UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2016/2017 Academic Session

June 2017

# MAT 101 - Calculus
## *[Kalkulus]*

Duration : 3 hours
*[Masa : 3 jam]*

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Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions**: Answer **all six** [6] questions.

*[**Arahan:*** *Jawab **semua enam** [6] soalan.]*

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

**QUESTION 1**

(a) Suppose $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ 4 & , \quad x = 2 \\ x+2 & , \quad 0 < x < 2 \\ \dfrac{|x|}{x} & , \quad x < 0. \end{cases}$

(i) Find $\lim_{x \to 2} f(x)$.

(ii) Why is $f$ continuous at $2$?

(iii) Find $\lim_{x \to 0^-} f(x)$.

[ 35 marks ]

(b) Find the following limits.

(i) $\lim_{x \to 3} \dfrac{x^2 - 9}{2x^2 - 7x + 3}$

(ii) $\lim_{t \to 0} \dfrac{\tan 7t}{\sin 2t}$

[ 40 marks ]

(c) Suppose $\lim_{x \to a} f(x) = 0$. Show that $\lim_{x \to a} \left( [f(x)]^2 \sin \dfrac{1}{x} \right) = 0$ using the Squeeze Theorem.

[ 25 marks ]

***SOALAN 1***

*(a)* *Andaikan* $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ 4 & , \quad x = 2 \\ x+2 & , \quad 0 < x < 2 \\ \dfrac{|x|}{x} & , \quad x < 0. \end{cases}$

*(i)* *Dapatkan* $\lim_{x \to 2} f(x)$.

*(ii)* *Nyatakan sebab* $f$ *selanjar pada* $2$.

*(iii)* *Dapatkan* $\lim_{x \to 0^-} f(x)$.

*[ 35 markah ]*

*(b)* *Dapatkan had yang berikut.*

*(i)* $\lim\limits_{x\to 3}\dfrac{x^2-9}{2x^2-7x+3}$

*(ii)* $\lim\limits_{t\to 0}\dfrac{\tan 7t}{\sin 2t}$

*[ 40 markah ]*

*(c)* *Andaikan* $\lim\limits_{x\to a} f(x)=0$. *Tunjukkan bahawa* $\lim\limits_{x\to a}\left([f(x)]^2\sin\dfrac{1}{x}\right)=0$ *dengan menggunakan Teorem Himpitan.*

*[ 25 markah ]*

**QUESTION 2**

(a) Suppose $f(x)=\begin{cases} x\sin\dfrac{1}{x} & , \quad x>0 \\ 0 & , \quad x<0 \end{cases}$. Determine whether $f'(0)$ exists using the definition of derivative.

[ 30 marks ]

(b) Find the derivative of the following functions. **Do not simplify your answer.**

(i) $y=x^{\pi}+\pi$

(ii) $y=\dfrac{\sec x}{\sqrt[3]{x}}$

(iii) $y=(e^x+3)^5$

(iv) $y=xe^x\ln x$

(v) $y=\sin\left(xe^x\right)$

[ 40 marks ]

(c) Prove that $\lim\limits_{x\to 1}(3x-2)=1$ using the $\varepsilon$-$\delta$ definition.

[ 30 marks ]

***SOALAN 2***

*(a)* *Andaikan* $f(x)=\begin{cases} x\sin\dfrac{1}{x} & , \quad x>0 \\ 0 & , \quad x<0 \end{cases}$. *Tentukan sama ada* $f'(0)$ *wujud dengan menggunakan takrif untuk terbitan.*

*[ 30 markah ]*

*(b)* *Dapatkan terbitan bagi fungsi berikut.* ***Jangan ringkaskan jawapan anda.***

*(i)* $y = x^{\pi} + \pi$

*(ii)* $y = \dfrac{\sec x}{\sqrt[3]{x}}$

*(iii)* $y = (e^x + 3)^5$

*(iv)* $y = xe^x \ln x$

*(v)* $y = \sin\left(xe^x\right)$

*[ 40 markah ]*

*(c)* *Buktikan bahawa* $\lim_{x\to 1}(3x-2)=1$ *dengan menggunakan takrif* $\varepsilon$-$\delta$.

*[ 30 markah ]*

**QUESTION 3**

(a) Show that the equation $2x+\cos x=0$ has a (real) root between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$.

[ 30 marks ]

(b) State the Mean Value Theorem.

[ 15 marks ]

(c) Suppose $a,b\in$ ¡ with $a<b$. Prove that $|\sin b-\sin a|<|b-a|$.

[ 40 marks ]

(d) Show that $\lim_{x\to a}[f(x)+g(x)]$ may exist although neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists.

[ 15 marks ]

**SOALAN 3**

*(a)* *Tunjukkan bahawa persamaan* $2x + \cos x = 0$ *mempunyai punca nyata antara* $-\frac{\pi}{2}$ *dan* $\frac{\pi}{2}$.

*[ 30 markah ]*

*(b)* *Nyatakan Teorem Nilai Min.*

*[ 15 markah ]*

*(c)* *Andaikan* $a, b \in ¡$ *dengan* $a < b$. *Buktikan bahawa* $|\sin b - \sin a| < |b - a|$.

*[ 40 markah ]*

*(d)* *Tunjukkan bahawa* $\lim_{x \to a}[f(x) + g(x)]$ *boleh wujud walaupun kedua-dua* $\lim_{x \to a} f(x)$ *dan* $\lim_{x \to a} g(x)$ *tidak wujud.*

*[ 15 markah ]*

**QUESTION 4**

(a) Suppose $\int_1^7 f(x)dx = 10$, $\int_7^4 f(x)dx = 1$, and $\int_4^1 g(x)dx = 4$. Compute

$$\int_1^4 f(x) + 3g(x)dx.$$

[ 30 marks ]

(b) Find $F'(x)$,, where $F(x) = \int_{\sin x}^{x^3} t^2\, dt$.

[ 25 marks ]

(c) Show that $f(x) = \frac{x^3}{1+x^2}$ is an odd function. Hence, deduce the value of $\int_{-3}^{3} \frac{x^3}{1+x^2}$.

[ 30 marks ]

(d) Why does the definite integral $\int_0^7 \frac{x^3 + x}{x^4 + x^2 + 1}\, dx$ exist?

[ 15 marks ]

***SOALAN 4***

*(a)* *Andaikan* $\int_1^7 f(x)dx = 10$, $\int_7^4 f(x)dx = 1$, *dan* $\int_4^1 g(x)dx = 4$. *Hitungkan*

$\int_1^4 f(x) + 3g(x)dx.$

*[ 30 markah ]*

*(b)* *Dapatkan* $F'(x)$, *diberi* $F(x) = \int_{\sin x}^{x^3} t^2\, dt$.

*[ 25 markah ]*

*(c)* *Tunjukkan bahawa* $f(x) = \dfrac{x^3}{1+x^2}$ *ialah fungsi ganjil. Justeru itu, deduksikan nilai* $\int_{-3}^{3} \dfrac{x^3}{1+x^2}$.

*[ 30 markah ]*

*(d)* *Nyatakan sebab kamiran tentu* $\int_0^7 \dfrac{x^3 + x}{x^4 + x^2 + 1}\, dx$ *wujud.*

*[ 15 markah ]*

**QUESTION 5**

(a) The region bounded by the graph of $y^2 = x$ and the line $x = 4$ is rotated about the $y$-axis. Compute the volume of the solid obtained using the washer method.

[ 50 marks ]

(b) Find the following integrals.

(i) $\int \dfrac{x\cos(x^2)}{1+\sin(x^2)}dx$

(ii) $\int x^2 e^x\, dx$

[ 50 marks ]

***SOALAN 5***

*(a)* *Rantau yang dibatasi oleh graf bagi* $y^2 = x$ *dan garis* $x = 4$ *dikisarkan sekitar paksi* $y$. *Hitungkan isipadu kisaran dengan menggunakan kaedah cakera.*

*[ 50 markah ]*

*(b)* *Dapatkan kamiran yang berikut.*

*(i)* $\int \frac{x\cos(x^2)}{1+\sin(x^2)}\,dx$

*(ii)* $\int x^2 e^x \, dx$

*[ 50 markah ]*

**QUESTION 6**

(a) Suppose $f(x) = \frac{4x-1}{2x+3}$. Show that $f$ is one-to-one.

[ 20 marks ]

(b) Suppose $f(x) = \int_3^x \sqrt{1+x^2}\,dx$. Assuming $f$ is one-to-one, find $(f^{-1})'(0)$.

[ 30 marks ]

(c) Is the following statement true or false? Just write down the correct answer, that is, either "TRUE" or "FALSE".

(i) Suppose $f : \mathbf{R} \to \mathbf{R}$. If $f(-2) = f(2)$, then $f$ is an even function.

(ii) If $\lim_{x\to a} f(x) = \infty$, then $\lim_{x\to a} f(x)$ exists.

(iii) If $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists, then f is differentiable at a.

(iv) Fermat's Theorem says that if $f$ has a local maximum or local minimum at $c$, then f is differentiable at $c$ and $f'(c) = 0$.

(v) The following two statements are equivalent.

- For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\delta < \varepsilon$.
- There exists $\delta > 0$ such that for every $\varepsilon > 0$, we have $\delta < \varepsilon$.

(vi) $\lim_{x\to 0} \sin \frac{1}{x}$ does not exist.

(vii) The function $f(x) = |x|$ is continuous but not differentiable at $0$.

(viii) The Riemann integral $\int_a^b f(x)\,dx$ is defined to be $\lim_{n\to\infty} R_n$, where $R_n$ is the corresponding right Riemann sum.

(ix) Suppose $f$ is continuous on an open interval $(a,b)$. The Extreme Value Theorem tells us that the absolute maximum of $f$ exists.

(x) If $\lim_{x\to 5} f(x) = \infty$, then the line $x = 5$ is a vertical asymptote of the graph of $f$.

[ 50 marks ]

**SOALAN 6**

*(a)* *Andaikan* $f(x) = \frac{4x-1}{2x+3}$. *Tunjukkan bahawa* $f$ *adalah satu-ke-satu.*

*[ 20 markah ]*

*(b)* *Andaikan* $f(x) = \int_3^x \sqrt{1+x^2}\,dx$. *Dengan anggapan* $f$ *adalah satu-ke-satu, dapatkan* $(f^{-1})'(0)$.

*[ 30 markah ]*

*(c)* *Adakah kenyataan berikut benar atau palsu? Cuma tuliskan jawapan yang betul, iaitu sama ada "BENAR" atau "PALSU".*

*(i)* *Andaikan* $f : \mathbf{R} \to \mathbf{R}$. *Jika* $f(-2) = f(2)$, *maka* $f$ *ialah fungsi genap.*

*(ii)* *Jika* $\lim_{x\to a} f(x) = \infty$, *maka* $\lim_{x\to a} f(x)$ *wujud.*

*(iii)* *Jika* $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ *wujud, maka* $f$ *terbezakan pada a.*

*(iv)* *Teorem Fermat menyatakan bahawa jika* $f$ *mempunyai maksimum setempat atau minimum setempat pada* $c$, *maka* $f$ *terbezakan pada c dan* $f'(c) = 0$.

*(v)* *Dua kenyataan berikut adalah setara.*

- *Bagi setiap* $\varepsilon > 0$, *wujud* $\delta > 0$ *sedemikian* $\delta < \varepsilon$.
- *Wujud* $\delta > 0$ *sedemikian bagi setiap* $\varepsilon > 0, \delta < \varepsilon$.

*(vi)* $\lim_{x\to 0} \sin \frac{1}{x}$ *tidak wujud.*

*(vii)* *Fungsi* $f(x) = |x|$ *adalah selanjar tetapi tak terbezakan pada* 0.

*(viii)* *Kamiran Riemann* $\int_a^b f(x)\,dx$ *ditakrifkan sebagai* $\lim_{n\to\infty} R_n$, *sedemikian bahawa* $R_n$ *ialah hasil tambah Riemann kanan yang sepadan.*

*(ix)* *Andaikan* $f$ *selanjar pada selang terbuka* $(a,b)$. *Teorem Nilai Lampau menyatakan bahawa maksimum mutlak bagi* $f$ *wujud.*

*(x)* *Jika* $\lim_{x\to 5} f(x) = \infty$, *maka garis* $x = 5$ *adalah asimptot tegak untuk graf* $f$.

*[ 50 markah ]*

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