UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2016/2017 Academic Session

June 2017

MSG 384 - Introduction to Geometric Modelling
[Pengenalan kepada Pemodelan Geometri]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

<u>Question 1</u>

(a) Find a quadratic polynomial $y(x)$ that interpolates the points $(-1, 1)$ and $(3, 2)$, and has maximum value at $x = 2$.

(b) Given two polynomials

$$\boldsymbol{P}(t) = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} t - \begin{pmatrix} 1 \\ 1 \end{pmatrix} t^2,$$

$$\boldsymbol{Q}(t) = \begin{pmatrix} 1 \\ a \end{pmatrix} (1-t)^2 + \begin{pmatrix} b \\ 1 \end{pmatrix} t(1-t) + \begin{pmatrix} c \\ 1 \end{pmatrix} t^2$$

where $t \in [0, 1]$. Suppose they join at a point with curvature continuity G^2, determine the values a, b and c.

[100 marks]

<u>Soalan 1</u>

(a) *Cari suatu polinomial kuadratik* $y(x)$ *yang menginterpolasi titik* $(-1, 1)$ *dan* $(3, 2)$, *dan mempunyai nilai maksimum pada* $x = 2$.

(b) *Diberi dua polinomial*

$$\boldsymbol{P}(t) = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} t - \begin{pmatrix} 1 \\ 1 \end{pmatrix} t^2,$$

$$\boldsymbol{Q}(t) = \begin{pmatrix} 1 \\ a \end{pmatrix} (1-t)^2 + \begin{pmatrix} b \\ 1 \end{pmatrix} t(1-t) + \begin{pmatrix} c \\ 1 \end{pmatrix} t^2$$

di mana $t \in [0, 1]$. *Andaikan mereka bergabung pada satu titik dengan keselanjaran kelengkungan* G^2, *tentukan nilai-nilai* a, b *dan* c.

[100 markah]

Question 2

Let the Bernstein polynomials of degree n be defined by

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i\,(1-t)^{n-i}, \qquad 0 \le t \le 1, \qquad \text{for } \; i = 0, 1, \ldots, n.$$

(a) Prove that the cubic Bézier curve always lies within the convex hull of its control polygon.

(b) Find a cubic Bézier curve that interpolates the points $(2, 0)$ and $(0, 2)$ with the respective tangent vectors $(1, 0)$ and $(0, 1)$. Sketch the curve.

[100 marks]

Soalan 2

Katakan polinomial Bernstein berdarjah n ditakrif sebagai

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i\,(1-t)^{n-i}, \qquad 0 \le t \le 1, \qquad \textit{bagi } \; i = 0, 1, \ldots, n.$$

(a) Buktikan bahawa lengkung Bézier kubik sentiasa berada di dalam hul cembung poligon kawalan.

(b) Cari suatu Bézier kubik yang menginterpolasi titik $(2, 0)$ dan $(0, 2)$ dengan vektor tangen $(1, 0)$ dan $(0, 1)$ masing-masing. Lakarkan lengkung berkenaan.

[100 markah]

Question 3

Let $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ be a non-decreasing knot vector where n and k are positive integers with $n \ge k-1$. The normalized B-spline basis functions of order k are defined recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \text{for} \quad k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

where $i = 0, 1, \ldots, n$.

(a) Suppose $\boldsymbol{u} = (0\,,\ 1\,,\ 2\,,\ 3\,,\ 4\,,\ 5)$, show that $N_1^3(u)$ is C^1 continuous on the interval $(0,\ 5)$.

(b) Suppose $\boldsymbol{u} = (-2,\ -1,\ 0\,,\ 1,\ 2\,,\ 3)$, show that $\sum_{i=0}^{2} N_i^3(u) = 1$, for $u \in [0,\ 1]$.

(c) Given $\boldsymbol{u} = (0\,,\ 1,\ 2\,,\ 3\,,\ 4\,,\ 5\,,\ 6)$, a B-spline curve $\boldsymbol{P}(u)$ of order 3 is defined with the ordered control points $(1,\ 1)$, $(h,\ h)$, $(1,\ h)$ and $(h,\ 1)$, where $h > 1$. Determine the values u and h such that the curve passes through the highest point $(2,\ 3)$. Sketch the curve and its control polygon.

[100 marks]

<u>Soalan 3</u>

Katakan $\mathbf{u} = (u_0,\ u_1,\ \ldots,\ u_{n+k})$ *ialah suatu vektor simpulan tak menyusut di mana* n *dan* k *adalah integer positif dengan* $n \geq k-1$. *Fungsi asas splin-B ternormal berperingkat* k *ditakrif secara rekursi oleh*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u)\,, \qquad \textit{bagi} \quad k > 1,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \textit{sebaliknya} \end{cases}$$

di mana $i = 0\,,\ 1,\ \ldots\,,\ n$.

(a) *Andaikan* $\mathbf{u} = (0\ ,\ 1\ ,\ 2\ ,\ 3\ ,\ 4\ ,\ 5)$, *tunjukkan bahawa* $N_1^3(u)$ *adalah berkeselanjaran* C^1 *pada selang* $(0,\ 5)$.

(b) *Andaikan* $\mathbf{u} = (-2\ ,\ -1\ ,\ 0\ ,\ 1\ ,\ 2\ ,\ 3)$, *tunjukkan bahawa* $\sum_{i=0}^{2} N_i^3(u) = 1$, *bagi* $u \in [0,\ 1]$.

(c) *Diberi* $\mathbf{u} = (0\,,\ 1,\ 2\,,\ 3\,,\ 4\,,\ 5\,,\ 6)$, *satu lengkung splin-B* $\boldsymbol{P}(u)$ *berperingkat* 3 *ditakrif dengan titik kawalan teratur* $(1,\ 1)$, $(h,\ h)$, $(1,\ h)$ *dan* $(h,\ 1)$, *di mana* $h > 1$. *Tentukan nilai-nilai* u *dan* h *supaya lengkung itu melalui titik tertinggi* $(2,\ 3)$. *Lakarkan lengkung itu dan poligon kawalannya.*

[100 markah]

Question 4

(a) Consider a biquadratic Bézier surface

$$S(x, y) = \sum_{i=0}^{2}\sum_{j=0}^{2} c_{i,j}\, B_j^2(y) B_i^2(x), \qquad 0 \le x \le 1, \quad 0 \le y \le 1,$$

where $B_s^2(t)$, $0 \le t \le 1$, $s = 0, 1, 2$, indicate the Bernstein polynomials of degree 2 and $c_{i,j}$ are the control ordinates

$$c_{0,0} = 1, \qquad c_{0,1} = 0, \qquad c_{0,2} = 1,$$
$$c_{1,0} = 2, \qquad c_{1,1} = 3, \qquad c_{1,2} = 2,$$
$$c_{2,0} = 1, \qquad c_{2,1} = 0, \qquad c_{2,2} = 1.$$

Suppose the surface is degree elevated to a bicubic surface, find the control ordinates of the cubic curve defined along the line $y = 0.5$.

(b) A bilinearly blended Coons patch $S(u, v)$, $0 \le u \le 1$, $0 \le v \le 1$, is considered with $S(0, 0) = 1$, $S(1, 0) = 2$, $S(0, 1) = 0$, $S(1, 1) = 2$, and the gradients at these vertices are $\frac{\partial S}{\partial u} = \frac{\partial S}{\partial v} = 0$. Formulate all the boundary curves with Hermite cubics, and then calculate the S at $(u, v) = (0.5, 0.5)$.

[100 marks]

Soalan 4

(a) Pertimbangkan satu permukaan Bézier dwikuadratik

$$S(x, y) = \sum_{i=0}^{2}\sum_{j=0}^{2} c_{i,j}\, B_j^2(y) B_i^2(x), \qquad 0 \le x \le 1, \quad 0 \le y \le 1,$$

di mana $B_s^2(t)$, $0 \le t \le 1$, $s = 0, 1, 2$, *menandakan polinomial Bernstein berdarjah* 2 *dan* $c_{i,j}$ *ialah ordinat kawalan*

$$c_{0,0} = 1, \qquad c_{0,1} = 0, \qquad c_{0,2} = 1,$$
$$c_{1,0} = 2, \qquad c_{1,1} = 3, \qquad c_{1,2} = 2,$$
$$c_{2,0} = 1, \qquad c_{2,1} = 0, \qquad c_{2,2} = 1.$$

Andaikan permukaan ini dinaik darjah kepada permukaan dwikubik, cari ordinat kawalan bagi lengkung kubik yang ditakrif sepanjang garis $y = 0.5$.

(b) *Satu tampalan Coons teraduan dwilinear* $S(u, v)$, $0 \le u \le 1$, $0 \le v \le 1$, *dipertimbangkan dengan* $S(0, 0) = 1$, $S(1, 0) = 2$, $S(0, 1) = 0$, $S(1, 1) = 2$, *dan kecerunan pada bucu-bucu ini ialah* $\frac{\partial S}{\partial u} = \frac{\partial S}{\partial v} = 0$. *Ungkapkan semua lengkung sempadan dengan kubik Hermite, dan kemudian kira* S *pada* $(u, v) = (0.5, 0.5)$.

[100 markah]

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