Second Semester Examination
Academic Session 2017/2018

May/June 2018

EMT 212 – Computational Engineering
[Kejuruteraan Pengkomputeran]

Duration :3 hours
[Masa : 3 jam]

Please check that this paper contains **EIGHT [8]** printed pages including appendix before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **LAPAN [8]** mukasurat bercetak beserta lampiran sebelum anda memulakan peperiksaan.]*

INSTRUCTIONS : Answer **ALL SIX [6]** questions.
*[**ARAHAN** : Jawab **SEMUA ENAM [6]** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. **If the ratio of l_1 to l_2 satisfies the golden ratio and $l_1 < l_2$, prove that**
Jika nisbah l_1 kepada l_2 memenuhi nisbah emas dan $l_1 < l_2$, buktikan bahawa

$$\frac{l_1}{l_2} = \frac{\sqrt{5}-1}{2} \approx 0.618034.$$

(10 marks/*markah***)**

2. **Suppose you are businessman who will travel to Ipoh, Melaka, and Kota Bharu. It takes 122 km to Ipoh, 237 km to Melaka, and 307 km to Kota Bharu. Dining and other expenses vary by city: RM 95 in Ipoh, RM 130 in Melaka, and RM 180 in Kota Bharu. A trip to Ipoh will generate RM 800 in sales, while a trip to Melaka and Kota Bharu will generate RM 1300 and RM 1800, respectively.**

You can travel up to 3000 km and your expense must not exceed RM 2000. Your goal is to determine the number of trips you should make to each city to maximize sales.

Andaikan anda seorang peniaga yang akan berjalan ke Ipoh, Melaka, dan Kota Bharu. Perjalanan memerlukan jarak 122 km ke Ipoh, 237 km ke Melaka, dan 307 km ke Kota Bharu. Perbelanjaan untuk makan dan lain-lain adalah berbeza bagi setiap bandar: RM 95 di Ipoh, RM 130 di Melaka, dan RM 180 di Kota Bharu. Perjalanan ke Ipoh akan menjana jualan sebanyak RM 800, manakala ke Melaka dan Kota Bharu masing-masing akan menjana RM 1300 dan RM 1800.

Anda hanya boleh berjalan sehingga 3000 km dan perbelanjaan anda tidak boleh melebihi RM 2000. Matlamat anda adalah untuk menentukan jumlah perjalanan ke setiap bandar yang memaksimakan jualan.

[a] **Write the mathematical statement of the problem with <u>EQUALITY</u> constraints.**

Tuliskan pernyataan matematik bagi masalah ini dengan kekangan-kekangan <u>KESAMAAN</u>.

(3 marks/*markah***)**

[b] **Set the initial tableau.**

Binakan tableau permulaan.

(4 marks/*markah***)**

[c] **Explain the meaning of a basic variable in the simplex method.**

Terangkan maksud pembolehubah asas dalam kaedah simpleks.

(3 marks/*markah***)**

3. **Consider a right triangular prism with sides *a* and length *l* as shown in Figure 3. The prism floats in water such that half of its height is in water.**

   Pertimbangkan sebuah prisma segi tiga sudut tegak air dengan sisi-sisi a dan panjang l seperti yang tertera di dalam Rajah 3. Prisma itu terapung di dalam air di mana setengah dari tingginya berada di dalam air.

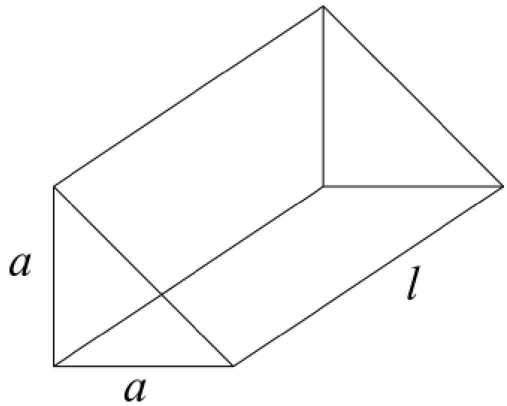

Figure 3
Rajah 3

[a] **Sketch the vector field of force due to the water pressure on the immersed part of the prism. You may show only the cross section of the body to simplify the sketch.**

Lakarkan medan vektor daya yang disebabkan oleh tekanan air ke atas bahagian prisma yang terendam. Anda boleh tunjukkan keratan rentas jasad sahaja bagi meringkaskan lakaran.

(5 marks/*markah***)**

[b] **Express the magnitude of the total force $\boldsymbol{F_f}$ on the prism due to the water pressure in terms of the surface integral over the affected area. DO NOT evaluate the integral.**

Ungkapkan magnitud daya keseluruhan $\boldsymbol{F_f}$ oleh tekanan bendalir ke atas prisma itu sebagai kamiran permukaan bagi kawasan yang berkaitan. JANGAN menghitung kamiran itu.

(5 marks/*markah***)**

[c] **Use the divergence theorem to express the magnitude of the buoyancy in terms of the dimensions a and l and the density of water ρ.**

Gunakan teorem pencapahan untuk mengungkapkan magnitud daya apung itu dalam sebutan-sebutan dimensi a dan l serta ketumpatan air ρ.

(10 marks/*markah***)**

4. **Consider a simplified design of a nuclear reactor where its cross section is depicted in Figure 4. At the center of the reactor are assembly of thermal rods that produce heat due to nuclear fission. An insulating material surrounds the thermal rods to prevent heat from escaping to environment through the vessel wall of the reactor.**

Pertimbangkan rekabentuk yang diringkaskan bagi sebuah reaktor nuklear dimana keratan rentasnya tertera pada Rajah 4. Pada pusat reaktor terdapat gabungan rod-rod terma yang menjana haba dari pembelahan nuklear. Suatu bahan penebat melitupi rod-rod terma bagi menghalang haba dari terbebas ke persekitaran melalui dinding kebuk reaktor itu.

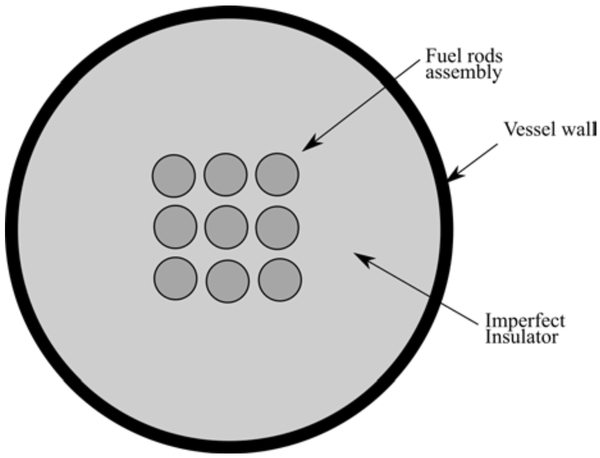

Figure 4
Rajah 4

[a] **Suppose you want to model the heat that may escape via conduction through the imperfect insulator and the vessel wall. State the information that you need to ensure the uniqueness of solution for the model.**

Andaikan anda ingin memodeli haba yang mungkin terbebas secara pengaliran melalui penebat tidak sempurna dan dinding kebuk. Nyatakan maklumat yang anda perlukan untuk memastikan penyelesaian unik bagi model ini.

(4 marks/*markah***)**

[b] **State the THREE (3) type of possible boundary conditions for the heat conduction model.**

Nyatakan TIGA (3) jenis syarat-syarat sempadan yang mungkin di dalam model pengaliran ini.

(3 marks/*markah***)**

[c] State why the problem cannot be solved as a 1D model.

Nyatakan mengapa masalah ini tidak boleh diselesaikan sebagai model 1D.

(4 marks/*markah***)**

[d] Describe the assumptions that must be made to ESTIMATE the solution with a 1D model.

Terangkan andaian-andaian yang mesti dibuat untuk MENGANGGARKAN penyelesaiannya dengan model 1D.

(4 marks/*markah***)**

5. Consider the following transient heat equation:

Pertimbangkan persamaan haba fana berikut:

$$-\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial t}=10 \qquad \text{for } x=[0,2]$$

where $\boldsymbol{u(0,t)}$ is prescribed with the Dirichlet boundary condition of 10 and $\boldsymbol{u(2,t)}$ is prescribed with homogenous Neumann boundary condition. Assume $\boldsymbol{u(x,0)=0}$

di mana $u(0,t)$ ditetapkan sebagai syarat sempadan Dirichlet sebanyak 10 dan $u(2,t)$ ditetapkan sebagai syarat sempadan Neumann homogen. Andaikan $u(x,0)=0$.

[a] Using the IMPLICIT method with THREE (3) spatial points of equal division and THREE (3) time levels where the time step $\boldsymbol{s=0.4}$ s, solve for all time levels. You may use minimum of 3 decimal places in the computation.

Dengan menggunakan kaedah TERSIRAT dengan TIGA (3) titik ruang dan TIGA (3) tahap masa di mana langkah masa $s=0.4$ s, selesaikan bagi setiap tahap masa. Anda boleh menggunakan minimum 3 tempat perpuluhan di dalam kiraan.

(14 marks/*markah***)**

[b] Sketch and label the discrete solutions of $\boldsymbol{u(x,t)}$.

Lakarkan dan labelkan penyelesaian diskret $u(x,t)$.

(8 marks/*markah***)**

[c] If the boundary condition at $x = 20$ is changed to

Jika syarat sempadan pada $x = 20$ ditukar kepada

$$\frac{\partial u}{\partial x} = 10,$$

sketch APPROXIMATELY the expected solution. DO NOT solve any equation.

lakarkan secara ANGGARAN penyelesaiannya. JANGAN selesaikan sebarang persamaan.

(8 marks/*markah*)

6. Consider Poisson's equation

Pertimbangkan persamaan Poisson

$$-\frac{\partial^2 u}{\partial x^2} = 10 \quad \text{for} \quad x = [0,1]$$

with the boundary conditions $u(0) = u(1) = 0$. The problem is discretized with FDM using FIVE (5) divisions of the domain.

Write the system matrix $\mathbf{A}$ and load vector $\mathbf{b}$ as MATLAB arrays. $\mathbf{A}$ and $\mathbf{b}$ must contain the boundary conditions data. DO NOT write code to solve the linear system.

dengan syarat-syarat sempadan $u(0) = u(1) = 0$. Masalah itu diungkap secara diskret dengan FDM menggunakan LIMA (5) bahagian domain itu.

Tuliskan matriks sistem $\mathbf{A}$ dan vektor beban $\mathbf{b}$ sebagai tatasusunan MATLAB. $\mathbf{A}$ dan $\mathbf{b}$ mesti mengandungi data syarat-syarat sempadan. JANGAN tulis kod bagi menyelesaikan sistem linear itu.

(15 marks/*markah*)

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APPENDIX 1
LAMPIRAN 1

USEFUL FORMULAS

1. Newton's Method

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

2. Formulas for first finite differences

$$f'(x_i) = \frac{f'(x_i) - f'(x_{i-1})}{h} + O(h)$$
$$f'(x_i) = \frac{f'(x_{i+1}) - f'(x_i)}{h} + O(h)$$
$$f'(x_i) = \frac{f'(x_{i+1}) - f'(x_{i-1})}{2h} + O(h^2)$$

3. Formulas for second finite differences

$$f''(x_i) = \frac{f'(x_{i+2}) - 2f'(x_{i+1}) + f'(x_i)}{h^2} + O(h)$$
$$f''(x_i) = \frac{f'(x_i) - 2f'(x_{i-1}) + f'(x_{i-2})}{h^2} + O(h)$$
$$f''(x_i) = \frac{f'(x_{i+1}) - 2f'(x_i) + f'(x_{i-1})}{h^2} + O(h^2)$$

4. Heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

5. Convective boundary condition

$$hu + ku' = hu_\infty$$

6. Discrete form of 1D Poisson's equation

$$-k\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} = f_i$$

7. Explicit and implicit methods for heat equation

$$-\lambda(u_{i+1}^l - 2u_i^l + u_{i-1}^l) = u_i^{l+1} - u_i^l - sf_i^{l+1}$$

$$-\lambda u_{i+1}^{l+1} + (1+2\lambda)u_i^{l+1} - \lambda u_{i-1}^{l+1} = u_i^l + sf_i^{l+1}$$

APPENDIX 2
LAMPIRAN 2

$$\lambda = \frac{\alpha s}{h^2}$$

8. Integrals of sine and cosine

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$$

9. Spherical coordinates

$$x = \rho \sin\varphi \cos\theta\,;\;\; y = \rho\sin\varphi\sin\theta\,;\; x = \rho\cos\varphi$$
$$\rho \geq 0; 0 \leq \;\varphi \leq\; \pi$$
$$dV = \rho^2 \sin\varphi\;\; d\rho d\theta d\varphi$$

10. Cylindrical coordinates

$$x = r\cos\theta\,;\;\; y = r\sin\theta\,;\; z = z$$
$$dV = rdzdrd\theta$$

11. Miscellaneous

$$\nabla \times \mathbf{u} = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)\mathbf{k}$$

$$\oint_C \; M(x,y)dx + N(x,y)dy = \iint_D \; \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$\oint_{\partial S} \mathbf{F}(x,y,z) \cdot d\mathbf{r} = \iint_S \; (\nabla \times \mathbf{F})\, dS$$

$$\oint_S \; (\mathbf{F} \cdot \mathbf{n})\, dS = \iiint_V \; \nabla \cdot \mathbf{F}\, dV$$

Volume of a sphere = $\frac{4}{3}\pi r^3$

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SULIT