



UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2016/2017 Academic Session

June 2017

**MAT 263 - Probability Theory**  
**[Teori Kebarangkalian]**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all seven** [7] questions.

**Arahan:** Jawab **semua tujuh** [7] soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

**Question 1**

Let  $X$  be a function defined as follows,

$$f_X(x) = \begin{cases} \binom{x+2}{x} \left(\frac{1}{2}\right)^{x+3} & x = 0, 1, 2, \dots \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find  $M_X[\ln(1.2)]$ . [ 2 marks ]
- (b) Find the second moment of this distribution. [ 4 marks ]
- (c) Given the following events,  $C$  and  $D$  are both sets of non-negative integer numbers,  $x$ , such that,  $C = \{x \in Z | x \geq 2\}$  and  $D = \{x \in Z | x < 4\}$ . From Find
- (i)  $P(C - D)$ .
- (ii)  $P(C \cap D)$ .
- (iii)  $P[(C - D) | C]$ . [ 13 marks ]

**Soalan 1**

Biarkan  $X$  sebagai suatu fungsi yang ditakrifkan seperti berikut,

$$f_X(x) = \begin{cases} \binom{x+2}{x} \left(\frac{1}{2}\right)^{x+3} & x = 0, 1, 2, \dots \\ 0 & \text{selainnya.} \end{cases}$$

- (a) Cari  $M_X[\ln(1.2)]$ . [ 2 markah ]
- (b) Cari momen kedua bagi taburan ini. [ 4 markah ]
- (c) Diberi peristiwa berikut,  $C$  dan  $D$  kedua-duanya merupakan set-set integer bukan negatif,  $x$ , iaitu,  $C = \{x \in Z | x \geq 2\}$  dan  $D = \{x \in Z | x < 4\}$ . Cari
- (i)  $Kb(C - D)$ .
- (ii)  $Kb(C \cap D)$ .
- (iii)  $Kb[(C - D) | C]$ . [ 13 markah ]

**Question 2**

Let  $A \cap C = C$ , and  $B \cap C \neq \emptyset$ , where  $A$  and  $B$  are independent. Also given that,  $P(A) = 0.5$ ,  $P(B) = 0.4$ ,  $P(C) = 0.3$  and  $P(B \cup C) = 0.6$ . Find

- (a)  $P(C|A)$ . [ 2 marks ]
- (b)  $P(B - C)$ . [ 3 marks ]
- (c)  $P[(B - C)|A]$ . [ 4 marks ]
- (d)  $P[(A \cap B \cap C)']$ . [ 3 marks ]

**Soalan 2**

Biar  $A \cap C = C$  dan  $B \cap C \neq \emptyset$ , yang mana  $A$  dan  $B$  adalah tidak bersandaran. Juga diberi,  $Kb(A) = 0.5$ ,  $Kb(B) = 0.4$ ,  $Kb(C) = 0.3$  dan  $Kb(B \cup C) = 0.6$ . Cari

- (a)  $Kb(C|A)$ . [ 2 markah ]
- (b)  $Kb(B - C)$ . [ 3 markah ]
- (c)  $Kb[(B - C)|A]$ . [ 4 markah ]
- (d)  $Kb[(A \cap B \cap C)']$ . [ 3 markah ]

**Question 3**

Random variable,  $A$  has the moment generating function as follows,

$$M_A(t) = \frac{1}{12}e^{2t} + \frac{5}{48}e^{2.5t} + \frac{1}{6}e^{4t} + \frac{5}{16}e^{7.5t} + \frac{1}{3}e^{8t}.$$

- (a) By using the equation above, find the mean and variance of  $A$ . [ 8 marks ]
- (b) State the distribution of  $A$ . [ 2 marks ]
- (c) Using Markov's inequality, find the highest value of  $P \left[ A \geq 10\sqrt{\text{Var}(A)} \right]$ . [ 3 marks ]

**Soalan 3**

Pembolehubah rawak,  $A$  mempunyai fungsi penjana momen seperti berikut,

$$M_A(t) = \frac{1}{12}e^{2t} + \frac{5}{48}e^{2.5t} + \frac{1}{6}e^{4t} + \frac{5}{16}e^{7.5t} + \frac{1}{3}e^{8t}.$$

- (a) Dengan menggunakan persamaan di atas, cari min dan varians bagi  $A$ .  
[ 8 markah ]
- (b) Nyatakan taburan bagi pemboleh ubah rawak  $A$ .  
[ 2 markah ]
- (c) Dengan menggunakan ketaksamaan Markov, cari nilai tertinggi bagi  $Kb [A \geq 10\sqrt{\text{Var}(A)}]$ .  
[ 3 markah ]

**Question 4**

Consider the jointly random variables,  $X$  and  $Y$ , where,  $X$  has Binomial distribution with parameters of  $n = 2$  and  $p = 3/4$ , whereas,  $Y|X = x$  is a Binomial distribution with the parameters of  $n = 2 - x$  and  $p = 4/5$ .

- (a) Determine the conditional probability of  $Y$  given  $X = x$ ,  $f_{Y|x}(y|x)$ .  
[ 2 marks ]
- (b) Compute,
- (i) The expectation,  $E(XY)$ ,  
[ 7 marks ]
- (ii) The mean and variance of  $X$  and  $Y$   
[ 11 marks ]
- (iii) The covariance between  $X$  and  $Y$ ,  $\text{Cov}(X, Y)$ .  
[ 2 marks ]
- (c) If  $Z = 3X + Y$ , then, find  $\text{Var}(Z)$ .  
[ 3 marks ]

**Soalan 4**

Pertimbangkan pemboleh ubah rawak tergabung,  $X$  dan  $Y$ , yang mana,  $X$  adalah taburan Binomial dengan parameter  $n = 2$  dan  $p = 3/4$ , manakala,  $Y|X = x$  adalah taburan Binomial dengan parameter  $n = 2 - x$  dan  $p = 4/5$ .

- (a) Tentukan kebarangkalian bersyarat bagi  $Y$  diberi  $X = x$ ,  $f_{Y|x}(y|x)$ . [ 2 markah ]
- (b) Kira
- (i) Jangkaan,  $E(XY)$ , [ 7 markah ]
- (ii) Min dan varians bagi  $X$  dan  $Y$ , [ 11 markah ]
- (iii) Kovarians antara  $X$  dan  $Y$ ,  $Kov(X, Y)$ . [ 2 markah ]
- (c) Jika  $Z = 3X + Y$ , maka, cari  $Var(Z)$ . [ 3 markah ]

**Question 5**

Find  $P[X \geq E(X)]$  where the random variable  $X$  has the moment generating function as follows,

- (a)  $\frac{2}{5t}(e^{5t} - e^{2.5t})$ . [ 6 marks ]
- (b)  $(0.4 + 0.6e^t)^8$ . [ 5 marks ]

**Soalan 5**

Cari  $Kb[X \geq E(X)]$  yang mana pemboleh ubah rawak  $X$  mempunyai fungsi penjana momen seperti berikut,

- (a)  $\frac{2}{5t}(e^{5t} - e^{2.5t})$ . [ 6 markah ]
- (b)  $(0.4 + 0.6e^t)^8$ . [ 5 markah ]

**Question 6**

Let  $X_1, \dots, X_{17}$  be independently normal distributed with mean  $\mu$ , and variance  $\sigma^2$ . Consider the following information,

$$\sum x = 4.33 \text{ and } \sum x^2 = 17.9093$$

- (a) Estimate the mean and standard deviation of the distribution above.  
[ 3 marks ]
- (b) Calculate a 95% confidence interval for the mean of this random variable.  
[ 4 marks ]
- (c) Calculate a 95% confidence interval for the standard deviation of this random variable.  
[ 5 marks ]

**Soalan 6**

Biarkan  $X_1, \dots, X_{17}$  sebagai taburan tak bersandar normal dengan min  $\mu$ , dan varians  $\sigma^2$ . Pertimbangkan maklumat seperti berikut,

$$\sum x = 4.33 \text{ dan } \sum x^2 = 17.9093$$

- (a) Anggarkan min dan sisihan piawai bagi taburan di atas.  
[ 3 markah ]
- (b) Kirakan 95% selang keyakinan min bagi pemboleh ubah rawak ini.  
[ 4 markah ]
- (c) Kirakan 95% selang keyakinan sisihan piawai bagi pemboleh ubah rawak ini.  
[ 5 markah ]

**Soalan 7**

Let  $X$  be a continuously uniform random variable under the domain of 0 and 1, i.e.  $x \in [0,1]$  Let  $Y = -3\ln X$ . Then, determine  $E(Y + Y^2)$ .

[ 8 marks ]

**Soalan 7**

Biarkan  $X$  sebagai taburan selanjar seragam di bawah domain 0 dan 1, iaitu,  $x \in [0,1]$ . Biar  $Y = -3\ln X$ . Maka, tentukan  $E(Y + Y^2)$ .

[ 8 markah ]

...7/-

Appendix

Random Variable, $X$	Probability distribution function, $f_X(x)$	Mean, $E(X)$	Variance, $Var(X)$	Moment Generating Function, $M_X(t)$
$bin(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \dots, n$	$np$	$np(1-p)$	$(pe^t + 1 - p)^n$
$Poisson(\lambda)$	$\frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots$	$\lambda$	$\lambda$	$e^{\lambda(e^t - 1)}$
$NB(r, p)$	$\binom{x+r-1}{x} p^r (1-p)^x, x = 0, 1, 2, \dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[ \frac{p}{1 - (1-p)e^t} \right]^r$
$uniform(a, b)$	$\frac{1}{b-a}, a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$
$N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], -\infty < x < \infty$	$\mu$	$\sigma^2$	$\exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$
$exp(\theta)$	$\frac{1}{\theta} e^{-x/\theta}, x > 0$	$\theta$	$\theta^2$	$(1 - \theta t)^{-1}$
$Gamma(\alpha, \theta)$	$\frac{1}{\Gamma(\alpha)} \frac{x^{\alpha-1}}{\theta^\alpha} \exp(-x/\theta), x > 0$	$\alpha\theta$	$\alpha\theta^2$	$(1 - \theta t)^{-\alpha}$

$(a + b)^n = \sum_{x=0}^n {}^n C_x a^x b^{n-x}$	$g(x) = \sum_{m=0}^{\infty} \frac{g^{(m)}(x_0)(x - x_0)^m}{m!}$	$Var(aX + bY)$ $= a^2 Var(X) + 2ab Cov(X, Y)$ $+ b^2 Var(Y)$
$P(X - Y) = P(X \cap \bar{Y})$		