UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2016/2017 Academic Session

June 2017

MAT 263 – Probability Theory
[Teori Kebarangkalian]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all seven** [7] questions.

*[**Arahan:** Jawab **semua tujuh** [7] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Let X be a function defined as follows,

$$f_X(x) = \begin{cases} \binom{x+2}{x}\left(\frac{1}{2}\right)^{x+3} & x = 0,1,2,\ldots \\ 0 & \text{otherwise.} \end{cases}$$

a. Find $M_X[ln(1.2)]$.

[2 marks]

b. Find the second moment of this distribution.

[4 marks]

c. Given the following events, C and D are both sets of non-negative integer numbers, x, such that, $C = \{x \in Z | x \geq 2\}$ and $D = \{x \in Z | x < 4\}$. FromFind

i. $P(C - D)$.

ii. $P(C \cap D)$.

iii. $P[(C - D)|C]$.

[13 marks]

1. Biarkan X sebagai suatu fungsi yang ditakrifkan seperti berikut,

$$f_X(x) = \begin{cases} \binom{x+2}{x}\left(\frac{1}{2}\right)^{x+3} & x = 0,1,2,\ldots \\ 0 & selainnya. \end{cases}$$

a. *Cari* $M_X[ln(1.2)]$.

[2 markah]

b. Cari momen kedua bagi taburan ini.

[4 markah]

c. Diberi peristiwa berikut, C dan D kedua-duanya merupakan set-set integer bukan negatif, x, iaitu, $C = \{x \in Z | x \geq 2\}$ *dan* $D = \{x \in Z | x < 4\}$. *Cari*

i. $Kb(C - D)$.

ii. $Kb(C \cap D)$.

iii. $Kb[(C - D)|C]$.

[13 markah]

2. Let $A \cap C = C$, and $B \cap C \neq \emptyset$, where A and B are independent. Also given that, $P(A) = 0.5$, $P(B) = 0.4$, $P(C) = 0.3$ and $P(B \cup C) = 0.6$. Find

a. $P(C|A)$.

[2 marks]

b. $P(B - C)$.

[3 marks]

c. $P[(B - C)|A]$.

[4 marks]

d. $P[(A \cap B \cap C)']$.

[3 marks]

2. Biar $A \cap C = C$ dan $B \cap C \neq \emptyset$, yang mana A dan B adalah tidak bersandaran. Juga diberi, $Kb(A) = 0.5$, $Kb(B) = 0.4$, $Kb(C) = 0.3$ dan $Kb(B \cup C) = 0.6$. Cari

a. $Kb(C|A)$.

[2 markah]

b. $Kb(B - C)$.

[3 markah]

c. $Kb[(B - C)|A]$.

[4 markah]

d. $Kb[(A \cap B \cap C)']$.

[3 markah]

3. Random variable, A has the moment generating function as follows,

$$M_A(t) = \frac{1}{12}e^{2t} + \frac{5}{48}e^{2.5t} + \frac{1}{6}e^{4t} + \frac{5}{16}e^{7.5t} + \frac{1}{3}e^{8t}.$$

a. By using the equation above, find the mean and variance of A.

[8 marks]

b. State the distribution of A.

[2 marks]

c. Using Markov's inequality, find the highest value of $P\left[A \geq 10\sqrt{Var(A)}\right]$.

[3 marks]

3. Pembolehubah rawak, A mempunyai fungsi penjana momen seperti berikut,

$$M_A(t) = \frac{1}{12}e^{2t} + \frac{5}{48}e^{2.5t} + \frac{1}{6}e^{4t} + \frac{5}{16}e^{7.5t} + \frac{1}{3}e^{8t}.$$

a. Dengan menggunakan persamaan di atas, cari min dan varians bagi A.

[8 markah]

b. Nyatakan taburan bagi pemboleh ubah rawak A.

[2 markah]

c. Dengan menggunakan ketaksamaan Markov, cari nilai tertinggi bagi $Kb\left[A \geq 10\sqrt{Var(A)}\right]$.

[3 markah]

4. Consider the jointly random variables, X and Y, where, X has Binomial distribution with parameters of $n = 2$ and $p = 3/4$, whereas, $Y|X = x$ is a Binomial distribution with the parameters of $n = 2 - x$ and $p = 4/5$.

a. Determine the conditional probability of Y given $X = x$, $f_{Y|x}(y|x)$.

[2 marks]

b. Compute,

i. The expectation, $E(XY)$,

[7 marks]

ii. The mean and variance of X and Y

[11 marks]

iii. The covariance between X and Y, $Cov(X, Y)$.

[2 marks]

c. If $Z = 3X + Y$, then, find $Var(Z)$.

[3 marks]

4. Pertimbangkan pemboleh ubah rawak tergabung, X dan Y, yang mana, X adalah taburan Binomial dengan parameter $n = 2$ dan $p = 3/4$, manakala, $Y|X = x$ adalah taburan Binomial dengan parameter $n = 2 - x$ dan $p = 4/5$.

a. Tentukan kebarangkalian bersyarat bagi Y diberi $X = x$, $f_{Y|x}(y|x)$.

[2 markah]

b. Kira

i. Jangkaan, $E(XY)$,

[7 markah]

ii. Min dan varians bagi X dan Y,

[11 markah]

iii. Kovarians antara X dan Y, $Kov(X, Y)$.

[2 markah]

c. Jika $Z = 3X + Y$, maka, cari $Var(Z)$.

[3 markah]

5. Find $P[X \geq E(X)]$ where the random variable X has the moment generating function as follows,

a. $\frac{2}{5t}(e^{5t} - e^{2.5t})$.

[6 marks]

b. $(0.4 + 0.6e^{t})^{8}$.

[5 marks]

5. Cari $Kb[X \geq E(X)]$ yang mana pembolehubah rawak X mempunyai fungsi penjana momen seperti berikut,

c. $\frac{2}{5t}(e^{5t} - e^{2.5t})$.

[6 markah]

d. $(0.4 + 0.6e^{t})^{8}$.

[5 markah]

6. Let $X_1, \dots, X_{17}$ be independently normal distributed with mean μ, and variance σ^2. Consider the following information,

$$\sum x = 4.33 \text{ and } \sum x^2 = 17.9093$$

a. Estimate the mean and standard deviation of the distribution above.

[3 marks]

b. Calculate a 95% confidence interval for the mean of this random variable.

[4 marks]

c. Calculate a 95% confidence interval for the standard deviation of this random variable.

[5 marks]

6. Biarkan $X_1, \dots, X_{17}$ sebagai taburan tak bersandar normal dengan min μ, dan varians σ^2. Pertimbangkan maklumat seperti berikut,

$$\sum x = 4.33 \text{ dan } \sum x^2 = 17.9093$$

a. Anggarkan min dan sisihan piawai bagi taburan di atas.

[3 markah]

b. Kirakan 95% selang keyakinan min bagi pembolehubah rawak ini.

[4 markah]

c. Kirakan 95% selang keyakinan sisihan piawai bagi pembolehubah rawak ini.

[5 markah]

7. Let X be a continuously uniform random variable under the domain of 0 and 1, i.e. $x \in [0,1]$ Let $Y = -3lnX$. Then, determine $E(Y + Y^2)$.

[8 marks]

7. Biarkan X *sebagai taburan selanjar seragam di bawah domain 0 dan 1, iaitu,* $x \in [0,1]$. *Biar* $Y = -3lnX$. *Maka, tentukan* $E(Y + Y^2)$.

[8 markah]

Appendix

Random Variable, X	**Probability distribution function,** $f_X(x)$	**Mean,** $E(X)$	**Variance,** $Var(X)$	**Moment Generating Function,** $M_X(t)$
$bin(n,p)$	$\binom{n}{x} p^x(1-p)^x,\ x = 0,1,\dots,n$	np	$np(1-p)$	$(pe^t + 1 - p)^n$
$Poisson(\lambda)$	$\frac{e^{-\lambda}\lambda^x}{x!},\ x = 0,1,2,\dots$	λ	λ	$e^{\lambda(e^t-1)}$
$NB(r,p)$	$\binom{x+r-1}{x} p^r(1-p)^x,\ x = 0,1,2,\dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$
$uniform(a,b)$	$\frac{1}{b-a},\quad a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], -\infty < x < \infty$	μ	σ^2	$exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$
$exp(\theta)$	$\frac{1}{\theta} e^{-x/\theta},\ x > 0$	θ	θ^2	$(1-\theta t)^{-1}$
$Gamma(\alpha,\theta)$	$\frac{1}{\Gamma(\alpha)} \frac{x^{\alpha-1}}{\theta^\alpha} exp(-x/\theta),\ x > 0$	$\alpha\theta$	$\alpha\theta^2$	$(1-\theta t)^{-\alpha}$

$(a+b)^n = \sum_{x=0}^{n} {}^nC_x a^x b^{n-x}$	$g(x) = \sum_{m=0}^{\infty} \frac{g^{(m)}(x_0)(x-x_0)^m}{m!}$	$Var(aX+bY) = a^2Var(X) + 2abCov(X,Y) + b^2Var(Y)$
$P(X-Y) = P(X \cap \bar{Y})$		