

**NONLINEAR CONTROL OF ROBOT  
MANIPULATOR USING SLIDING MODE AND  
COMPUTED TORQUE CONTROL TECHNIQUE**

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MODE AND COMPUTED TORQUE CONTROL TECHNIQUE**

**by**

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## LIST OF ABBREVIATIONS

AFC	Active Force Control
ATE	Average Tracking Error
AVR	Advanced Virtual Risc
C+PR	Constant plus Proportional Rate
CR	Constant Rate
CTC	Computed Torque Control
DC	Direct current
D-H	<i>Denavit–Hartenberg</i>
DOF	Degree of Freedom
FK	Forward Kinematics
HIL	Hardware in a Loop
IK	Inverse Kinematics
KE	Kinetic Energy
MATLAB	Matrix Laboratory
P	Proportional
PC	Personal Computer
PD	Proportional and Derivative
PE	Potential Energy
PID	Proportional, Integral and Derivative
PIDCTC	Proportional, Integral, Derivative Computed Torque Control
PO	Percent of Overshoot
PR	Power Rate
SMC	Sliding Mode Control

SMCTC	Sliding Mode Computed Torque Control
SSE	Steady State Error
TS	Settling Time
VSC	Variable Structure Control



## LIST OF SYMBOLS

${}^0T_2, A_1, A_2$	Homogeneous transformation matrix
$a$	Length of the link
$A$	Generalised inertia matrix
Amp	Amplitude of periodic position tracking
$B$	Matrix represent Coriolis force
$B'C'$	Combination of coriolis and centripetal term
$C$	Matrix represent centripetal force
$c$	Sliding surface parameter
$D$	Matrix represent grabitational acceleration
$d$	Joint offset
$e$	Position error
$\dot{e}$	First derivative of error
$\ddot{e}$	Second derivative of error
$g$	Gravitational acceleration
$k$	Constant rate reaching parameter
$K_d$	Derivative gain
$K_i$	Integral gain
$K_p$	Proportional gain
$l$	Length of link
$m$	Mass of link
$M$	Control input value
$n$	Number of order/link/joint
$P_f$	Coordinate of final position

$P_i$	Coordinate of initial position
$Q$	Proportional rate reaching parameter
$S$	Sliding manifold
$s$	Sliding surface
$\dot{s}$	Derivative of sliding surface
$t$	time
$T_d$	Derivative time constant
$T_i$	Integral time constant
$u$	Control input
$V$	<i>Lyapunov</i> function
$\dot{V}$	Derivative of <i>Lyapunov</i> function
$W_l$	Mass of link for servomotor robot manipulator
$W_M$	Mass of motor for servomotor robot manipulator
$X_f$	$x$ -axis final position
$X_i$	$x$ -axis initial position
$X_{int}$	$x$ -axis intermediate position
$x_T$	Final state
$Y_f$	$y$ -axis final position
$Y_i$	$y$ -axis initial position
$Y_{int}$	$y$ -axis intermediate position
$\alpha$	Power rate reaching parameter
$\beta$	Twist angle
$\theta$	Angular position
$\ddot{\theta}$	Angular acceleration
$\dot{\theta}$	Angular velocity

$\theta_d$	Desired position
$\theta_{Dist}$	Amplitude of disturbance
$\tau$	Applied torque
$\omega$	Angular frequency

# **KAWALAN TAK LINEAR BAGI PENGOLAH ROBOT MENGGUNAKAN TEKNIK KAWALAN MOD GELUNGSURAN DAN DAYA KILAS TERKIRA**

## **ABSTRAK**

Idea penciptaan robot adalah berdasarkan sifat manusia dan kehidupan alam sekitar. Tujuannya adalah untuk menggantikan pekerjaan manusia yang melelahkan, berulang-ulang dan pekerjaan yang berbahaya dalam industri atau kegunaan ketenteraan. Sistem robot pengolahan adalah yang paling banyak digunakan dan boleh didapati dalam industri pembuatan. Robot jenis pengolah menyerupai mekanisma sistem rangkaian lengan dan objektif kawalannya adalah untuk mengolah sesuatu bahan tanpa berhubung secara terus dengan menggerakkan rangkaian akhir bagi melaksanakan operasi yang diinginkan. Oleh itu, kajian mengenai hubungan ruang sendi dan Kartesian serta sistem kawalan adalah penting. Objektif kajian adalah untuk mengkaji kawalan linear Berkadaran, Kamiran dan Terbitan (PID), Kawalan Daya Kilas Terkira (CTC) dan Kawalan Mod Gelunsuran (SMC) dan algoritma kawalan telah dibina dengan menggunakan blok MATLAB Simulink. Seterusnya, algoritma kawalan PID, PIDCTC dan SMCTC telah dilaksanakan pada robot pengolah yang mana parameter kawalan telah ditentukan dalam julat yang ditetapkan. Prestasi sistem kawalan diuji terhadap tindak balas langkah, menjejak kedudukan dan anggaran model. Ujian tambahan bagi mengurangkan penggelatukan telah dilaksanakan untuk SMCTC dengan menggunakan hukum kawalan pencapai. Berdasarkan keputusan tersebut, prestasi setiap kawalan dibandingkan. Sistem kawalan PID menunjukkan prestasi yang baik dan memenuhi keperluan yang ditetapkan dalam kajian ini. Manakala, PIDCTC menghasilkan keputusan yang lebih baik daripada kawalan PID terutamanya bagi menjejak kedudukan. Walau bagaimanapun apabila sistem

mempamerkan gangguan luaran, kedua-dua pengawal tidak mampu menolak gangguan tersebut. SMCTC teguh terhadap gangguan luaran dan menunjukkan prestasi yang terbaik di kalangan semua kawalan. Pelaksanaan hukum kawalan mencapai bukan sahaja dapat mengurangkan masa penyelesaian tetapi juga mampu untuk menghapuskan fenomena penggelatukan. Keberkesanan SMCTC kemudiannya telah ditunjukkan dengan melaksanakannya pada robot pengolah motor servo. Berdasarkan hasil kajian, prestasi eksperimen telah menunjukkan bahawa kedua-dua sendi mampu mencapai kedudukan yang dikehendaki.

# **NONLINEAR CONTROL OF ROBOT MANIPULATOR USING SLIDING MODE AND COMPUTED TORQUE CONTROL TECHNIQUE**

## **ABSTRACT**

The idea of a robot is created based on human and biological nature. The purpose of creating robots is to replace human work that is tiresome, repetitive, or dangerous task in industries or military application. The most extensively used is the manipulation robot system which can be found in manufacturing industries. This type of robot is an arm-link mechanism system, and the control objective is to manipulate material without direct contact by commanding the end-effector motion to achieve the desired operation. For this purpose, the study on the relation of joint-space and Cartesian-space, together with the control system, is essential. The objectives of this research are to study the linear Proportional, Integrator and Derivative (PID) control, nonlinear Computed Torque Control (CTC) and Sliding Mode Control (SMC), and the control algorithm was built using MATLAB Simulink block. The control algorithm of the PID, PIDCTC and SMCTC were implemented into the robot manipulator, where the controller parameters were determined within a prescribed range. The performance of the control system was tested for step response, position tracking and modelled estimation. An additional test for chattering reduction has been carried out for SMCTC with reaching control law. Based on the results, the performance of each of the controllers was compared. The PID control system shows a relatively good performance within the requirements of this study. Also, the PIDCTC produced better results than the PID controller, especially for position tracking. However, when the system were subjected to external disturbance, both controllers were unable to reject the disturbance. The SMCTC is robust towards external disturbance, and has shown

the best performance. The implementation of reaching control law not only reduced the settling time, but was also able to eliminate the chattering phenomenon. The effectiveness of SMCTC has been shown by its implementation into a servomotor robot manipulator. Based on the results, the experimental performance has shown that both joints are able to reach the position as desired.

# CHAPTER 1

## INTRODUCTION

### 1.1 Background of research

A robot is an electro-mechanical device guided by computer programs and applications, and is a combination of mechanical engineering, electrical engineering and computer science. A robot is generally constructed and designed based on human and biological nature. The purpose of creating robots is to replace human work that is tiresome, repetitive, or dangerous. This could be military and police work, such as the manipulation of explosive devices or the access to places that are difficult to reach by humans, such as space (Moosavian & Papadopoulos, 2007) or the bottom of the sea (Salvador et al., 2013), due to extreme environments which humans are unable to survive. Due to the limited working space, the first robotic surgery was successfully performed by two main robots called McSleepy and DaVinci, which allowed the surgeon to work with delicate and precise hand movements of fingers that would be impossible to be done by humans alone (Science Daily, 2010).

Robots can generally be categorized based on three basic applications (Stonecypher, 2009). The first application is mobile robots, which are automated platforms usually used to carry objects from one location to another. The second application is data acquisition and control robot system, which are used to acquire, process and transmit data into important information signals. The third category, which is the most extensively used, is the manipulation robot system, which is mostly found in manufacturing industries. The manipulator type robot is an arm-link mechanism system, and the control objective is to manipulate material without direct contact by



commanding the end-effector motion to achieve desired responses (Braganza et al., 2005).

The study on controlling the end-effector is related to kinematics and dynamics of robot manipulators (Hemami & Labonville, 1988; Rocha et al., 2011). Kinematics is the study of geometry and motion without the consideration of force and torque that give rise to the motion. While dynamics deal with forces or torque that cause the manipulator motion. Practically, the robot is commonly controlled in joint space, and follows a particular pre-set tracking trajectory. Thus, the feedback obtained is in terms of joint space, which means that the position error of the end effector is not directly obtained (Soltanpour & Fateh, 2009). The tracking performance of joint space is inheritable to the end-effector, which is influenced by the robot manipulator's modelling and control system.

The controller is a device that manages, commands, directs or regulates the behaviour of robots system. The force, or torque, is applied so that the system moves according to the commanded instructions. However, due to difficulty in accurately computing the robot manipulator parameters and changes in the payload, and non-consideration of friction in the mathematical model of the joint angle, the end-effector will not accurately follow the desired trajectory (Piltan et al., 2012a). Therefore, the actual joint angle and its derivative are returned as feedback to the control system to rectify it.

The robot manipulator control system can be classified into linear and nonlinear control systems. A system is considered linear if the differential describing it is linear and its components behave in a linear fashion (Niku, 2011), else, the system

is considered nonlinear. All practical systems are nonlinear, but the system component may be assumed to be linear or linearized for a small range in order to simplify the analysis. Thus, the linear control technique is valid for small changes in the angle, and when the robot moves in a slow manner (Saha, 2008).

One of the conventional linear controllers still in use is the Proportional, Integrator and Derivative (PID) controller. This controller is well known for its simple control structure, which does not require any component of the robot dynamics into its control law (Kelly, 1998). The controller uses an error signal to generate the proportional, integral and derivative action with a resulting signal gain and sum to form a control signal applied to the system (Aydogdu & Korkmaz, 2011). Since the PID is a linear control technique, the linear tuning methods cannot be applied to robot manipulator PID control directly, because robot manipulator dynamic is nonlinear and tuning the PID parameters are required to guarantee good performance of the system (Yu et al., 2013), which is time consuming (Alassar et al., 2010).

The nonlinearities of the robot manipulator nonlinear system can be classified into natural and artificial nonlinearities. The centripetal force in the rotational motion between each link, and the Coulumb friction between the contacting surface on the robot manipulator design contribute to the natural nonlinearities (Slotine & Li, 1991), while artificial nonlinearities are introduced by the designer. It is essential to express the dynamic model precisely so that the compensation in the control system is accurate (Brandtstaedter, 2009). Thus, applying the nonlinear control system with the consideration of relevant nonlinearities is crucial so that the system possess sufficient control performance despite a large range and high speed of motion (Slotine & Li, 1991).

One of the nonlinear controls is the Computed Torque Control (CTC), which is a special application of feedback linearization of nonlinear systems (Lewis et al., 2003). Precise knowledge of the model is required to design the CTC. However, it is well-known for its potentially high tracking accuracy (Nguyen-Tuong et al., 2008). Another nonlinear control technique is the Sliding Mode Control (SMC), which takes into consideration the nonlinearities and the disturbance of the system.

SMC can be construed as a variable structure control method due to the behaviour of the control input that is able to switch from one continuous function to another based on the current position of the state space. There are two main parts involve in the designing of the SMC (Jezernik et al., 1994). The first part is the design of a sliding surface with the state variable of the plant dynamic restricted to another set of equations. The second part deals with the construction of a switched feedback gain in order to drive the plant's state trajectory to the sliding surface. The SMC consists of two main phases. The reaching phase, where the state trajectory of the system is driven from any initial state to reach the switching manifolds, or the predetermined sliding surface in finite time. The sliding-mode phase is where the system is induced or slides into the sliding motion on the switching manifolds (Bartoszewicz & Zuk, 2010).

## **1.2 Problem statement**

The PID control offers the simplest and most efficient solution to many real-world control problems (Ang et al., 2005), as no other controllers match the simplicity, clear functionality, applicability, and ease of use offered by this type of controller (Wang et al., 1995). However, when the nonlinearities of the plant are considered, and dealing with fluctuated parameter and disturbance rejection, the PID controller must

be retuned regularly. The conventional controller is no longer worthwhile, as obtaining the PID gain will be tedious (Lee et al., 2002), and the simple physical meaning of PID gain will be lost (Yu et al., 2013). To overcome this problem, several nonlinear and robust control techniques have been developed.

One of them are the CTC, where the controller exhibits a good tracking performance, however, it requires precise knowledge of the robot model (Jezernik et al., 1994; Nguyen-Tuong et al., 2008). Thus, it is preferable to implement a robust control technique for a robot manipulator. It is well-known that the SMC is a nonlinear control technique that is robustness against model uncertainty and external disturbance (Mohammad & Ehsan, 2008; Slotine & Li, 1991). Moreover, Lee et al. (1992) stated extra advantages, which are the output performance can be predetermined by sliding surface, and there is no overshoot in regulations. Thus, in this study, the CTC and the SMC is used in order to obtain good tracking performance and robustness capability for the system.

However, SMCs have often encountered chattering phenomenon, which is a drawback of the system (Lee et al., 1992; Sefriti et al., 2012). It is due to the high frequency oscillation of the control input, which causes the motion trajectory of the system to change frequently in the vicinity of a sliding surface before it reaches and slides along the sliding surface. This may excite un-modelled high frequency modes, which degrades the performance of the system and may even lead to instability. Chattering may also lead to a high wear for moving mechanical parts, and high heat losses in the electrical power circuit.

Thus, numerous research works on SMCs have the main aim to overcome this drawback. (Slotine & Sastry, 1983) eliminated the chattering problem based on a boundary layer solution. However, this method provides a solution for small uncertainties only. Wang et al. (2002) eliminated the high control activity and chattering by incorporating an auto-tuning neuron into the SMC. Also, (Fallaha et al. (2011) stated that the method proposed by using the reaching control law is able to deal with chattering performance. Thus, in this study, the Sliding Mode Computed Torque Control (SMCTC) will be implemented with the reaching control law which is introduced by Gao & Hung (1993) in order to reduce the chattering phenomenon.

### **1.3 Research objective**

The purpose of this research is to study the implementation of a nonlinear control system in order to control the position robot manipulator system as desired. The sub-objectives of this research are:

1. to design a nonlinear robust sliding mode type controller and implement it by considering various operating and loading condition
2. to compare the effectiveness of the designed nonlinear controller with other controllers and
3. to verify the designed controller into a servomotor robot manipulator system in real-time.

#### **1.4 Research scope**

There are several limitations of the proposed research. The plant of the controller system is a two link robot manipulator in which will be moved simultaneously and the controller system focuses on the linear PID control, nonlinear PID Computed Torque Control (PIDCTC) and nonlinear SMCTC. Although the PID and PIDCTC have been studied previously by other researchers, in this study, these controller will be used for comparing the performance of designed SMCTC. Where the SMCTC will be designed with the consideration of reaching control law. Each controller parameter focuses on the prescribed range. From that range, three different values are tested on the robot manipulator system in order to produce the desired performance.

The proposed controller system focuses only on the two type of position angle tracking, which is the constant position and a sinusoidal position tracking. From the output feedback of the system, only measurement of joint displacement is available. The velocity and acceleration are obtained through single and double differentiation of the joint displacement angle, but are usually contaminated by measuring noise.

For verification of the controller, SMCTC scheme will be implemented into a servomotor robot manipulator system. The structure of the robot is assumed as a planar robot with a slender link. The movement is restricted by two Degree of Freedom (DOF) caused by a revolute joint. In this study, the elasticity and damping of the joints and the backlash introduced by the gear pairs of the transmission mechanism are not taken into consideration. The controller is not designed to eliminate all of the factors mentioned above.

## **1.5 Research approach**

The research is initiated by studying the fundamental concepts and relevant topics. Topics include the mathematical modelling of the robot manipulator, which consists of kinematics and dynamics models. Recent research works and studies in the existing literature on linear and nonlinear control systems were reviewed carefully. The application of the control systems on robot manipulators was the main focus.

From the objectives previously stated, there are several tasks to be achieved. First, the robot manipulator is modelled based on mathematical equation of motion. The behaviour of the robot manipulator is studied and comprehended before any controller system is implemented into the system.

Using the equation of motion, the linear PID control, nonlinear CTC and SMC are studied. Based on that, the combination of CTC and SMC to become SMCTC is designed to control each link of the robot manipulator to follow the desired task. The control algorithms of PID, the combination PID and CTC known as PIDCTC and SMCTC were built to obtain the comparison results and effectiveness for each of the controllers.

After the SMCTCs manage to control the robot manipulator, the controller part is adjusted to suit the hardware part for implementation into the designed servomotor robot manipulator. Several tests were conducted and the results and data were used to evaluate the performance of step response and position tracking robot manipulators for both simulations and practical experiments.

## **1.6 Thesis outline**

This thesis is arranged in accordance with the objectives and approaches, as previously mentioned. Chapter 2 provides an in-depth literature review of the related subjects including the enhancement on previous studies and explanation of previous concepts and knowledge. The topics discussed include the development of robot manipulators, linear and nonlinear control systems, such as PID, CTC, SMC and the development of other control systems that attract researcher's interest.

Chapter 3 provides a description of the proposed research methodology. The methods of kinematics and dynamics of the robot manipulator are shown. The conventional PID controller, CTC and SMC are discussed in more detail, including the control algorithms. Next, a detailed development of the servomotor robot manipulator, including the control system, is explained.

In Chapter 4, the results achieved based on the gain value of PID, PIDCTC and SMCTC are discussed, along with a reasonable justification to support the results. The study concludes with a summary, conclusion and contribution of the overall research, which are presented in Chapter 5. Recommendations for future research are presented as well.



## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

This chapter presents the overview of the topic covered in this study. Four major topics are focused, which are the modelling the robot manipulator, conventional control systems, sliding mode control system and chattering phenomenon. The review on the modelling of the robot manipulator includes kinematics and dynamics modelling. Next, the linear PID control system and the nonlinear CTC system are discussed. After that, the concept of nonlinear SMC is presented. Lastly, one of the common problems regarding the SMC, which is chattering, and the method to reduce chattering, is discussed.

#### **2.2 Previous Study on Modelling Robot Manipulator**

Modelling a robot system can be categorized into kinematics and dynamics modelling. Kinematics and dynamics of robot manipulators are fundamental to robot technologies. Both models are widely used in the simulation of motion, analysis of robot manipulator structures and design of control algorithms (Lee, 1982).

##### **2.2.1 Review on Kinematics Equation**

Kinematics is the study of the motion of a system without consideration of the forces or moments that cause the motion. Kinematic modelling is crucial for analysing the behaviour of robot manipulators. It can be divided into forward and inverse kinematics. Forward kinematics (FK) is usually applied for the design and simulation

of robotics. Deriving the FK equation is more straightforward and less complex when compared to inverse kinematics problems. On the other hand, inverse kinematics (IK) is more difficult to solve due to the singularities and nonlinearities within the system. Nevertheless, IK is crucial for motion planning resolution algorithms (Kucuk & Bingul, 2006; Rocha et al., 2011). Normally, before formulating the IK equation, the FK equation is obtained first. From that, the IK equation is calculated.

There were three types of methods for formulation of FK that had been studied by Aspragathos & Dimitros (1998), namely, homogeneous transformation, Lie algebra and the screw theory via-dual quaternion algebra. Nevertheless, for the three methods, the robot parameters definition is based on the well-known *Denavit-Hartenberg* notation which the concept was first introduced in a study by *Denavit, Hartenberg* and *Evanston*. Since then, this technique was modified by many researchers in order to ease the parameter identification. The detailed formulation of such a technique had been documented by Niku (2011).

On the other hand, there are two types of techniques to solve the IK problem, which are the analytical and numerical methods. This study focuses on the analytic methods which are classified by geometric and algebraic solutions (Kucuk & Bingul, 2006). The application of a geometric solution is suitable for a simple robot structure. For robots that constitute of complicated structures, an algebraic solution is preferred (Lee & Ziegler, 1984).

### **2.2.2 Review on Dynamics Equation**

A dynamic equation represents the relationship between robot motions towards the applied torque. This equation is essential for the simulation of robot manipulator

motion and also for the design of a control algorithm. In order to obtain the dynamics equation of motion for robot manipulators, there are two methods that are commonly used, which are the *Lagrangian* and the *Newtonian* methods. Both methods have been studied and compared by Silver (1982). The author showed that the formulation of the *Lagrangian* and *Newtonian* methods is indeed equivalent, and there is in fact no fundamental difference in computational efficiency between them.

The concept of *Lagrangian* was introduced by Joseph Louis Lagrange, which is defined as the kinetic energy minus the potential energy of the system (Zefran & Bullo, 2004). The *Lagrangian* concept leads to obtaining the equation of motion of the system by substituting such equation into the *Euler-Lagrange* equation. The robot system is treated as a whole and provided systematic procedures for eliminating the constraints from the dynamic equations. A detailed derivation of the dynamic equation for the *Lagrange* method has been established in numerous studies. Niku (2011) provided a systematic procedure to obtain the dynamics model of the robot manipulator, not only for a simple structure, but also for multiple DOF robot manipulators.

### **2.2.3 Perturbation**

Modelling is basically a process of constructing a mathematical description for a physical system to be controlled. Modelling a system can be divided into two parts, the nominal model and the model uncertainties. The model uncertainties are the differences between the nominal model and the real system, and can be categorized as parametric uncertainties and non-parametric uncertainties, or un-modelled dynamics. The parametric uncertainties are due to the imprecise model and the variation in the

load, while the non-parametric uncertainties are usually neglected, such as the motor dynamics, measurement noise and sensor dynamics (Slotine & Li, 1991). The term perturbation indicates the combination of modelling uncertainties and external disturbance (Elmali & Olgac, 1996).

## **2.3 Controller System**

Various controller schemes have been developed for robotic manipulators. Two types of conventional controllers, PID control and CTC, have been widely used. The PID control is a linear controller, and the CTC is a nonlinear controller. Most of them are based on the assumption that a complete state measurement (position and velocity) of the robot manipulator is available (Mien et al., 2013).

### **2.3.1 Review on Linear PID Controller**

The PID control algorithm is one of the most commonly used algorithms in the control system area. This is due to its simple structure and clear physical meaning of control parameters, which makes it easier to implement in control systems (Patel & Chaphekar, 2012). Even though PID control law has already been established and implemented in industrial robot manufacturing, there still exists an open problem that attracts researchers in this area. The early application of the PID controller on robot manipulators has been studied by Takegaki & Arimoto (1981). Later on, Wen & Murphy (1990) extended the research by Takegaki & Arimoto (1981) using the modified Lyapunov function for stability trajectory tracking of robot manipulators.

One of the issues that often arises during implementation of PID control techniques is insufficiency to guarantee the desired performance of the system.

Therefore, the PID must be appropriately tuned. Manually tuning the PID gain for the robot manipulator is time consuming due to the nonlinearities of the system. Tuning using nonlinear method caused the great advantage of the clear physical meaning disappears, as the method to obtain control gain becomes complicated (Yu et al., 2013). The study by Kelly & Carelli (1996) extended the previous results of linear gain PD controllers with the nonlinear functions of the gains. The authors also provided sufficient conditions on proportional and derivative gain in order to guarantee a global asymptotically based system.

Most of the mentioned studies on control are tested on computer simulations, and a few of them consider case studies. Research by Agrawal et al., (2012) implemented a discrete PID control technique into a DC motor using an Advanced Virtual Risc (AVR) (Atmega 16/24) microcontroller of a robot arm to replace the complex electronic circuitry. The simple PID controller presented applied the *Ziegler-Nichols* tuning method in order to determine the PID gains values. *Ziegler and Nichols* is one of the well-known simple tuning techniques that use heuristics method. However, it is written that this technique is applicable on linear systems only, and cannot be applied for nonlinear robot systems (Yu et al., 2013).

### **2.3.2 Review on Nonlinear CTC**

The CTC is a special application of feedback linearization for nonlinear systems. The design control problem consists of a feed-forward loop model and a feedback loop model. The feed-forward model is used to predict the feed-forward control input in order for the robot to follow the given desired trajectory. The dynamic

model of the robot can be used as a feed-forward model. The feedback control input is used as a compensator for tracking errors (Nguyen-Tuong & Peters, 2008).

For good tracking performance, a feed-forward model requires the accurate model of the robot. However, such a condition is difficult to achieve due to the presence of disturbances and the variance of the manipulator's parameters. Thus, to compensate the performance, Piltan et al. (2012b) applied a proportional gain and derivative gain type feedback control input. These gain feedback parameters must be tuned in a way to compensate for the difference between the nominal parameters and the perturbed parameters. Piltan et al. (2012a) later improved the overall performance of the CTC by substituting the linear PD type feedback control input with a discontinuous feedback control input. The authors showed a comparison of the performance between both feedbacks. The results proved that the discontinuous feedback control input is more robust than the PD CTC.

The requirement for the complete knowledge of robot dynamics and physical parameters is a well-known issue in CTC. A less accurate model requires high feedback gains and caused the robot to be less safe for the environment, and degrades the performance of the system. The study by Chen et al. (1988) proposed an improved dynamic model for two types of robust CTC in order to enhance the accuracy of the model by compensating the uncertainty of the system. Firstly, the non-adaptive robust CTC is applied for a system where the bounded uncertainty is available. Secondly, the adaptive robust CTC is applied for a system in which the bounded uncertainty is unavailable. The adaptive scheme is used to estimate the uncertainty bounded. The control action is then based on the estimated uncertainty. The simulation results proved

that the robust computed torque guarantees a zero error convergence when system uncertainty is considered.

### **2.3.3 Review on Robust Controller**

The robustness of a system is an important criteria for a good control system. The study by Mailah et al. (2006) proposed an approach for a robust motion control for mobile robot manipulator. The part of designed controller consists proportional-integral active force control to compensate the dynamic effects including the bounded known/unknown disturbances and uncertainties. The effectiveness and robustness of the proposed scheme are investigated through deliberately introduce a number of disturbances in the form of vibratory and impact forces.

Later on, Sabzehmeidani et al. (2010) designed and integrated three different types of control algorithms into the robot controller system which are the PID controller, active force control (AFC) and SMC. This hybrid scheme is to be known as AFC+SMC+PID. The primary objective to ensure accurate and robust trajectory tracking control of the micro robot system is achieved. The performances of the control system under different types of disturbances are evaluated through a simulation study. The obtained results clearly demonstrate an effective trajectory tracking capability of the wormlike micro robot in spite of the negative effects of the external disturbances.

## **2.4 Nonlinear SMC**

A SMC system is a kind of Variable Structure Control (VSC) proposed and elaborated by Emelyanov and several co-researchers including Utkin and Itkis from the Soviet Union in the early 1950s (Piltan et al., 2011). Since then, the SMC technique

has been extensively studied for the class of nonlinear systems due to its special characteristics that provide robust behaviour towards model uncertainty and external disturbance.

Due to a simple structure in implementing the control law, while at the same time maintaining good performance, the SMC is an ideal candidate for robot manipulator control (Ge & Ye, 2011). SMC was first implemented into robot manipulators by Young (1978), and then by Slotine & Sastry (1983). The authors presented the control design methodology with approximate SMC to remedy the chattering effect.

The design methodology of a SMC is composed of a two-step procedure. The first step is to design a sliding surface where the state trajectory is restricted to such a surface in order to obtain the desired response. The second step is to construct a control action that takes the system into such surface and keeps it there (Kurfess, 2010; Liu & Wang, 2011). The control action is constructed by switching control and equivalent control (steady state control).

The design procedure mentioned above yields two type of phase as shown in Figure 2.1. The first phase is the reaching phase; this is the step where the state with an error vector is attracted to a sliding manifold,  $S$  or sliding surface,  $s = 0$ . The second phase is the sliding mode phase; the state with error vector slides on the surface until it reaches the equilibrium point or final state,  $x_T = 0$  or error,  $e = 0$ , which is known as steady state (Gao & Hung, 1993).



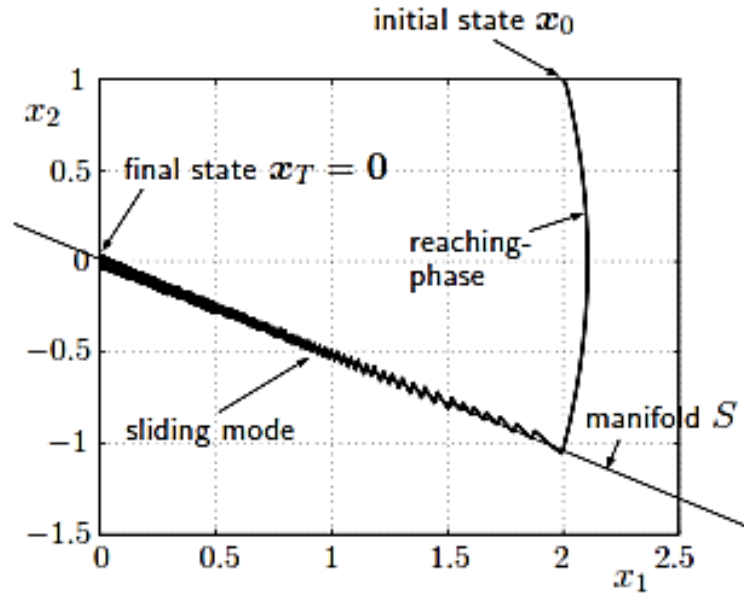


Figure 2.1 Sliding mode mechanism in phase plane (Fallaha et al., 2011).

The system behaviour of the SMC can be analysed in the phase plane ( $e = x_1$ , derivative error,  $\dot{e} = x_2$ ) as shown in Figure 2.1 and Equation (2.1). The discontinuities of the control input,  $u$  occur at the  $s = 0$ , which consists of two functions. The first function is when state trajectory is at the upper semiplane,  $s > 0$ , thus  $u = -M$ . The second function is when the state trajectory is at the lower semiplane,  $s < 0$ , and thus  $u = +M$ . Where  $M$  is a control value action. When the state trajectory reaches the switching line at time  $t_1$ , the state remains in the switching line for  $t > t_1$ , and this motion in the sliding line is called the sliding mode. This is due to the state trajectory being interpreted as the motion equation (Utkin et al., 1999),

$$\dot{e} + ce = 0 \quad (2.1)$$

To guarantee the convergence, the sliding surface parameter,  $c$  must be selected such that Equation (2.1) is Hurwitz (Yilmaz & Hurmuzlu, 2000). The real part of its eigenvalue must be negative  $\dot{e} = -ce$ . Thus,  $c > 0$ .

A simple derivation of SMC algorithm is presented by Chen et al. (1990), which does not required the inverse of inertia matrix. The methodology shows the application of *Lyapunov* function in order to guarantee that the constructed control input  $u(t)$  converges the  $s(t)$  to be 0.

Myszkorowski (1989) proposed a feed-forward SMC for trajectory tracking of a robot manipulator system. The feed-forward part is the compensator for the know dynamics of the robot, while the feedback SMC is the variable structure regulator. The author also proved that the proposed control law is locally stable for the whole system in the presence of parameter uncertainty and bounded disturbances.

Comparing the nonlinear type controls between CTC and SMC, the CTC is unable to eliminate the nonzero steady state error in the presence of the parametric uncertainty system. Tzafestas et al. (1996) study the comparison between the conventional CTC and SMC, which robustified the CTC. The finding shows that SMC is superior to CTC, and the superiority is strengthened when the uncertainty level is increased.

For the next section, two different phases of SMC are discussed, including a brief introduction on inherent robustness.

### 2.4.1 Reaching Phase

SMC usually make a distinction between two different phases, which are the reaching phase and sliding mode phase. The reaching phase is also known as a transient phase while approaching the sliding mode phase. It lasts until the system state reaches the sliding surface.

During this period, the structure of reaching control law does not contain the discontinuous term and consequently, does not suffer from chattering (Bartoszewicz & Lesniewski, 2014). However, the system is unable to control the tracking error directly and is sensitive to parameter uncertainties and noise (Yilmaz & Hurmuzlu, 2000). Therefore, reducing, or even eliminating the reaching phase is an interesting issue in the SMC.

The reaching control law approaches were first introduced by Gao & Hung (1993). The purpose of its implementation is to improve the performance of the reaching mode and amplitude of the chattering. Three types of reaching laws are introduced by the authors. The constant plus proportional rate reaching is implemented in a case study for controlling the robot systems. The results obtained from this work show that the response during the reaching phase is able to improve by modifying the parameter of the reaching control law.

The Lyapunov method in the SMC only warrants the reachability to the sliding surface in a finite time. However, the system behaviour during the reaching phase is not specified. Furthermore, the trajectory error of the system is unable to be directly controlled. Thus, Chang & Hurmuzlu (1993) proposed a modified SMC which eliminate the reaching phase. It is known that once the state reaches the sliding surface,

it will never leave it. Based on such statement, the tracking error is modified so that the state response begin on the sliding surface regarding arbitrary initial conditions.

Later on, Yilmaz & Hurmuzlu (2000) presented a reaching phase elimination by modifying the sliding surface through the use of an exponential function, as the exponential form is the most preferable choice for a good convergence. The proposed method ensured the optimal convergence parameter with respect to the tracking error and control input.

The reaching control law approach is also used to reduce chattering. Fallaha et al. (2011) intended to reduce the chattering effect and at the same time, maintain a good tracking performance. The authors designed a nonlinear reaching control law by using the exponential function. The exponential reaching law is dynamically adapted to a variation of controlled systems in order to achieve the desired performance. The result of this work shows that the chattering and the tracking performance are positively improved compared to conventional SMCs.

#### **2.4.2 Sliding Mode Phase**

The sliding mode phase starts when the state trajectory reaches the sliding surface (Bartoszewicz & Zuk, 2010). Once the state reaches the sliding surface, it will slide and remain on the sliding surface,  $s \approx 0$  (Harashima et al., 1987; Yilmaz & Hurmuzlu, 2000). During this phase, the dynamics of the state are determined by the sliding line parameter and the order of the equation of the original system is reduced following the sliding line (Bartoszewicz & Zuk, 2010; Sage et al., 1999).

The sliding mode phase on the sliding surface is carried out due to the discontinuity of the controller (Piltan et al., 2011), which is very sensitive to small deviations and provides an infinitely high gain as a corrective action. Therefore, this causes the system to be insensitive towards model uncertainty and external disturbance.

During the sliding mode phase, although the system is robust towards invariance, the discontinuous control action to correct the trajectory error supposedly switches quickly. This action exhibits a serious drawback of high frequency oscillations, which inevitably result in chattering (Bartoszewicz & Zuk, 2010). The chattering phenomenon is discussed in the next section.

## **2.5 Chattering**

The SMC is an attractive nonlinear control technique due to its robustness towards parameter variation and external disturbance. However, there are a few problems that arise within the SMC, which receive a great deal of attention among researchers. One of the common problems in the application of the SMC is the chattering phenomenon.

Chattering is high frequency oscillation that appear about the desired equilibrium point and causes a decrease in the system's performance, which causes the system to become unstable. This is due to un-modelled dynamics, switching gain value, discontinuous function in the SMC, idle time or delay due to computer calculation limitation of physical actuators, among other reasons (Hung, Gao, & Hung, 1993).

Many different approaches exist that attempt to reduce or completely eliminate the chattering effect. They can be categorized as continuous approximation, observer design and higher order SMC (Aguilar-Ibañez et al., 2013). A review of these approaches are discussed in the following sections.

### **2.5.1 Continuous Approximation**

One technique to reduce the chattering phenomenon is to replace the discontinuous control law with a continuous type. Slotine & Sastry (1983) proposed a continuous control law which approximates discontinuous control law in order to obtain the insensitive tracking system towards parameter variation and disturbance, and thus, improves the chatter along the sliding mode. The detailed design methodology of the continuous control law can be found in the study by Kurfess (2010).

In order to surmount the chattering phenomenon, the discontinuous function is replaced with the smooth function. The widely used smooth function is related to the boundary layer technique. However, Huang & Chang (2005) stated that the boundary layer technique does not guarantee the elimination of steady state errors. The authors introduced the self-tuning law in SMCs, which have been observed to control input chattering, and as a result the steady state error do not occur.

Sulaiman et al. (2014) proposed an improved technique of state the dependent auto-tuning of sigmoid function and the switching gain. The authors introduced a control algorithm without the use of a complex algorithm so that it is easy to implement into the hardware system. The findings reveal that the proposed technique can maintain the robust performance of the SMC to suppress the chattering phenomenon.

### **2.5.2 Observer or Estimation of Uncertainty**

Conventional SMC require the knowledge of model uncertainties and external disturbance. The exact knowledge is impossible to obtain, consequently, these perturbations are assumed to be bounded. The control action based on this knowledge negatively affects the tracking performance and causes undesired oscillations. Elmali & Olgac (1996) proposed a perturbation estimator to improve such issues. An on-line estimator is used to estimate the perturbation, which relieve the burden of guessing the upper bound from the previous technique. The authors proved that the tracking performance can be improved.

The control gain of the boundary layer technique is a counterbalance between chattering and robustness. Lower control gain is required to reduce or eliminate chattering. In order to maintain the robustness of the system, it requires all states to be modelled, including the un-modelled part, which are difficult to model. A neural network structure is proposed by Sefriti et al. (2012) to estimate the unknown parts of the model. The neural network weight is adjusted during the online implementation by using the gradient descent (GD) method.

### **2.5.3 Higher order Sliding Mode Design**

The boundary layer technique does not guarantee that the oscillations will disappear. Bartolini et al. (1998) proposed a high-order SMC that translates the discontinuity produced by the sign function to the higher order derivatives, producing the continuous control signal. However, this technique requires great computing efforts.