will always remain. However, with the advancement of DMU and virtual reality, models are interrogated using computers with high resolution screens. Hence, the reduction of the number of models to reach a final shape is achievable with less effort and is cost effective [7].

1.3.2 FIORES-II

The aim of the latter project is to investigate and identify the links between emotional shape perception and geometry in the field of the automotive (BMW, FORMTECH, PININFARINA and SAAB) and household industries (ALLESI, EIGER and FORMTECH). By understanding the underlying principals of the emotional aspects and the mappings between verbal styling descriptions and engineering parameters, the marketing success of a product can be increased [27].

The definition of free form shapes for consumer appliances in terms of the stylists, designers and model makers has been thoroughly investigated in [28]. The term styling and design can be differentiated as follows:

- Styling: concentrates on modelling the outer appearance of a product. It uses emotional and aesthetic values and does not serve any technical or functional purpose.
- Design: is the creative integration of technology, function and form (mathematical formulation and its notation falls under this category).

For more details, see [25, 29, 30].

1.4 Motivation

Kansei is a Japanese term which means a high-order function of the brain, including inspiration, intuition, pleasure and pain, taste, curiosity, aesthetics, emotion, sensitivity, attachment and creativity [31, 32]. Kansei engineering refers to the translation of consumers' psychological feeling about a product into perceptual design elements. It was invented in the 1970s by Nagamachi (Dean of Hiroshima International University). Kansei engineering is also sometimes referred to as 'kansei ergonomics','sensory engineering' or 'emotional usability'. This technique involves determining which sensory attributes elicit particular subjective responses from people, and then designing a product using the attributes which elicit the desired responses.

According to the Oxford dictionary, "aesthetic" means a branch of philosophy dealing with the principles of beauty and tastefulness. The customer decides to buy a product after having carefully considered four factors; efficiency, quality, price and its appearance. Pugh [33] has stated that customers judge the aesthetic appeal of a product before the physical performance. This clearly indicates the importance of aesthetic shapes for the success of marketing an industrial product. According to Bézier & Sioussiou [34], the objects that are manufactured can be classified into three categories:

1. Parts designed based on its technical functionalities and their shapes are determined by rigorous tests and successive approximation. Examples include propellers, aircraft wings, turbine foils and ship hulls.

2. Parts designed based on its functionalities, often hidden and the only requirement is these parts do not collide with other parts. These parts are often designed in such a way to ease manufacturing process (stamping, forging, casting or machining). Examples are inner panels of electrical appliances and car bodies.

3. Parts which are apparent and have to fulfil aesthetic requirements. Examples include the skin of car bodies, household products, sports and leisure equipment, glassware and the outer part of electrical appliances.

They further stated that the solution of the third category must be primarily interactive. The reason is that stylists with strong intuition about shape but limited knowledge of mathematical models must be able to control complex mathematical representation of curves and surfaces. An example of a solution to this problem is the Unisurf CAD system [35]. It has been widely used by the French car industry. In other words, the stylist and designer must be able to spend more time on the ergonomic shape of the model instead of going through repeated modification of the prototype in order to arrive at an aesthetically appealing shape. Hence, an ultimate system for industrial product design must be geometrically interactive for stylists and designers and the shape interrogation tools must work in hand in order to propose other probable aesthetic shapes.

Aesthetic plastic surgery is a field which revolves around cosmetic surgery rejuvenation and aesthetic enhancement of the patient's appearance. The use of "aesthetic" in this context simply stresses the capability of cosmetic surgery to enhance the facial and bodily appearance. A group of Taiwan researchers investigated identifying the ideal female leg which can be used as a guide for doctors and patients involved in plastic surgery

[36]. This indicates that the general features of what makes a female leg aesthetic can specified, which leads to the fact that the aesthetic features of female legs are not subjective. In similar fashion, this research attempts to identify ways to define high quality curves used for design intent. The term "aesthetic" in this thesis refers to high quality planar curves used to design visually pleasing products, e.g., automobiles and electrical appliances.

There are many studies that indicate the importance of the curvature profile to characterize planar curves [37, 38, 39]. In [40], the curvature profile has been highlighted as the shape interrogation tool to fair B-spline curves and surfaces. Since then, it has been the de facto standard to verify the fairness of a curve. By interactive/automated tweaking of control points and concurrently inspecting the curvature plot, the designer arrives to the desired curve.

A different kind of approach has been proposed by Harada et. al to analyze the characteristics of planar curves with monotonic curvature [41, 42]. The relationship between the length frequency of a segmented curve with regards to its radius of curvature is plotted in a log-log coordinate system and called Logarithmic Distribution Diagram of Curvature (LDDC). These type of graphs can be used to identify the aesthetic value of a curve [43, 41]. Harada et. al first used the LDDC as a tool to characterize the curves used for automobile design. To note, the generation of an LDDC is through quantitative methods.

The notion behind generating an LDDC is to mathematically obtain the locus of the interval of radius of curvature and its corresponding length frequency. Thus, two curves of different length would generate a distinct LDDC regardless of the similarities of the shape of the curvature profile. For example, two circular arcs with the same radius and different lengths would generate the same curvature profile, but the LDDC would generate different shapes [41].

The method prescribed by Kanaya et. al to obtain the LDDC is an analytical way of obtaining LDDC, in which they produced an equation called $\mathcal{K}$-vector [44]. $\mathcal{K}$-vector was also known as Logarithmic Curvature Histogram (LCH) by other Japanese researchers. In 2008, the name for the stated graph was standardized to Logarithmic Curvature Graph (LCG). The LCG is proposed as a tool for smart CAD implementation.

1.5 Objectives

The main objective of this research is to produce visually pleasing planar curves. Every chapter is constructed with detailed objectives that are outlined in the beginning of the chapter. The five main objectives are (in accord with each chapter in this thesis):

1. to formulate a piecewise circle involute via evolute-involute process,
2. to improve the LDDC's algorithm, the LCG's formulation and to derive the LCG's gradient function for shape interrogation purposes,
3. to investigate the Generalized Cornu Spiral (GCS) via the LCG and its gradient function and, to elucidate what makes a curve aesthetic,
4. to propose a new type of curve which covers all the natural spirals and stands as the general representation of aesthetic curves in CAD and CAGD environment, and
5. to propose a method to numerically evaluate the aesthetic value of planar curves using the modified Birkhoff's formula.

1.6 Organisation of Thesis

The first chapter introduces a broad field of study that involves CAx and its history. The background theory and related ideas of constructing high quality curves are discussed in Chapter two. The mathematical formulation and notation of aesthetic curves is the core of this research, therefore it is stated in this chapter.

Chapter three introduces two types of curves which are generated from an evolute-involute process. The first type of involute curve(s) is generated using straight line(s) as the evolute(s) and is called IFSL. The second type of involute curve(s) is generated based on circular arc(s) as well as a straight line and is called IFCA. The IFSL and IFCA splines consist of circular arcs and spirals respectively. These curves can be used for free form curve design where the curvature of proposed curves can be controlled at the end points.

Chapter four investigates the algorithm of Logarithmic Distribution Diagram of Curvature (LDDC) and it is renamed as Logarithmic Distribution of Graph Curvature (LDGC) upon the improvement of the algorithm. The analytical formulation of the LDDC is Logarithmic Curvature Graph (LCG). The LCG and its gradient function are derived in a compact form which enables the user to use them as a shape interrogation tool. For numerical examples, two types of planar curves have been studied; planar curves with constant gradient (clothoid, circle involute and Logarithmic spiral) and planar curves with almost constant gradient (parabola and logarithmic curve). The examples show that planar curves can easily be analyzed by using the developed formula. The foundation of this research relies on the formulated LCG and its gradient.

Chapter five focuses the investigation of the Log- Aesthetic Curve (LAC) and the Generalized Cornu Spiral (GCS). Both curves were developed via curve synthesis where the curves are formulated from given curvature function. However, the LAC segment was developed directly by manipulating the LCG and its gradient whereas the GCS segment was developed via a bilinear curvature function (BLINCE). Hence, the GCS is thoroughly investigated by means of the LCG and its gradient function. The results were used to elucidate what makes a planar curve aesthetic.

The formation of a new kind of curve called Generalized Log Aesthetic Curve (GLAC) in line with the definition proposed in Chapter five is carried out in Chapter six. The two methods involved in deriving GLAC segments are ρ-shift and κ-shift. The formulation indicates that κ-shift is better than ρ-shift as the direction of angle for κ-shift can be obtained explicitly. Hence, the formulation of κ-shift method is used to define the GLAC segment. Numerical results are shown in the last section of the chapter for better understanding.

In 1933, Birkhoff proposed the aesthetic measure of an object as the quotient between order and complexity where it has a high aesthetic value on orderliness and low value on complexity [45]. Aesthetic measure increases as complexity decreases. The process of modifying Birkhoff's formula to measure the aesthetic value of planar curves has been carried out in chapter seven. First, the fundamental entities of planar curves; the occurrence of inflection points, curvature maximum or minimum, monotonicity of curvature, gradient of Logarithmic curvature histogram and the existence of cusp or loop are identified. Then, the classification of the entities either order or complexity is determined. Finally, a customized equation to numerically evaluate the aesthetic value of planar curves is proposed. In the last section, numerical examples are shown to illustrate the usage of the proposed formula. Chapter eight concludes the research and looks into future areas of development.

CHAPTER 2

LITERATURE REVIEW

2.1 Notation, Conventions and Some Formulae

For typographical convenience, a column vector is written as a row in braces:

$$\begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix} = \{a_1, a_2, \dots, a_n\}$$

Row vectors are indicated by parentheses: $\{a_1, a_2, \dots, a_n\}^t = (a_1, a_2, \dots, a_n)$, where ^{t} indicates the transpose of a matrix.

A point is simply denoted in Euclidean space as: $b_1 = \{b_x, b_y\}$ and $b_2 = \{b_x, b_y, b_z\}$, where b_1 and b_2 represents the locations in 2D and 3D respectively.

The dot product of two vectors is denoted as $b \bullet d = b_x d_x + b_y d_y$. The length of vector b is $\|b\| = \sqrt{b \bullet b}$. The cross product of two vectors (in 3D) is denoted as $b \wedge d = \{b_y d_z - b_z d_y, b_z d_x - b_x d_z, b_x d_y - b_y d_x\}$. Its length is $\|b \wedge d\| = \|b\|\|d\| \sin(\theta)$, where θ is the anti-clockwise angle from b to d.

Let $C(t) = \{x(t), y(t)\}$ be a parametric plane curve defined with parameter

t on a unit interval, where $t \in \mathbb{R}$. Thus, the corresponding points to $t \in \mathbb{R}$ are $x(t) = (x_1, x_2, \ldots, x_n)$ and $y(t) = (y_1, y_2, \ldots, y_n)$. $Max[x(t)]$ and $Min[x(t)]$ indicates the maximum and the minimum values for $x(t)$. If there exist complex numbers, then $Re[C(t)]$ and $Im[C(t)]$ represents the real and imaginary component of $C(t)$ respectively. In this case, the complex plane is used to represent the geometry of a planar curve.

The curve $C(t)$ is said to be C^k-continuous (or just C^k) if the first k derivatives of $x(t)$ and $y(t)$ exist and are continuous. Suppose $C(t)$ is C^1 curve defined on an interval I, then (2.1) is called the speed of the curve $C(t)$. If $v(t) \neq 0$ for $\forall t \in \mathbb{R}$, then $C(t)$ is said to be a regular curve. If $v(t) = 1$ for $\forall t \in \mathbb{R}$, then $C(t)$ is said to be a unit speed curve [46].

$$v(t) = \|C'(t)\| = \sqrt{(x'(t))^2 + (y'(t))^2} \tag{2.1}$$

The tangent vector for $C(t)$ is $C'(t) = (x'(t), y'(t))$ and the unit tangent vector and the unit binormal vector are defined in equation (2.2) and (2.3) respectively:

$$\mathbf{t}(t) = \frac{(x'(t), y'(t))}{v(t)} \tag{2.2}$$

$$\mathbf{b}(t) = \frac{C'(t) \wedge C''(t)}{\|C'(t) \wedge C''(t)\|} \tag{2.3}$$

The normal vector of $C(t)$ is obtained by rotating $C'(t)$ through an angle $\frac{\pi}{2}$ radians (anticlockwise) and denoted as $(-y'(t), x'(t))$ and the unit normal vector is:

$$\mathbf{n}(t) = \frac{(-y'(t), x'(t))}{v(t)} \tag{2.4}$$

The arc length of a curve is denoted as $s(t)$ and it is calculated as shown below:

$$s(t) = \int_{t}^{t_0} \|C'(t)\| dt \tag{2.5}$$

The turning angle $\theta(t)$ to a curve is the angle from the positive x-axis to the tangent vector $C'(t)$. The turning angle is related to the components of $C(t)$ by the differential equations as shown below:

$$\frac{dx}{dt} = \cos\theta(t)\frac{ds}{dt} \tag{2.6}$$

$$\frac{dy}{dt} = \sin\theta(t)\frac{ds}{dt} \tag{2.7}$$

The amount of bending of the curve at any point on the curve where $\theta(t)$ is differentiable is called curvature denoted as κ. The curvature is positive when the curve bends to left and it is negative when the curve bends to right at parameter t. Mathematically κ is the derivative of the turning angle θ with respect to arc length:

$$\kappa(s) = \frac{d\theta}{ds} = \frac{d\theta}{dt}/\frac{ds}{dt} \tag{2.8}$$

Thus, the intimate connection between turning angle θ and the curvature is [37]:

$$\theta(s) = \int \kappa(s)ds \tag{2.9}$$

The curvature of a parametric curve $C(t)$ is defined as follows:

$$\kappa(t) = \frac{C'(t) \wedge C''(t)}{\|C'(t)\|^3} = \frac{x'(t)y''(t) - y'(t)x''(t)}{(v(t))^3} \tag{2.10}$$

If a curve is given in the form of $y = y(x)$, then the curvature formula becomes:

$$\kappa(x) = \frac{y''(x)}{(1 + (y'(x))^2)^{\frac{3}{2}}} \tag{2.11}$$

The curvature profile for a parametric curve can be obtained by plotting parameter t and against $\kappa(t)$. It indicates how the curvature varies as a point progresses along the curve. The absolute value of the reciprocal of curvature equals to radius of curvature and denoted as $\rho(t) = |\frac{1}{\kappa(t)}|$.

The curvature value is positive if the circle of curvature is on the left of the curve in accord to the direction of travel and is negative if the circle is of curvature is on the right of the curve. A point where the curvature changes sign is called an inflection point. A point where the first derivative of the curvature changes sign is called curvature extremum [47].

An impulse can be represented as $\kappa(s) = \delta(s - s_1)$, where $\delta(s - s_1)$ is a Dirac delta function. It appears as the limiting form of a rectangle of unit area with width tending to zero and height tending to infinity in a curvature profile [37].

The area under a curvature profile for a planar curve is equal to its turning angle as shown in equation (2.9). This indicates that different shapes of curves can be represented by given value of turning angle.

The curvature is curve intrinsic and it is geometric invariant [48]. The shape of the curve is not affected by translation, reflection about an arbitrary axis, rotation and uniform scaling. Even though uniform scaling changes the curvature, the number and relative positions of curvature extrema, cusps, inflection points and self-intersections are preserved [49].

The importance of the curvature profile to design engineers for product design and manufacture had been highlighted even in 1970s. In [50], it is stated that design engineers prefer to specify the shape of a curve in terms of its curvature profile instead of the curve's equation itself.

In the field of rapid manufacturing, curvature value determines the preprocess error. It occurs during the conversion of CAD to standard tessellation language (STL) format (as machine input) where the outer surface of the part is approximated by triangles. As the value of curvature increases, this type of error increases. Even though meshing with smaller triangles may diminish the error, it increases file processing time, and forms more complicated laser trajectory [51].

Planar curves which are visually pleasing have been denoted with many terms, examples are fair curves, beautiful curves, aesthetic curves and monotonic curvature curves. In differential geometry, a spiral segment means a curve with monotone curvature of constant sign (pg.48,[52]). Farin defined a fair curve as a curve which generates continuous curvature profile and consists of only a few monotonic pieces (pg.364, [48]). Although he admits that the definition of fairness is subjective, this term has been widely accepted to denote visually pleasing curves.

The construction of high quality curves especially planar curves with monotonic curvature profile, has been an on going process. A review of the literature gives the following types of construction:

1. **Curve synthesis**: The idea relies on creating a planar curve from the given curvature profile.

2. **Fairing process**: The process of tweaking control points with the help of shape interrogation techniques, for example curvature profile to obtain the desired curve. The fundamental principle is to reduce the occurrence of wriggles in curvature profile either interactively or automatically using a particular algorithm.

3. **Improvement in control of natural spiral**: The formulation of natural spirals (clothoid and Logarithmic spiral) to suit design intent, for example, an algorithm to produce G^2 planar curves from given points by using clothoid, circular arcs and staight lines. To note, even though circle involute has monotonic curvature profile, there is only one notable research carried out.

4. **Construction of New Type of Planar Curves**: This involves the manipulation of the parameter variables meant for controlling the shape of the curve in order to reach a monotonic curvature profile. Hence, the end product of this research will be a curve with less degree of freedom (DOF) compared to its original DOFs with monotonic curvature profile.

5. **Natural spiral fitting and approximation techniques**: The process of approximating natural spirals using flexible curves (rational Bézier cubic and B-spline curves) falls in this group. The main idea of this field of study is to represent natural spirals (which are usually in transcendental form) in polynomial form so that it can be incorporated into commercial CAD systems.

2.2 Curve Synthesis

In 1970s, the CAD research group at the Engineering department of Cambridge University investigated the techniques involved for integrating curvature profiles for planar curves [50]. The idea behind this method is to construct a curve based on a given curvature profile. The following cases were formulated during 1970s:

1. $\kappa(t) = 0$; the curve generated is a straight line.
2. $\kappa(t) = constant = 1/r$; a circle of radius of r is generated.
3. An abrupt step change in curvature from zero to constant value; a straight line connected to a circular arc.
4. An abrupt step change in curvature from one constant ($\kappa(t) \neq 0$) to another constant ($\kappa(t) \neq 0$); the generation of two circular arcs connected with different radii.
5. $\kappa(t) = at$; a segment of clothoid is generated.

Pal & Nutbourne [53] extended the stated findings to generate a smooth curve with given data points and curvature, and tangent direction at those points. Again, a clothoid has been used for curve synthesis purpose and a complete package has been proposed for the generation of a linear curvature spline package for general use of curve synthesis.

2.2.1 The General Formulation for Curve Synthesis

Let a curve be represented by $C(s) = \{x(s), y(s)\}$, for $0 \leq s \leq S$ in which $C(s)$ is a planar curve with arc length parameterization, S corresponds to the total arc length of $C(s)$ and $\kappa(s)$ is its signed curvature. Unit tangent vectors ($\mathbf{t}(s)$), unit normal vectors ($\mathbf{n}(s)$) and the signed curvature ($\kappa(s)$) can be calculated directly as stated below respectively:

$$\mathbf{t}(s) = \left\{ \frac{dx(s)}{ds}, \frac{dy(s)}{ds} \right\} \tag{2.12}$$

$$\mathbf{n}(s) = \left\{ -\frac{dx(s)}{ds}, \frac{dy(s)}{ds} \right\} \tag{2.13}$$

$$\kappa(s) = \left(\frac{dx(s)}{ds}\frac{d^2y(s)}{ds^2} - \frac{dy(s)}{ds}\frac{d^2x(s)}{ds^2} \right) \tag{2.14}$$

The signed angle $\theta(s)$ is measured in radians from the positive x axis to $\mathbf{t}(s)$: $\mathbf{t}(s) = \{\cos(\theta), \sin(\theta)\}$, where

$$\theta(s) = \theta(0) + \int_0^s \kappa(t)dt \tag{2.15}$$

The parametric equation of the planar curve derived from its given curvature is:

$$C(s) = P_a + \left\{ \int_0^s \cos\left[\theta(0) + \int_0^t \kappa(u)du\right] dt, \int_0^s \sin\left[\theta(0) + \int_0^t \kappa(u)du\right] dt \right\} \tag{2.16}$$

where $P_a = \{x(0), y(0)\}$ is the starting point of the curve and the integrals contained in equation (2.16) are called the Fresnel integrals and cannot be integrated directly, thus numerical computing software must be utilized to generate the points on the curve. Equation (2.16) can be represented in a complex plane as follows:

$$C(s) = P_a + \boldsymbol{e}^{i\theta(0)} \int_0^s \boldsymbol{e}^{i\left(\int_0^S \kappa(u)du\right)}\, dv \tag{2.17}$$

where $\boldsymbol{e}$ represents the exponential function and the curve can be generated in a cartesian coordinate by representing x-axis with real component and y-axis with imaginary component. For typographical convenience, equation (2.17) is used to represent planar curves formulated in Chapter six.

Linear Curvature Profile

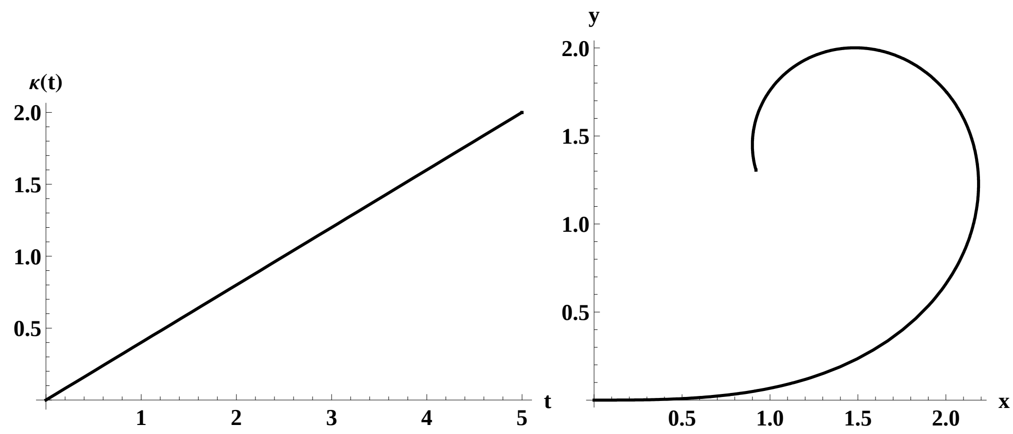
A curvature function which is equivalent to a constant value is the simplest form of curvature function and it produces a segment of a circular arc. A linear curvature element function or LINCE is represented by equation (2.18) and corresponds to a segment of a clothoid:

$$\kappa(s) = a + bs \tag{2.18}$$

where $\{a, b\} \in \mathbb{R}$ are constants and if equation (2.18) satisfies the given value of end curvatures, denoted by κ_0 and κ_1, then the corresponding curvature function is:

$$\kappa(s) = \left(1 - \frac{s}{S}\right)\kappa_0 + \frac{s}{S}\kappa_1 \tag{2.19}$$

Figure 2.1 illustrates an example of a LINCE and its corresponding clothoid with $\kappa_0 = 0$, $\kappa_1 = 2$ and $S = 5$.



(a) LINCE with $\kappa_0 = 0$, $\kappa_1 = 2$ and $S = 5$ (b) Corresponding clothoid segment

Figure 2.1: Curve Synthesis using LINCE with $P_a = \{0, 0\}$.

Quadratic Curvature Profile

A quadratic curvature function defined by $\kappa(s) = as^2 + bs + c$ (where $a \neq 0$ and $a, b, c \in \mathbb{R}$). When it satisfies the end curvature values, the representation of quadratic curvature function is:

$$\kappa(s) = as^2 + \frac{\kappa_1 - \kappa_0 - aS^2}{S}s + \kappa_0 \tag{2.20}$$

Since oscillations may occur in a quadratic curvature function, it is not preferable. Equation (2.20) can be further modified with the introduction of total turning angle (which can be derived from end tangents and given total arc length) and one may obtain a reasonably acceptable curve with less oscillations. However, a quadratic curvature function cannot be reduced to a Logarithmic spiral [54].

Bilinear Curvature Profile

A bilinear curvature element or BLINCE is a curvature function in the form of rational linear function and it produces a GCS segment. It was first introduced by Ali [55]. The GCS curve segment is claimed to be a high quality planar curve which is suitable for aesthetic design and it can interpolate any configuration of end points with tangent vectors which makes it feasible for a CAD environment [56]. The rest of this section outlines the formulation of the GCS curve segment.

Let a GCS curve segment be defined in the interval of $0 \leq s \leq S$, and its curvature function is represented by BLINCE as proposed in [55]:

$$\kappa_{GCS}(s) = \frac{p + qs}{S + rs} \tag{2.21}$$

where p, q, r and S are free parameters of the curve. To ensure $\kappa_{GCS}(s)$ is continuous for $0 \leq s \leq S$, r must be greater than -1. The resultant curve upon curve synthesis is a family of GCS. It is noted that the GCS contains straight lines $(p = q = 0)$, circular arcs $(q = r = 0)$, Logarithmic spirals $(q = 0, r \neq 0)$ and clothoids $(q \neq 0, r = 0)$. An inflection occurs at the most once at $s = -p/q$. To note, GCS cannot represent circle involute.

Let the arc length of the GCS curve segment be S and the end curvatures are κ_0 and κ_1, then at $s = 0$ and $s = S$, we obtain $\kappa(0) = \kappa_0$ and $\kappa(S) = \kappa_1$. Thus, equation

(2.21) can be further reduced to the following set of equations:

$$q = S\kappa_0 \tag{2.22}$$

$$pS + q = S(1+r)\kappa_1 \tag{2.23}$$

Solving for p in terms of r, we get $p = (1+r)\kappa_1 - \kappa_0$. Finally, by substituting p and q in equation (2.21), the curvature function becomes [55]:

$$\kappa(s) = \frac{(\kappa_1 - \kappa_0 + r\kappa_1)s + \kappa_0 S}{rs + S} \tag{2.24}$$

By substituting equation (2.24) in equation (2.16), a GCS segment, $GCS(s) = \{x_{GCS}(s), y_{GCS}(s)\}$ can be obtained as follows:

$$\begin{aligned} x_{GCS}(s) &= x_0 + \int_0^s \cos[\omega] \ dt \\ y_{GCS}(s) &= y_0 + \int_0^s \sin[\omega] \ dt \end{aligned} \tag{2.25}$$

where $\omega = \frac{1}{r^2}(rt(-\kappa_0 + \kappa_1 + r\kappa_1) + (1+r)S(\kappa_0 - \kappa_1)(-\log[S] + \log[S+rt]))$, $\{x_0, y_0\}$ is the first point of the GCS segment and its parameter is defined in the interval of $0 \le s \le S$ with $r > -1$. The monotonicity of the curvature function is always preserved which indicates that the GCS curve segments are classified as high quality curves. Thus, the investigation of GCS using the LCG may further offer insights for the identification of aesthetic curves.

The Formulation of A LAC Segment

Miura [57] proposed a new type of curve called log-aesthetic Curves (LAC). This curve was the result of the extension of Harada et.al's work on LDDC [41]. Log Aesthetic Curves (LAC) is a family of planar curves which has constant gradient value of LCG. Letting the arc length function represented with $s(t)$ and the radius of curvature function represented with $\rho(t)$, the LCG gradient can be simplified as:

$$\begin{aligned} Log\left(\frac{ds(t)}{d(log\rho(t))}\right) &= \left(Log\frac{ds/dt}{d(Log\rho)/dt}\right) \\ &= Log\left(\rho\frac{ds/dt}{d\rho/dt}\right) \\ &= Log\rho + Log\frac{ds}{dt} - Log\frac{d\rho}{dt} \end{aligned} \tag{2.26}$$

If in the limit $\Delta Log\rho \to 0$, then the LCG gradient can be represented as a straight line:

$$Log\left(\rho\frac{ds}{d\rho}\right) = \alpha Log\rho + C \tag{2.27}$$

where $\{C, \alpha\} \in \mathbb{R}$. Upon algebraic simplification, equation (2.27) becomes:

$$\begin{aligned} \frac{1}{\rho^{\alpha-1}}\frac{ds}{d\rho} &= C_0 \\ \Rightarrow \frac{ds}{d\rho} &= C_0\rho^{\alpha-1} \end{aligned} \tag{2.28}$$

where $\boldsymbol{e}^C = C_0$. The case of $\alpha = 0$ is not considered in the formulation of the LAC segment because the resultant formula may lead to the generation of straight lines and circles in which the radius of curvatures are constant. If $\alpha \neq 0$, equation (2.27) can further be simplified to:

$$\begin{aligned} s &= \frac{C_0}{\alpha}\rho^\alpha + C_1 \\ \Rightarrow \rho^\alpha &= C_2 s + C_3 \end{aligned} \tag{2.29}$$

where C_1 is the integral constant, $C_2 = \alpha/C_0$ and $C_3 = -(C_1\alpha)/C_0$. Thus, the LAC segment is given as (2.30) where variable C_2 and C_3 are now renamed as c_0 and c_1 respectively:

$$\rho(s)^\alpha = c_0 s + c_1 \tag{2.30}$$

Equation (2.30) indicates that the α^{th} power of radius of curvature ρ is a linear function composed of the arc length function of s; denoted as a general radius of curvature for the LAC segment. In addition, the principal character of logarithmic spirals and clothoids in