ENTROPY-STABLE RESIDUAL DISTRIBUTION METHODS FOR SYSTEM OF EULER EQUATIONS

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FOR SYSTEM OF EULER EQUATIONS

by

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This thesis is dedicated to my grandfather who always believed in every doubt and doubted in every believe...

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LIST OF ABBREVIATIONS

PDE	Partial Differential Equation
CFD	Computational Fluid Dynamics
RD	Residual Distribution
FV	Finite Volume
N	Narrow-scheme
PSI	Positive Streamwise Invariant
LDA	Low Diffusion Advection
TE	Truncation Error
1D	One-Dimensional
2D	Two-Dimensional
3D	Three-Dimensional
C-N	Classic N
C-LDA	Classic LDA
C-PSI	Classic PSI
ES-P	Entropy Stable Positive
L-N	Linearized N
L-LDA	Linearized LDA

L-PSI	Linearized PSI
ES-N	Entropy Stable N
ES-LDA	Entropy Stable LDA
ES-PSI	Entropy Stable PSI
ES-N-S	Entropy Stable Simplified N
ES-LDA-S	Entropy Stable Simplified LDA
ES-PSI-S	Entropy Stable Simplified PSI
ES-LOW	Alternative Entropy Stable Low Order
ES-HIGH	Alternative Entropy Stable Low Order
ES-LIM	Alternative Entropy Stable Limited

LIST OF SYMBOLS

$\vec{n}_i$	Scaled normal i^{th}-edge vector
A_i	Dual point area
l_n	Normal distance of a point from characteristic line
u	Scalar conservative variables
$\vec{f}$	Scalar flux vector
(f, g)	Scalar fluxes
U	Entropy function
v	Scalar entropy variable
S	Physical entropy
$\vec{F}$	Entropy flux vector
(F, G)	Entropy fluxes
$\dot{U}$	Entropy generation rate
k_i	Scalar scaled jacobian with length for edge i
$k_i^{\pm}$	Scalar scaled positive or negative jacobian with length for edge i
ϕ_T	Scalar total residual
$\vec{\lambda}$	Scalar characteristic vector
Ω_i^e	Signal portion going to point i from element e

$\mathbf{u}$	Euler conservative variables
$\mathbf{v}$	Euler entropy variables
$\vec{\mathbf{f}}$	Euler flux vector
$(\mathbf{f}, \mathbf{g})$	Euler fluxes
K_i	Euler scaled jacobian with length for edge i
$K_i^{\pm}$	Euler scaled positive or negative jacobian with length for edge i
R	Right eigenvector matrix
Λ	Diagonal eigenvalue matrix
L	Left eigenvector matrix
S	Diagonal scaled matrix
$\boldsymbol{\phi}_T$	Euler total residual

KAEDAH-KAEDAH ENTROPI-STABIL PENGEDARAN SISA UNTUK SISTEM PERSAMAAN EULER

ABSTRAK

Kaedah pengedaran-sisa (RD) mempunyai pelbagai manfaat asas berbanding dengan kaedah isipadu terhingga (FV) atau kaedah perbezaan terhingga (FD) secara khususnya daripada segi permodelan fizik pelbagai dimensi, mencapai ketepatan yang tinggi menggunakan stensil yang lebih kecil dan kurang sensitif terhadap perubahan grid. Penyelidikan ini akan membangunkan kaedah RD pelbagai-dimensi yang mempunyai sifat entropi-stabil untuk menyelesaikan sistem persamaan hiperbolik. Pertama, suatu kaedah RD alternatif dicadangkan yang memenuhi pemuliharaan pembolehubah utama secara semulajadi. Kemudian, suatu kaedah RD pelbagai-dimensi RD yang memenuhi entropi-dipulihara dan entropi-stabil dibangunkan bermula dengan persamaan Burgers dua dimensi. Ini diikuti dengan pembangunan kaedah yang sama untuk persamaan Euler dua dimensi. Analisis terperinci akan dijalankan ke atas kaedah tersebut daripada segi entropi-stabil, keadaan positif pelbagai-dimensi dan kajian ralat pemangkasan untuk menentukan ketepatan. Tambahan pula, kaedah baru ini akan dibuktikan sebagai memenuhi syarat pemuliharaan secara automatik berbanding dengan kaedah RD yang sedia ada yang memerlukan syarat purata ciri-ciri tertentu dalam setiap elemen dan berbeza mengikut persamaan yang diselesaikan. Pembangunan kaedah entropi-stabil RD yang terhad juga dilaksanakan dalam kajian ini. Eksperimen-eksperimen berangka yang dijalankan untuk persamaan Burgers merangkumi aliran pengembangan dan aliran kejut diikuti dengan masalah-masalah dinamik gas secara subsonik, transonik dan

supersonik. Malahan, kaedah-kaedah klasik RD seperti N, LDA dan PSI juga digunakan dalam kajian sebagai perbandingan kepada kaedah entropi-stabil RD yang baru ditemui. Keputusan eksperimen menunjukkan bahawa kaedah RD yang baru ini adalah keseluruhannya sama baik dengan kaedah-kaedah klasik RD tetapi adalah lebih teguh untuk pelbagai kes ujian.

ENTROPY-STABLE RESIDUAL DISTRIBUTION METHODS FOR SYSTEM OF EULER EQUATIONS

ABSTRACT

Residual-distribution (RD) methods have fundamental benefits over finite volume (FV) or finite difference (FD) methods particularly in mimicking multi-dimensional physics, achieving higher order accuracy with much smaller stencils and less sensitivity to grid changes. The aim of this study is to develop a multi-dimensional entropy-stable residual distribution method to solve the hyperbolic system of equations. First, an alternative residual-distribution method is proposed to ensure conservation of primary variables is obtained by default. This is followed by introducing a new signal distribution and multi-dimensional entropy-conserved and entropy-stable RD method starting with the two-dimensional Burgers' equation. The development is extended to the two-dimensional Euler equations. There will be rigorous mathematical analyses on entropy-stability, multi-dimensional positivity, and truncation error study to determine the formal order-of-accuracy for the entropy stable methods. In addition, it will also be shown that conservation is automatic with the new RD method unlike with the current RD methods where conservation requires a strict set of characteristic-averaging within the elements and different systems of equations would require a different type of averaging. The developments of limited entropy-stable RD methods would also be included herein. Numerical experiments for the Burgers' equation include an expansion and a shock-tree problem followed by subsonic, transonic and supersonic gas dynamics problem over various geometries for the Euler equations. Moreover, the