



UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2016/2017 Academic Session

June 2017

MSG 265 – DESIGN AND ANALYSIS OF EXPERIMENTS
[REKABENTUK DAN ANALISIS UJIKAJI]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **FOUR** (4) questions.

Arahan: Jawab **semua empat** (4) soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Question 1

Consider the three-factor model

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{ijk} \quad \begin{cases} i = 1, 2, K, a \\ j = 1, 2, K, b \\ k = 1, 2, K, c \end{cases}$$

- (a) Assuming all the factors are fixed, write down the analysis of variance table, including the expected mean squares. What would you use as the “experimental error” to test hypotheses?
- (b) Assuming that all the factors are random. Develop the analysis of variance table, including the expected mean squares. Propose appropriate test statistics for all effects.
- (c) An engineer studied methods for improving the ability to detect targets on a radar scope. Two factors she considered to be important were the amount of background noise or “ground clutter” on the scope and the type of filter placed over the screen. An experiment is designed using three levels of ground clutter and two levels of filter types. If the experimenter decided to have two randomization restrictions in this study:
- (i) State the outline of the design used in this experiment.
- (ii) Write the linear statistical model for this design. Define all the parameters in your model.

[100 marks]

Soalan 1

Pertimbangkan model tiga faktor

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{ijk} \quad \begin{cases} i = 1, 2, K, a \\ j = 1, 2, K, b \\ k = 1, 2, K, c \end{cases}$$

- (a) *Andaikan bahawa semua faktor adalah tetap, tuliskan jadual analisis varians, termasuk jangkaan min kuasa dua. Apakah yang akan anda gunakan sebagai “ralat eksperimen” bagi ujian hipotesis?*
- (b) *Andaikan bahawa semua faktor adalah rawak. Bangunkan jadual analisis varians, termasuk jangkaan min kuasa dua. Cadangkan ujian statistik yang bersesuaian bagi semua kesan.*

- (c) Seorang jurutera sedang mengkaji kaedah bagi meningkatkan keupayaan mengesan pada skop radar. Dua faktor dianggap penting dipertimbangkan iaitu bilangan hingar atau "ground clutter" pada skop dan jenis penapis yang diletakkan pada skrin. Reka bentuk ujikaji ini menggunakan tiga tahap "ground clutter" dan dua jenis penapis. Jika pengujikaji memutuskan terdapat dua kekangan perawakan dalam kajian ini:
- (i) Nyatakan kerangka reka bentuk yang digunakan dalam ujikaji ini.
- (ii) Tulis model statistik linear bagi ujikaji ini. Takrifkan semua parameter dalam model anda.

[100 markah]

Question 2

Consider the data below:

Standard Order	Run Order	A	B	C	D	E=ABC	F=BCD	Pitch
1	5	-	-	-	-	-	-	74
2	7	+	-	-	-	+	-	190
3	8	-	+	-	-	+	+	133
4	2	+	+	-	-	-	+	127
5	10	-	-	+	-	+	+	115
6	12	+	-	+	-	-	+	101
7	16	-	+	+	-	-	-	54
8	1	+	+	+	-	+	-	144
9	6	-	-	-	+	-	+	119
10	9	+	-	-	+	+	+	188
11	14	-	+	-	+	+	-	135
12	13	+	+	-	+	-	-	170
13	11	-	-	+	+	+	-	126
14	3	+	-	+	+	-	-	175
15	15	-	+	+	+	-	+	126
16	4	+	+	+	+	+	+	193

- (a) What is the name of this design? List all the treatment combinations involved in this experiment.
- (b) Identify the complete defining relation and the alias relationships of the main effects.

(c) Given that the percent contributions of the following factors are:

$$\begin{aligned}
 A &= 41.8777 & AB &= 0.262737 & CD &= 3.45252 \\
 B &= 0.016421 & AC &= 0.102631 & DE &= 7.94778 \\
 C &= 2.77515 & AD &= 0.262737 & ABD &= 0.00410 \\
 D &= 22.4804 & AE &= 0.016421 & ABF &= 0.591157 \\
 E &= 19.5451 & BD &= 0.332526 & & \\
 F &= 0.332526 & & & &
 \end{aligned}$$

Which factors seem to have large effects? Estimate the factors that have large effects. State a tentative model.

(d) Based on (c), perform appropriate statistical tests on the model. Give your conclusion.

[100 marks]

Soalan 2

Pertimbangkan data di bawah:

Susunan Piawai	Susunan Larian	A	B	C	D	E=ABC	F=BCD	Pitch
1	5	-	-	-	-	-	-	74
2	7	+	-	-	-	+	-	190
3	8	-	+	-	-	+	+	133
4	2	+	+	-	-	-	+	127
5	10	-	-	+	-	+	+	115
6	12	+	-	+	-	-	+	101
7	16	-	+	+	-	-	-	54
8	1	+	+	+	-	+	-	144
9	6	-	-	-	+	-	+	119
10	9	+	-	-	+	+	+	188
11	14	-	+	-	+	+	-	135
12	13	+	+	-	+	-	-	170
13	11	-	-	+	+	+	-	126
14	3	+	-	+	+	-	-	175
15	15	-	+	+	+	-	+	126
16	4	+	+	+	+	+	+	193

(a) Apakah nama reka bentuk ini? Senaraikan semua kombinasi rawatan yang terlibat dalam eksperimen ini.

(b) Kenalpasti hubungan pentakrif lengkap dan hubungan alias bagi faktor utama.

(c) *Diberikan peratus sumbangan bagi faktor-faktor berikut adalah:*

$$\begin{array}{lll}
 A = 41.8777 & AB = 0.262737 & CD = 3.45252 \\
 B = 0.016421 & AC = 0.102631 & DE = 7.94778 \\
 C = 2.77515 & AD = 0.262737 & ABD = 0.00410 \\
 D = 22.4804 & AE = 0.016421 & ABF = 0.591157 \\
 E = 19.5451 & BD = 0.332526 & \\
 F = 0.332526 & &
 \end{array}$$

Faktor manakah yang menunjukkan kesan yang besar? Anggarkan kesan faktor tersebut. Nyatakan model tentatif.

(d) *Berdasarkan (c), jalankan ujian statistik yang bersesuaian analisis data terhadap model yang dinyatakan. Berikan kesimpulan anda.*

[100 markah]

Question 3

- (a) (i) List down and explain three basic principles of experimental design.
- (ii) Give two important properties of replication.

[40 marks]

- (b) An agricultural experiment, to test the effectiveness of certain crop stimulants on certain crops is carried out as follows. Three 100 hectare plots of land are selected and randomly labeled as 1, 2 and 3. In plot 1, wheat is grown. In plot 2, rye is grown and in plot 3, canola is grown. Then each plot is divided into four 25 hectare fields and the four possible stimulants are applied, one to a field, in random order. Each replicate of a factorial experiment requires 12 observations, and the experimenter has decided to run four replicates.

However, due to time constraint, the experimenter is only capable of making 12 runs per day. So, the experimenter decides to run one replicate on each of the four days and to consider the days or replicates as blocks.

- (i) What is the name of the design being used here?
- (ii) Write down the linear model which you would use to analyze these data. Define all the parameters involved in this model.
- (iii) Give the general analysis of variance for the data described in this experiment. Your ANOVA table should consist of source of variation, degrees of freedom, expected mean squares and test statistics, F_o .

[60 marks]

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Soalan 3

- (a) (i) *Senaraikan dan huraikan tiga prinsip asas bagi analisis ujikaji.*
- (ii) *Berikan dua ciri penting pereplikaan.*

[40 markah]

- (b) *Suatu eksperimen pertanian bagi menguji keberkesanan perangsang tanaman tertentu ke atas beberapa tanaman dijalankan seperti berikut. Tiga plot tanah 100 hektar dipilih dan dilabelkan 1,2 dan 3 secara rawak. Dalam plot 1, gandum ditanam. Manakala dalam plot 2, 'rye' ditanam dan plot 3, kanola ditanam. Seterusnya, setiap plot dibahagikan kepada empat padang seluas 25 hektar dan empat perangsang diberikan secara rawak di setiap padang. Setiap replika bagi eksperimen faktorial memerlukan 12 cerapan, dan pengujikaji menetapkan untuk menjalankan empat replika. Walau bagaimanapun, disebabkan oleh kekangan masa, pengujikaji hanya boleh menjalankan 12 larian per hari. Maka, pengujikaji memutuskan untuk menjalankan satu replika setiap hari dan menganggap hari atau replika sebagai blok.*

- (i) *Apakah nama rekabentuk yang digunakan?*
- (ii) *Tuliskan model linear yang akan anda gunakan bagi menganalisis data tersebut. Takrifkan semua parameter yang terlibat dalam model ini.*
- (iii) *Berikan jadual analisis varians umum bagi data yang dijalankan dalam eksperimen ini. Jadual ANOVA anda perlu mengandungi sumber variasi, darjah kebebasan, jangkakan min kuasa dua dan ujian statistik, F_0 .*

[60 markah]

Question 4

Consider a 2^4 factorial design to identify the effect of pH, concentration, time and pressure on the yield in an industrial process. The experiment were replicated twice. The data are given below:

Treatment	pH	Concentration	Time	Pressure	Yield (Rep I)	Yield (Rep II)
1	3	10	0.2	0	28.64	31.06
2	3	10	0.07	0	27.93	28.71
3	10	10	0.2	0	8.86	10.844
4	10	10	0.07	0	17.99	19.05
5	3	100	0.2	0	3.86	5.02
6	3	100	0.07	0	24.91	23.67
7	10	100	0.2	0	11.31	10.17
8	10	100	0.07	0	12.32	12.3
9	3	10	0.07	3000	36.69	36.03
10	3	10	0.2	3000	23.96	21.76
11	10	10	0.07	3000	23.38	24
12	10	10	0.2	3000	13.71	12.31
13	3	100	0.2	3000	8.68	10.9
14	3	100	0.07	3000	20.09	22.33
15	10	100	0.07	3000	17.45	17.29
16	10	100	0.2	3000	10.43	10.95

Given that:

$$\begin{aligned}
 \text{contrast}_A &= -121.876 & \text{contrast}_{AB} &= 87.396 \\
 \text{contrast}_B &= -143.244 & \text{contrast}_{AC} &= 31.284 \\
 \text{contrast}_C &= -141.676 & \text{contrast}_{AD} &= 20.036 \\
 \text{contrast}_D &= 33.316 & \text{contrast}_{BC} &= -16.404 \\
 \text{contrast}_{BD} &= -4.196 & \text{contrast}_{CD} &= -27.444 \\
 \text{contrast}_{ABC} &= 60.796 & \text{contrast}_{ABD} &= -9.076 \\
 \text{contrast}_{ACD} &= -1.044 & \text{contrast}_{BCD} &= 40.724 \\
 \text{contrast}_{ABCD} &= -53.116 & &
 \end{aligned}$$

- (a) Analyze the data. Draw your conclusion.
- (b) Due to some restrictions, it is impossible to perform a complete replicate of a factorial design in one block. To overcome this problem, the experimenter decided to run the experiment in two blocks for each replicate.
- (i) Give the outline of the design. What is the name of the design?
- (ii) Analyze the data and draw conclusion.

[100 marks]

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Soalan 4

Pertimbangkan reka bentuk faktor 2^4 bagi mengenalpasti kesan pH, kepekatan, masa dan tekanan ke atas hasil dalam suatu proses industri. Eksperimen diulang sebanyak dua kali. Data bagi eksperimen diberikan di bawah:

Rawatan	pH	Kepekatan	Masa	Tekanan	Hasil (Rep I)	Hasil (Rep II)
1	3	10	0.2	0	28.64	31.06
2	3	10	0.07	0	27.93	28.71
3	10	10	0.2	0	8.86	10.844
4	10	10	0.07	0	17.99	19.05
5	3	100	0.2	0	3.86	5.02
6	3	100	0.07	0	24.91	23.67
7	10	100	0.2	0	11.31	10.17
8	10	100	0.07	0	12.32	12.3
9	3	10	0.07	3000	36.69	36.03
10	3	10	0.2	3000	23.96	21.76
11	10	10	0.07	3000	23.38	24
12	10	10	0.2	3000	13.71	12.31
13	3	100	0.2	3000	8.68	10.9
14	3	100	0.07	3000	20.09	22.33
15	10	100	0.07	3000	17.45	17.29
16	10	100	0.2	3000	10.43	10.95

Diberikan:

$$\begin{aligned}
 \text{kontrast}_A &= -121.876 & \text{kontrast}_{AB} &= 87.396 \\
 \text{kontrast}_B &= -143.244 & \text{kontrast}_{AC} &= 31.284 \\
 \text{kontrast}_C &= -141.676 & \text{kontrast}_{AD} &= 20.036 \\
 \text{kontrast}_D &= 33.316 & \text{kontrast}_{BC} &= -16.404 \\
 \text{kontrast}_{BD} &= -4.196 & \text{kontrast}_{CD} &= -27.444 \\
 \text{kontrast}_{ABC} &= 60.796 & \text{kontrast}_{ABD} &= -9.076 \\
 \text{kontrast}_{ACD} &= -1.044 & \text{kontrast}_{BCD} &= 40.724 \\
 \text{kontrast}_{ABCD} &= -53.116 & &
 \end{aligned}$$

- (a) Analisa data tersebut. Berikan kesimpulan anda.
- (b) Disebabkan beberapa kekangan, adalah mustahil bagi menjalankan reka bentuk ujikaji faktor dengan replika penuh dalam satu blok. Bagi mengatasi masalah tersebut, pengujikaji memutuskan untuk menjalankan ujikaji dalam dua blok bagi setiap replika.
- (i) Beri gambaran bagi reka bentuk ujikaji ini. Apakah nama reka bentuk ini?
- (ii) Analisa data dan berikan kesimpulan.

[100 markah]

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FORMULA**Expected Mean Square: Three-factors Fixed Effects Model (A, B and C Fixed):**

$$E[MS_A] = \sigma^2 + bc \sum_{i=1}^a \frac{\tau_i^2}{a-1}$$

$$E[MS_B] = \sigma^2 + ac \sum_{j=1}^b \frac{\beta_j^2}{b-1}$$

$$E[MS_C] = \sigma^2 + ab \sum_{k=1}^c \frac{\gamma_k^2}{c-1}$$

$$E[MS_{AB}] = \sigma^2 + c \sum_{i=1}^a \sum_{j=1}^b \frac{(\tau\beta)_{ij}^2}{(a-1)(b-1)}$$

$$E[MS_{BC}] = \sigma^2 + a \sum_{j=1}^b \sum_{k=1}^c \frac{(\beta\gamma)_{jk}^2}{(b-1)(c-1)}$$

$$E[MS_E] = \sigma^2$$

Expected Mean Square: Three-factors Random Effects Model (A, B and C Random):

$$E[MS_A] = \sigma^2 + c\sigma_{\tau\beta}^2 + bc\sigma_{\tau}^2$$

$$E[MS_B] = \sigma^2 + c\sigma_{\tau\beta}^2 + a\sigma_{\beta\gamma}^2 + ac\sigma_{\beta}^2$$

$$E[MS_C] = \sigma^2 + a\sigma_{\beta\gamma}^2 + ab\sigma_{\gamma}^2$$

$$E[MS_{AB}] = \sigma^2 + c\sigma_{\tau\beta}^2$$

$$E[MS_{BC}] = \sigma^2 + a\sigma_{\beta\gamma}^2$$

$$E[MS_E] = \sigma^2$$

Expected Mean Square : Split-Plot Design

$$\tau_i : \sigma^2 + ab\sigma_\tau^2$$

$$\beta_j : \sigma^2 + b\sigma_{\tau\beta}^2 + \frac{rb \sum \beta_j^2}{a-1}$$

$$\gamma_k : \sigma^2 + \frac{ra \sum \gamma_k^2}{ab-1}$$

$$(\tau\beta)_{ij} : \sigma^2 + b\sigma_{\tau\beta}^2$$

$$(\beta\gamma)_{jk} : \sigma^2 + \frac{r \sum \sum (\beta\gamma)_{jk}^2}{(a-1)(b-1)}$$

$$\varepsilon_{ijk} : \sigma^2$$