UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2016/2017 Academic Session

June 2017

MAT 161 - Elementary Statistics
[Statistik Permulaan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of ELEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEBELAS *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all nine** [9] questions.

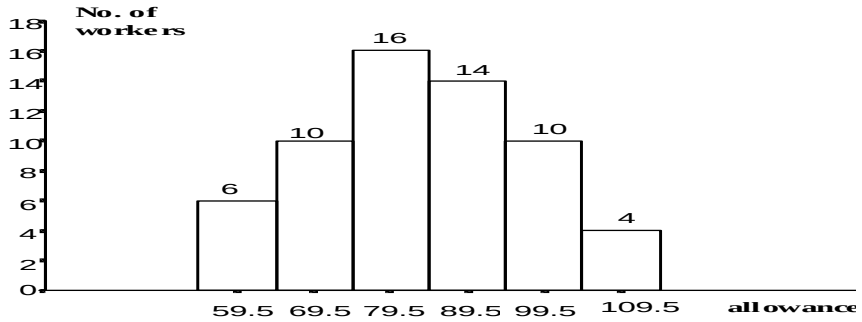
*[**Arahan:** Jawab **semua sembilan** [9] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

The following histogram shows the distribution of the daily allowances (in RM) of a sample of 60 workers of a company. The number on the *x*-axis are midpoints of each class.



(i) Construct a frequency table for the above data.

(ii) Determine the mean daily allowance.

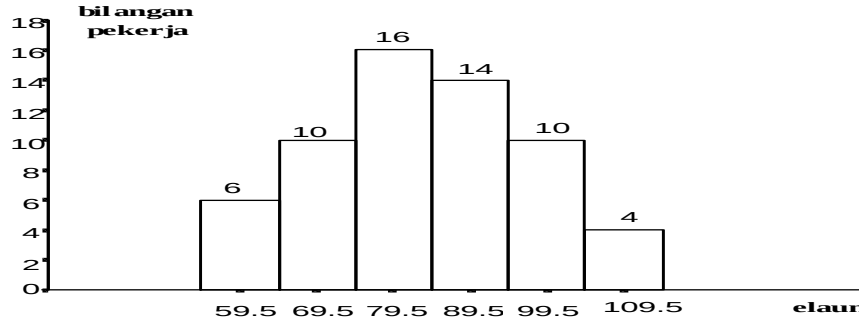
(iii) The variance in the daily allowance is RM196. Use Chebyshev's rule to construct an interval that contains at least 60% of the data.

(iv) Estimate the proportion of company worker who gets an allowance of more than RM95 per day.

[20 marks]

Soalan 1

Histogram yang berikut menunjukkan taburan elaun harian (dalam RM) bagi suatu sampel 60 orang pekerja di sebuah syarikat. Nombor pada paksi-x adalah titik tengah bagi setiap kelas.



(i) Bina suatu jadual kekerapan bagi data di atas.

(ii) Tentukan min elaun harian.

(iii) Varians elaun harian ialah RM196. Guna petua Chebyshev untuk membina suatu selang yang mengandungi sekurang-kurangnya 60% daripada data.

(iv) Anggarkan kadaran pekerja syarikat yang mendapat elaun lebih dari RM95 sehari.

[20 markah]

Question 2

Suppose that events *A, B* and *C* are non-empty and mutually independent. Show that

(i) *A, B* and *C* are not mutually exclusive.

(ii) $P\left(\bar{A}\cap\bar{B}\cap\bar{C}\right)=P\left(\bar{A}\right)P\left(\bar{B}\right)P\left(\bar{C}\right)$.

[15 marks]

Soalan 2

Andaikan peristiwa A, B dan C adalah tidak kosong dan saling tak bersandar. Tunjukkan bahawa

(i) A, B dan C bukan saling eksklusif.

(ii) $P\left(\bar{A}\cap\bar{B}\cap\bar{C}\right)=P\left(\bar{A}\right)P\left(\bar{B}\right)P\left(\bar{C}\right)$.

[15 markah]

Question 3

A housewife makes 6 cakes everyday and sells them at RM30 per cake. The demand for her cakes follows a Poisson distribution with mean 0.8 cake every two hours. The housewife runs her business everyday in a week for 10 hours per day.

(i) Calculate the probability that the demand for her cakes in a day cannot be met.

(ii) Find the probability that in any one week, there is at most one day with no demand for her cakes.

(iii) Write the probability distribution for the daily demand for her cakes.

(iv) Calculate the housewife's expected daily income from sales of her cakes.

[25 marks]

Soalan 3

Seorang surirumah membuat 6 biji kek setiap hari dan menjualnya pada harga RM30 sebiji. Permintaan untuk keknya tertabur secara Poisson dengan min 0.8 kek setiap dua jam. Surirumah tersebut menjalankan perniagaannya setiap hari dalam seminggu selama 10 jam setiap hari.

(i) Hitung kebarangkalian bahawa permintaan untuk keknya dalam satu hari tidak dapat dipenuhi.

(ii) Dapatkan kebarangkalian bahawa dalam sebarang satu minggu, terdapat sebanyak-banyaknya satu hari tanpa permintaan untuk keknya.

(iii) Tuliskan taburan kebarangkalian bagi permintaan harian untuk keknya.

(iv) Hitung pendapatan harian jangkaan bagi surirumah tersebut daripada jualan keknya.

[25 markah]

Question 4

The amount of rice (in kg) eaten by a family in a week is a random variable *W*, with probability density function given as follows

$$f(w)=\begin{cases} kw^3(5-w) & 0\le w\le 5 \\ 0 & otherwise \end{cases}$$

where *k* is a positive constant.

(i) Show that the value of $k=\frac{20}{5^5}$.

(ii) Given that on the average, the family eats 3.33kg of rice per week, find the variance.

(iii) Determine the mode of the distribution.

[20 marks]

Soalan 4

Jumlah beras (dalam kg) yang dimakan oleh sebuah keluarga dalam satu minggu ialah suatu pemboleh ubah rawak W, dengan fungsi ketumpatan kebarangkalian seperti diberikan berikut

$$f(w)=\begin{cases} kw^3(5-w) & 0\le w\le 5 \\ 0 & sebaliknya \end{cases}$$

dengan k pemalar positif.

(i) Tunjukkan bahawa nilai $k=\frac{20}{5^5}$

(ii) Diberikan bahawa pada puratanya, keluarga tersebut makan 3.33kg beras dalam satu minggu, dapatkan variansnya.

(iii) Tentukan mod taburan.

[20 markah]

Question 5

An experiment is proposed to determine whether a coin is biased. The coin will be tossed 10 times and it will be considered biased if at least 8 heads or at least 8 tails are obtained.

(i) State a suitable null and alternative hypotheses to be tested for the problem.

(ii) State the critical region and hence, determine the significance level of the test.

(iii) Determine the probability of making a type II error when the probability of obtaining a head on each toss is actually 0.6.

[20 marks]

Soalan 5

Suatu ujikaji dicadangkan untuk menentukan sama ada sekeping syiling adalah berat sebelah. Syiling tersebut dilambung 10 kali dan dianggap berat sebelah jika sekurang-kurangnya 8 kepala atau sekurang-kurangnya 8 bunga diperolehi.

(i) Nyatakan hipotesis nol dan hipotesis alternatif yang sesuai untuk diuji bagi masalah di atas.

(ii) Nyatakan kawasan kritikal dan seterusnya, tentukan aras keertian ujian tersebut.

(iii) Tentukan kebarangkalian melakukan ralat jenis II apabila kebarangkalian memperoleh kepala pada setiap lambungan adalah, sebenarnya, 0.6.

[20 markah]

Question 6

A supermarket requires that all eggs from its suppliers meet its specifications of a mean diameter of 4.0cm and a standard deviation of no more than 0.2cm. A random sample of 50 eggs from farm *A*, a potential egg supplier, shows a mean diameter of 3.95cm with a standard deviation of 0.24cm.

(i) Test whether the sample sufficiently meets the specification with regard to the variance of the diameter. Use a significance level of 0.05.

(ii) Of the 50 eggs sampled from supplier *A*, 3 are found to be out of specification. From another potential egg supplier, *B*, 80 eggs are sampled and 6 are out of specification. Conduct a test to determine whether the proportion of out-of specification eggs from farm *A* is smaller than that from farm *B*. Use the 0.05 significance level.

(iii) Eggs that arrived at the supermarket from approved suppliers are further inspected randomly. Determine the number of eggs to be sampled if the supermarket would like to be 95% confident that the sample proportion of eggs that do not meet the store's specifications is within 0.01 of the true proportion.

[25 marks]

Soalan 6

Sebuah pasaraya memerlukan supaya semua telur daripada pembekal memenuhi spesifikasi min diameter 4.0cm dan sisihan piawai tidak melebihi 0.2cm. Suatu sampel rawak 50 biji telur daripada ladang A, seorang bakal pembekal, menunjukkan min diameter 3.95cm dengan sisihan piawai 0.24cm.

(i) Uji sama ada sampel tersebut memenuhi spesifikasi dengan secukupnya dari segi varians diameter. Guna aras keertian 0.05.

(ii) Daripada 50 biji telur yang disampelkan daripada pembekal A, 3 biji didapati tidak memenuhi spesifikasi. Daripada 80 biji telur yang disampelkan daripada seorang lagi bakal pembekal B, 6 biji didapati tidak memenuhi spesifikasi. Jalankan suatu ujian bagi menentukan sama ada kadaran telur yang di luar spesifikasi daripada ladang A lebih kecil daripada kadaran kilang B. Guna aras keertian 0.05.

(iii) Telur-telur yang tiba daripada pembekal yang diluluskan diperiksa lagi secara rawak. Tentukan bilangan telur yang perlu disampelkan jika pasaraya tersebut ingin 95% yakin bahawa kadaran telur yang tidak memenuhi spesifikasi dalam sampel adalah dalam sekitar 0.01 daripada kadaran sebenar.

[25 marks]

Question 7

In order to study the harmful effects of DDT poisoning, the pesticide was fed to 13 randomly chosen rats out of a group of 26 rats. The other 13 rats were used as the control group. The following table summarizes the measurements of the amount of tremor detected in the bodies of each rat after the experiment: The more tremor, the more harmful.

	mean	variance
Poisoned Group (X)	17.6	39.9
Control Group (Y)	12.8	36.3

(i) If the population variances are known to be σ_X^2 = 37.5 and σ_Y^2 = 35.2, respectively, construct a 95% confidence interval for the difference between the mean amount of tremor detected in the bodies of poisoned rat and the controlled group. Comment briefly on this confidence interval.

(ii) Suppose now that the population variances σ_X^2 and σ_Y^2 are unknown. Test whether the population variance in the amount of tremor detected in the bodies of poisoned rat and the population variance in the controlled group are significantly different at the 5% level.

(iii) Using the result from part (ii), perform a test to investigate whether there is a difference between the mean amount of tremor detected in the bodies of poisoned rat and the mean amount of tremor in the controlled group. Use the 0.05 significance level.

(iv) What assumptions are made about the distribution of tremor for the two populations?

[30 marks]

Soalan 7

Bagi mengkaji kesan bahaya keracunan DDT, racun tersebut diberikan kepada 13 ekor tikus yang dipilih secara rawak daripada suatu kumpulan 26 ekor tikus. 13 ekor tikus yang lain digunakan sebagai kumpulan kawalan. Jadual berikutnya meringkaskan ukuran amaun gegaran yang dikesan pada badan setiap tikus selepas ujikaji tersebut. Lebih banyak gegaran bermakna lebih merbahaya.

	min	*varians*
Kumpulan Racun	*17.6*	*39.9*
Kumpulan Kawalan	*12.8*	*36.3*

(i) *Jika varians populasi masing-masing diketahui* σ_X^2 *= 37.5 dan* σ_Y^2 *= 35.2, bina suatu selang keyakinan 95% bagi perbezaan di antara min amaun gegaran yang dikesan pada tikus yang diracun dan kumpulan kawalan. Komen secara ringkas mengenai selang keyakinan yang dibina.*

(ii) *Andaikan sekarang bahawa varians populasi* σ_X^2 *dan* σ_Y^2 *tidak diketahui. Uji sama ada varians populasi amaun gegaran yang dikesan pada tikus yang diracun dan varians populasi kumpulan kawalan berbeza secara bererti pada aras keertian 0.05.*

(iii) *Dengan menggunakan keputusan dalam bahagian (ii), jalankan suatu uijan untuk menyiasat sama ada terdapat perbezaan di antara min amaun gegaran yang dikesan pada tikus yang diracun dan min amaun gegaran bagi kumpulan kawalan. Guna aras keertian 0.05.*

(iv) *Apakah andaian yang dibuat mengenai taburan gegaran bagi kedua-dua populasi?*

[30 marks]

Question 8

Twenty overweight executives take part in an experiment to compare the effectiveness of two exercise methods, A (isometric), and B (isotonic). They are allocated at random to the two methods, ten to isometric, ten to isotonic methods. After several weeks, the reductions in abdomen measurements (in centimeters) are recorded as follows:

A (isometric)	3.1	2.1	3.3	2.7	3.4	2.7	2.7	3.0	3.0	1.6
B (isotonic)	4.5	4.1	2.7	2.2	4.7	2.2	3.6	3.0	3.3	3.4

(i) Name a parametric test to determine if the isotonic method is more effective than the isometric method, assuming that the reductions in abdomen measurements are normally distributed? Why do you choose this test?

(ii) What nonparametric test can be used if we are unsure about the distribution of the samples?

(iii) Perform the nonparametric test named in part (ii) at the 0.05 level of significance.

[20 marks]

Soalan 8

Dua puluh orang eksekutif dengan berat badan berlebihan mengambil bahagian dalam suatu uji kaji untuk membandingkan keberkesanan dua kaedah senaman, A (isometrik) dan B (isotonik). Mereka diperuntukkan secara rawak kepada kedua-dua kaedah, 10 kepada kaedah isometrik dan 10 kepada kaedah isotonik. Selepas beberapa minggu, penurunan dalam ukuran abdomen (dalam sentimeter) direkodkan seperti berikut:

A (isometrik)	*3.1*	*2.1*	*3.3*	*2.7*	*3.4*	*2.7*	*2.7*	*3.0*	*3.0*	*1.6*
B (isotonik)	*4.5*	*4.1*	*2.7*	*2.2*	*4.7*	*2.2*	*3.6*	*3.0*	*3.3*	*3.4*

(i) Namakan suatu ujian berparameter untuk menentukan sama ada kaedah isotonik lebih berkesan daripada kaedah isometrik, dengan andaian bahawa penurunan dalam ukuran abdomen tertabur secara normal. Kenapa anda pilih ujian ini?

(ii) Apakah ujian tak berparameter yang dapat digunakan jika anda tidak pasti mengenai taburan bagi sampel?

(iii) Jalankan ujian yang dinamakan dalam bahagian (ii) pada aras keertian 0.05.

[20 markah]

Question 9

A man believes that his lucky coin is a fair one. To test his belief, he tosses the coin 60 times and counts the number of tails between successive heads. If the coin is a fair one, then P(r tails) = $\left(\frac{1}{2}\right)^{r+1}$ for $r = 1, 2, 3, \ldots$ The results from his tosses are as follows:

Number of tails, r	0	1	2	3	4	≥ 5
Frequency, f	25	18	9	6	2	0

Determine whether there is significant evidence that the coin is a fair one. Test at the 1% significance level.

[25 marks]

Soalan 9

Seorang lelaki percaya bahawa syiling bertuahnya adalah syiling adil. Untuk menguji kepercayaannya, dia melambung syiling tersebut 60 kali dan mengira bilangan bunga yang muncul antara dua kepala yang berturutan. Jika syiling tersebut syiling adil, maka $P(r \text{ bunga}) = \left(\frac{1}{2}\right)^{r+1}$ *bag* $r = 1, 2, 3, \ldots$ *Hasil daripada lambungannya adalah seperti berikut:*

Bilangan bunga, r	*0*	*1*	*2*	*3*	*4*	*≥5*
Kekerapan, f	*25*	*18*	*9*	*6*	*2*	*0*

Tentukan sama ada terdapat bukti yang bererti bahawa syiling tersebut adalah syiling adil. Uji pada aras keertian 1%.

[25 markah]

FORMULA

$\bar{x} = \frac{\sum xf}{\sum f}$ $s^2 = \frac{\sum (x^2 f) - \frac{(\sum xf)^2}{\sum f}}{\sum f - 1}$	$S_p{}^2 = \frac{(n_x - 1)s_x{}^2 + (n_y - 1)s_y{}^2}{n_x + n_y - 2}$ $\bar{p} = \frac{X + Y}{n_x + n_y}$
Confidence Intervals: $\overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $\overline{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$(\overline{X} - \overline{Y}) \pm z_{\alpha/2} \sqrt{\frac{\sigma_x{}^2}{n_x} + \frac{\sigma_y{}^2}{n_y}}$ $(\overline{X} - \overline{Y}) \pm t_{\alpha/2} \sqrt{S_p{}^2 \left(\frac{1}{n_x} + \frac{1}{n_y}\right)}$ $(\hat{p}_x - \hat{p}_y) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}}$

Test Statistics: $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ $T = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$ $T = \frac{\overline{d} - \mu_d}{\frac{s_d}{\sqrt{n_d}}}$ $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$	$Z = \frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x{}^2}{n_x} + \frac{\sigma_y{}^2}{n_y}}}$ $T = \frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p{}^2 \left(\frac{1}{n_x} + \frac{1}{n_y}\right)}}$ $Z = \frac{(\hat{p}_x - \hat{p}_y) - (p_x - p_y)}{\sqrt{\frac{p_x(1-p_x)}{n_x} + \frac{p_y(1-p_y)}{n_y}}}$	$Z = \frac{(\hat{p}_x - \hat{p}_y) - (p_x - p_y)}{\sqrt{\bar{p}(1-\bar{p}) \left(\frac{1}{n_x} + \frac{1}{n_y}\right)}}$ $F = \frac{s_1^2}{s_2^2}$ $\chi^2 = \sum \frac{(O-E)^2}{E}, \quad E = np$

Nonparametric Statistics:

Wilcoxon Signed-rank: $W = \sum R^+$, $W = \sum R^-$

$$Z = \frac{T - \mu_W}{\sigma_W} \ , \quad \mu_W = \frac{n(n+1)}{4} , \ \sigma_W = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

Wilcoxon Rank Sum Test: $U = R - \frac{n(n+1)}{2}$

$$Z = \frac{T - \mu_T}{\sigma_T} \ , \quad \mu_T = \frac{n_1(n_1 + n_2 + 1)}{2} , \ \sigma_T = \sqrt{\frac{n_1 n_2 (n+1)(n_1 + n_2 + 1)}{12}}$$

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