UNIVERSITI SAINS MALAYSIA

1st. Semester Examination
2006/2007 Academic Session

October / November 2006

EAS 661/4 – Advanced Structural Mechanics

Duration: 3 hours

Instructions to Candidates:

1. Ensure that this paper contains **SIX (6)** printed pages before you start your examination.
2. This paper contains **FIVE (5)** questions. Answer **ALL FIVE (5)** questions.
3. Each question carries equal mark.
4. All questions **CAN BE** answered either in English or Bahasa Malaysia.
5. Each question **MUST BE** answered on a new sheet.
6. Write the answered question numbers on the cover sheet of the answer script.

1. (a) Explain the meaning of a homogeneous isotropic body. Figure 1 shows an infinitesimal volume in a three dimensional body under stressed condition. Derive the constitutive equation $\boldsymbol{\varepsilon} = \boldsymbol{D\sigma}$ for the case of a homogeneous isotropic elastic body where :

$$\boldsymbol{\sigma} = [\ \sigma_x\ \sigma_y\ \sigma_z\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}\ ]^T$$

and

$$\boldsymbol{\varepsilon} = [\ \varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx}\ ]^T$$

are vectors representing the Cartesian components of stress and the corresponding strain, respectively; and $\boldsymbol{D}$ is the elasticity matrix. Indicate clearly the meaning of all symbols/notations used (other than those already given in the question) in your derivation.

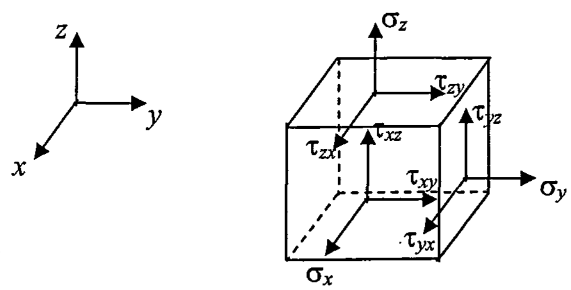

Figure 1

(6 marks)

(b) The sets of equilibrium equations and strain-displacement equations for an infinitesimal volume in a three dimensional body as shown in Figure 1 are given as follows, respectively:

Equilibrium equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + R_x = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x} + R_y = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + R_z = 0$$

Where R_x, R_y and R_z are body forces per unit volume in x, y and z-directions, respectively;

Strain-displacement equations:

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad , \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad , \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad , \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad , \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

where u, v and w are components of displacement in x, y and z-directions, respectively, of a point within the three dimensional body.

Using the above sets of equations together with the general constitutive equations for a homogeneous isotropic body derived in (a) above, specialize them to the case of a plane stress problem. Justify clearly all assumptions made in the process of specialization.

(10 marks)

(b) Derive the governing differential equation for the 1D bar problem shown in Figure 2. Make use of the sets of equations in 1(b) together with the general constitutive equations for a homogeneous isotropic body derived in 1(a).

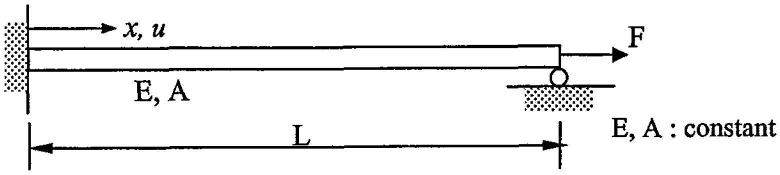

Figure 2

(4 marks)

2. (a) Prove that the statement of Principle of Virtual Displacement (PvD) when specialized to the case of conservative problem can be expressed as follows:

$\delta W_e = \delta U_p$

where δW_e : variation in external work and δU_p : variation in strain energy. Explain clearly the meaning of all symbols/notations used. Next, derive the equilibrium equation in terms of f, k and Δ (where f is load applied on the spring, k is elastic constant for the spring and Δ is elongation of the spring) for the linearly elastic spring shown in Figure 3 by using PvD.

Figure 3

(8 marks)

2. (b) Figure 4 shows a linearly elastic stepped beam with fixed end conditions subjected to a uniformly distributed load, w. Flexural rigidities of the beam are 3EI, 2EI and EI for portions AB, BC and CD, respectively. A spring with elastic constant k is attached to the beam at the mid-span. Show the procedures used in the analysis of the beam by the use of piecewise Rayleigh-Ritz method together with principle of minimum potential energy. Use quartic polynomial for the displacement field in your explanation of the solution procedures. You are required to show clearly all equations and principle involved in the solution process. Your explanation must indicate clearly how the solution will be obtained. Detailed solution for the displacement is not required.

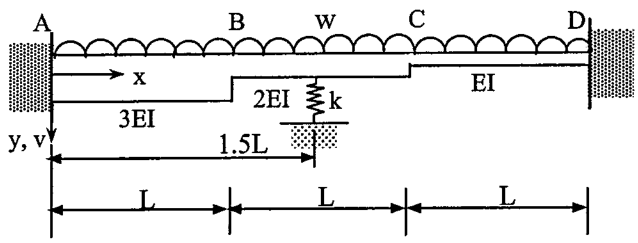

Figure 4

(12 marks)

3. (a) Briefly define the differences between a triangular and rectangular finite element in plane elasticity.

(5 marks)

(b) Show clearly in a step by step manner the development process of a stiffness matrix, $[K]^e$, for a triangular element in a state of plane stress as shown in Figure 5. Given $E = 200$ GN/m^2, $\nu = 0.3$ and $t = 1$ cm. Details of $[K]^e$ is not required.

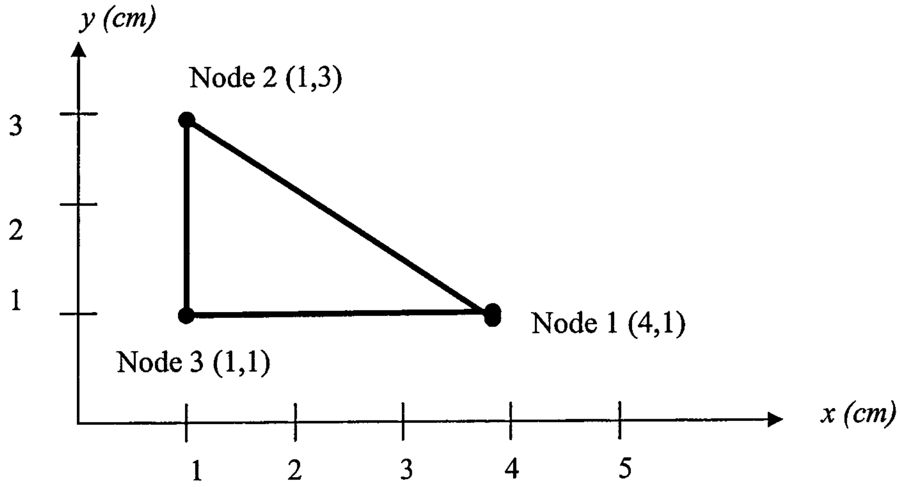

Figure 5

(15 marks)

5. (a) Explain the importance of boundary conditions in the modeling procedures for Finite Element Method.

(5 marks)

(b) A plate with a hole is subjected to tension, p = 25.0 N/mm^2 as shown in Figure 7. It is a 2D plane stress problem. The plate is modeled with linear four-noded rectangular elements and three-noded triangular element. Given the Elastic Modulus E = 7.0 x 10^4 N/mm^2, thickness, t = 1 mm and Poisson ratio, ν = 0.25.

i) Explain and perform simplification through symmetry in modeling the plate using Finite Element Analysis.
ii) State the boundary conditions that apply to the model in i).
iii) Sketch a suitable meshing for the model with rectangular and triangular element. Justify the choices of the elements.
iv) Sketch the stress in z direction near the hole.
v) Indicate point of maximum tension and maximum deflection.

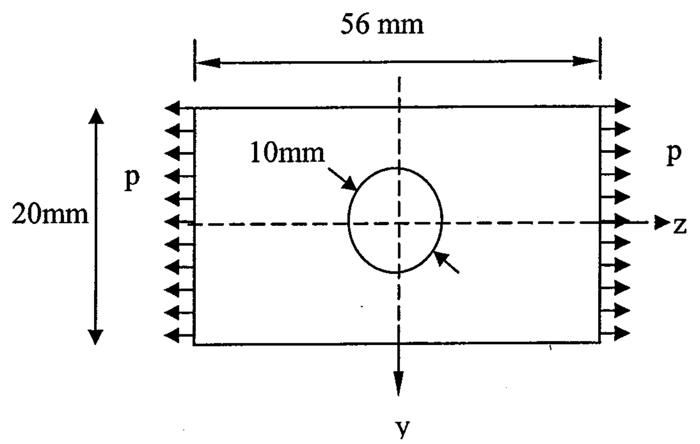

Figure 7

(10 marks)

(c) What will happen to the value of maximum stress and maximum displacement in z direction if the problem in (b) is assumed as plane strain instead of plane stress problem?

(5 marks)

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