UNIVERSITI SAINS MALAYSIA

1st Semester Examination
2006/2007 Academic Session

October / November 2006

EAS 663/4 – Dynamics and Stability of Structures

Duration: 3 hours

Instructions to Candidates:

1. Ensure that this paper contains **TEN (10)** printed pages including appendices before you start your examination.
2. This paper contains **FIVE (5)** questions. Answer **ALL FIVE (5)** questions.
3. Each question carries equal mark.
4. All questions **CAN BE** answered either in English or Bahasa Malaysia.
5. Each question **MUST BE** answered on a new sheet.
6. Write the answered question numbers on the cover sheet of the answer script.

1. (a) Define viscous damping. Sketch the displacement response, (v) versus time, (t) of undamped and damped free vibration for SDOF systems. Does the natural period of vibration, T, change with the presence of damping?

(6 marks)

(b) Figure 1 shows a simple frame. The support of the frame is subjected to a sinusoidal ground motion, V_s (t) = 5x10^{-3} Cos 5.3t m, to support a rotating machine. Assume the damping of the system is equal to 5% of critical damping and the value of E = 200 x 10^3 MPa, determine:

(i) natural circular frequency , ω
(ii) frequency ratio, r
(iii) static deflection, V_o
(iv) maximum relative displacement , $u_{\max} = \dfrac{r^2 V_o}{\sqrt{(1-r^2)+(2\zeta\, r)^2}}$
(v) maximum shearing force in each column by assuming k = 3EI/L^3
(vi) maximum moment in the column,
(vii) maximum stress in the column. Assume the cross section of the columns is square.

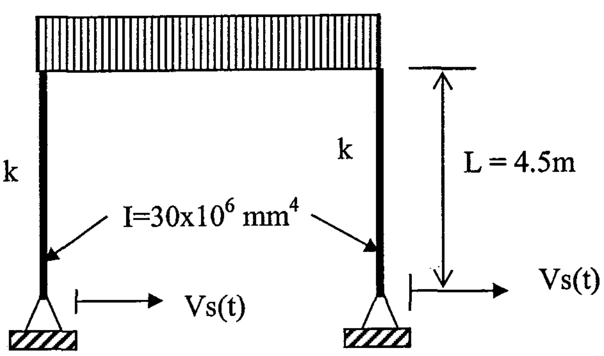

Figure 1

(8 marks)

1. (c) Figure 2 shows a spring-mass model for undamped SDOF system under harmonic excitation p(t). Assume the system is initially at rest. Given the value of m = 20 kg and k = 7000 N/m. Determine the resulting motion for the system. The total response, $v = \frac{V_o}{1-r^2}\cos\Omega t + A_1 \cos\omega t + A_2 \sin\omega t$

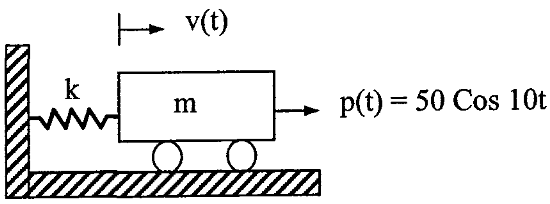

Figure 2

(6 marks)

2. (a) Define response spectra in structural dynamic problems.

(4 marks)

(b) Figure 3(a) shows a model of column-mass for a SDOF system. The system is subjected to two rectangular loads, p(t) as shown in Figure 3(b). The weight of the mass block is 2500 kN and the column stiffness, k = 2000 kN/mm. Assuming it is an undamped system, predict the maximum displacement response, $v_{\max} = R_{\max}(\frac{Po}{K})$ and the maximum total elastic forces developed in the system for both p(t). The value of the maximum response ratio, R_{max} can be obtained from the displacement response spectra as shown in Figure 3(c). Give comments on your observations of the results for both rectangular loads.

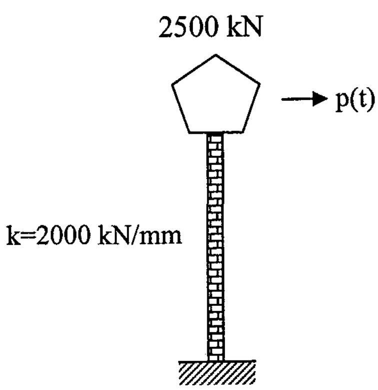

Figure 3(a)

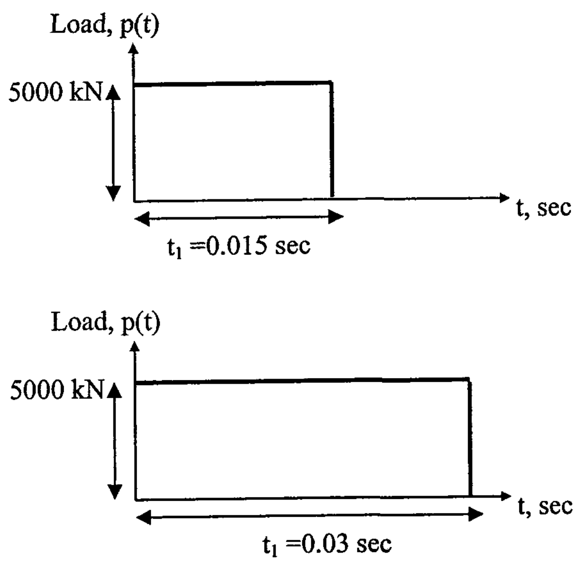

Figure 3(b)

(6 marks)

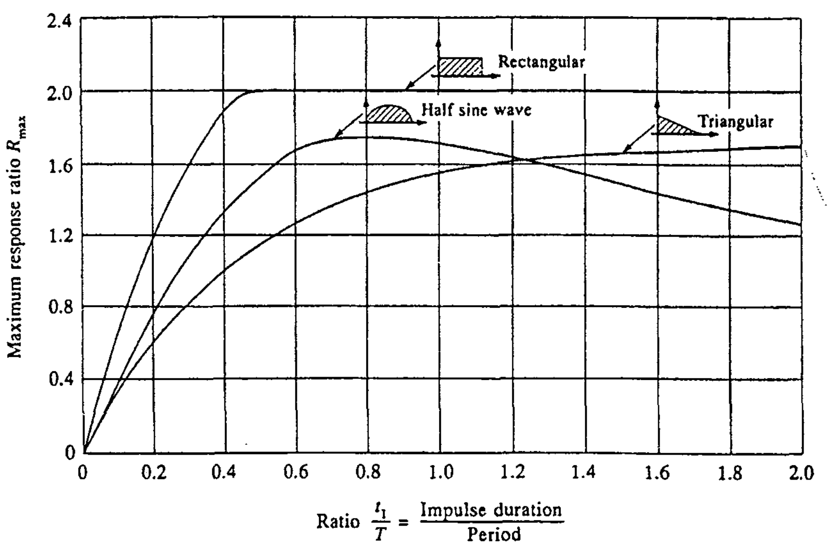

Figure 3(c)

2. (c) Briefly discuss the response of a SDOF system to impulsive load. Sketch **THREE (3)** typical graphs of force, (P) versus time, (t).

(5 marks)

(d) Figure 4 shows a model for 2DOF system under free vibration. Derive the equations of motion for the system.

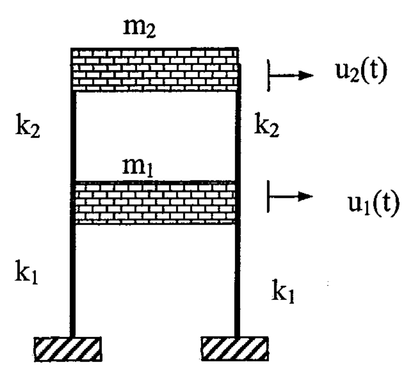

Figure 4

(5 marks)

3. (a) Equilibrium state of a structural system can be classified as stable, neutral and unstable. By using an axially loaded perfectly straight column with both ends pinned, explain the concepts of stable, unstable and neutral equilibrium. Explain also the associated concept of bifurcation point.

(6 marks)

(b) Explain the meaning of an imperfect column. Figure 5 shows an initially straight column subjected to an axial load P which acts at an eccentricity e from the centroidal axis of the column. Derive the following relation between mid-height deflection δ and ratio P/P_E where P_E is Euler buckling load ($\pi^2 EI/L^2$):

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right) - 1\right]$$

Sketch the graph of P/P_E versus δ for three different values of e.

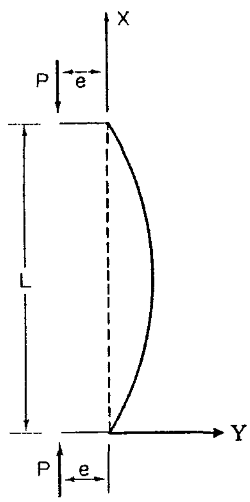

Figure 5

Sketch also on the same graph the behaviour of a perfect column with the same end conditions. Based on the graph plotted, discuss the difference in behaviour between a perfect and an imperfect column.

(14 marks)

4. (a) Derive the following fourth order differential equation for a beam-column carrying a distributed load w as shown in Figure 6 :

$$y^{iv} + k^2 y'' = \frac{w}{EI}\ , \ k^2 = \frac{P}{EI}$$

where y : lateral displacement of beam-column, P : axial force acting at both ends of beam-column, EI : flexural rigidity, $(\ ...)'' = d^2(...)/dx^2$ $(\ ...)^{iv} = d^4(...)/dx^4$. Next, explain how the above fourth order differential equation is used to determine the critical load of a fixed-fixed beam-column without any distributed load. You are required to specifically point out in your explanation how to arrive at the eigen value problem which can be used to solve for the critical load of a fixed-fixed beam-column. Detailed solution for the critical load is not required.

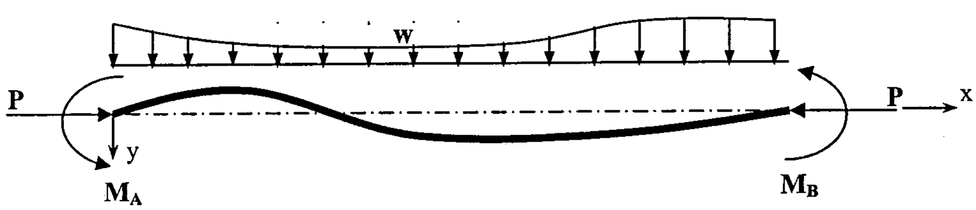

Figure 6

(12 marks)

4. (b) A simple frame as shown in Figure 7 is subjected to two axial loads, P. Both supports A and D are fixed. It is given that the frame is properly braced against lateral sway. Obtain the effective length, L_e for columns in the frame by using the following equation for an elastically retrained column:

$$(1 - \lambda_1 - \lambda_2 - \lambda_1\lambda_2\Phi^2)\Phi \sin\Phi + (2 + \lambda_1\Phi^2 + \lambda_2\Phi^2)\cos\Phi - 2 = 0$$

where $\lambda_1 = EI/(\alpha_1 L)$, $\lambda_2 = EI/(\alpha_2 L)$, $\Phi = kL$, $k^2 = P/EI$, EI : flexural rigidity , L : length of column, α_1, α_2 : rotational stiffness of end 1 and 2 of column being studied, respectively. If it is found that bracing system of the frame is not effective, comment on the value of the effective length relative the value obtained above. Provide proper justification to support your comments.

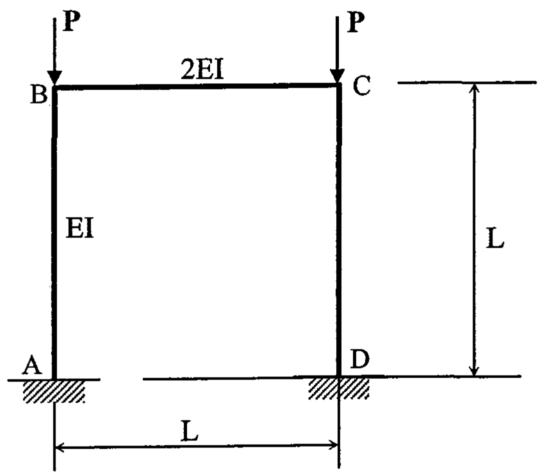

Figure 7

(8 marks)

5. (a) Explain the meaning of P-δ and P-Δ effects. Use suitable sketches in your explanation.

(4 marks)

(b) Slope deflection equations for a beam-column are given as follows:

$$M_A = \frac{EI}{L}\left(s_{ii}\theta_A + s_{ij}\theta_B\right)$$
$$M_B = \frac{EI}{L}\left(s_{ji}\theta_A + s_{jj}\theta_B\right)$$

where s_{ii}, $s_{ij}(=s_{ji})$, s_{jj} are stability functions as shown in the following equations :

$$s_{ii} = s_{jj} = \frac{kL\sin kL - (kL)^2\cos kL}{2 - 2\cos kL - kL\sin kL}$$

$$s_{ij} = s_{ji} = \frac{(kL)^2 - kL\sin kL}{2 - 2\cos kL - kL\sin kL}$$

$$k^2 = \frac{P}{EI}$$

and M_A, M_B, θ_A and θ_B are shown in Figure 8.

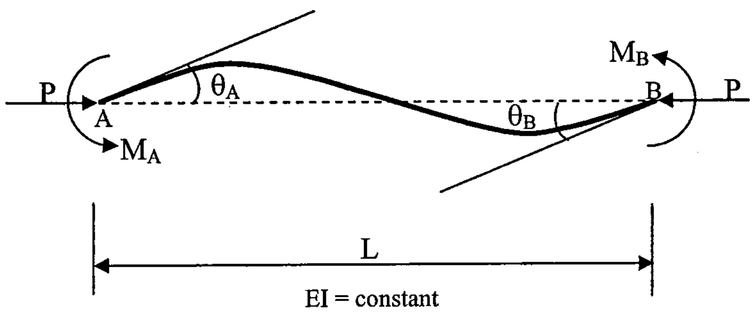

Figure 8

Making use of the above set of slope-deflection equations, obtain the equation representing the eigen value problem which must be solved to yield the critical load for column B-C shown in Figure 9. You are required to show clearly the steps involved in obtaining the eigen value problem and contents of the equations. Detailed solution for the critical load is not required.

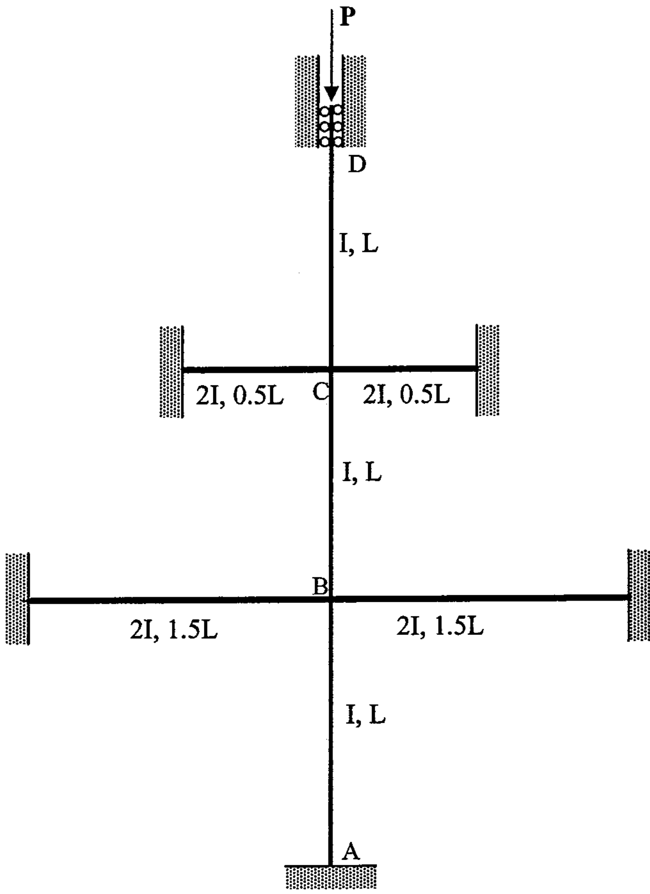

Figure 9

(16 marks)