UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2015/2016 Academic Session

June 2016

MGM 563 – Statistical Inference
[Pentaabiran Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **FIVE** (5) questions.

*[**Arahan:** Jawab **LIMA** (5) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) A shop sells a computer hardware. The weekly demand for the hardware follows a Poisson distribution with mean 3. Find the probability that the shop sells.

(i) at least 3 in a week.

(ii) at most 7 in a week.

(iii) more than 20 in a month.

(a) Suatu kedai menjual perkakasan komputer. Permintaan mingguan perkakasannya mengikut taburan Poisson dengan min 3. Cari kebarangkalian bahawa kedai itu menjual:

(i) sekurang-kurangnya 3 dalam satu minggu.

(ii) paling banyak 7 dalam satu minggu.

(iii) lebih daripada 20 dalam masa sebulan.

(b) Stocks are refurnished only at the beginning of each month. Find the minimum number that should be in stock at the beginning of a month so that the shop can be at least 95% save of being able to meet the demand during the month.

(b) Stok akan ditambah pada awal setiap bulan. Cari bilangan minimum yang perlu ada dalam stok pada awal suatu bulan supaya kedai tersebut adalah sekurang-kurangnya 95% selamat dalam memenuhi permintaan sepanjang bulan itu.

2. (a) Let X_1 and X_2 be independent random variables from the normal distribution $N(\mu , \sigma^2)$. Show that $W = X_1 + X_2$ and $Z = X_1 - X_2$ are independent by using the Jacobian transformation, $f_{W,Z}(w , z)$.

(a) Biarkan X_1 dan X_2 sebagai pembolehubah rawak tak bersandar yang bertaburan normal $N(\mu , \sigma^2)$. Tunjukkan bahawa $W = X_1 + X_2$ dan $Z = X_1 - X_2$ adalah tidak bersandar dengan menggunakan transformasi Jacobian, $f_{W,Z}(w , z)$.

[20 marks/*markah*]

(b) Let X_1 and X_2 be two independent random variables having gamma distributions with parameters $\alpha_1 = 3$, $\theta_1 = 3$ and $\alpha_2 = 5$, $\theta_2 = 1$ respectively. Find the moment generating function of $Y = 2X_1 + 6X_2$. What is the distribution of *Y*?

(b) Biarkan X_1 *dan* X_2 *sebagai dua pembolehubah rawak tidak bersandar yang bertaburan gamma masing-masing dengan parameter* $\alpha_1 = 3$, $\theta_1 = 3$ *dan* $\alpha_2 = 5$, $\theta_2 = 1$. *Cari fungsi penjana momen bagi* $Y = 2X_1 + 6X_2$. *Apakah taburan Y?*

3. (a) Suppose that $Y_1, Y_2, ..., Y_n$ is a random sample of size *n* from the Poisson distributed population with mean λ. Assume that *n* = 2*k* for some integer *k* and $\hat{\lambda} = \frac{1}{2}\sum_{i=1}^{k}(Y_{2i} - Y_{2i-1})^2$. Show that $\hat{\lambda}$ is an unbiased estimate for λ.

(a) Biarkan $Y_1, Y_2, ..., Y_n$ *sebagai sampel rawak daripada taburan Poisson bersaiz n dengan min* λ. *Andaikan bahawa n = 2k bagi beberapa integer k dan* $\hat{\lambda} = \frac{1}{2}\sum_{i=1}^{k}(Y_{2i} - Y_{2i-1})^2$. *Tunjukkan bahawa* $\hat{\lambda}$ *adalah suatu anggaran saksama bagi* λ.

(b) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from a uniform $(0, \theta)$ distribution, where θ is unknown. Define the estimator

$$\hat{\Theta}_n = \max\{X_1, X_2, ..., X_n\}$$

(i) Find the bias of $\hat{\Theta}_n$.

(ii) Find the mean square error of $\hat{\Theta}_n$.

(iii) Is $\hat{\Theta}_n$ a consistent estimator of θ?

[20 marks/*markah*]

(b) *Biarkan* $X_1, X_2, X_3, ..., X_n$ *sebagai suatu sampel rawak daripada taburan seragam yang mana* θ *tidak diketahui. Takrifkan penganggar*

$$\hat{\Theta}_n = \max\{X_1, X_2, ..., X_n\}$$

(i) *Cari kepincangan bagi* $\hat{\Theta}_n$

(ii) *Cari ralat kuasa dua min bagi* $\hat{\Theta}_n$

(iii) *Adakah* $\hat{\Theta}_n$ *suatu penganggar konsisten bagi* θ *?*

4. (a) Let $X_1, ..., X_n$ be a random sample from an exponential distribution with mean θ. We are interested in the estimation of the variance of the distribution which is θ^2.

(i) What is the maximum likelihood estimator of θ^2?

(ii) Find the Cramer-Rao lower bound for the variance of an unbiased estimator of θ^2.

(a) *Biarkan* $X_1, ..., X_n$ *sebagai suatu sampel rawak daripada taburan eksponen dengan min* θ. *Kita berminat dalam anggaran varians untuk taburan itu iaitu* θ^2.

(i) *Apakah penganggar kebolehjadian maksimum bagi* θ^2 *?*

(ii) *Cari batas bawah Cramer-Rao untuk varians penganggar saksama* θ^2.

(b) Suppose that X is a discrete random variable with $P(X=0) = \frac{2}{3}\theta$, $P(X=1) = \frac{1}{3}\theta$, $P(X=2) = \frac{2}{3}(1-\theta)$, $P(X=3) = \frac{1}{3}(1-\theta)$. Find the method of moments estimate of θ.

(b) *Andaikan bahawa* X *adalah suatu pembolehubah rawak diskret dengan* $P(X=0) = \frac{2}{3}\theta$, $P(X=1) = \frac{1}{3}\theta$, $P(X=2) = \frac{2}{3}(1-\theta)$, $P(X=3) = \frac{1}{3}(1-\theta)$. *Cari anggaran kaedah momen bagi* θ.

[20 marks/*markah*]

5. (a) X is normally distributed with unknown mean μ and standard deviation 16. A random sample of n = 16 is selected with the probability of committing a Type I error at $\alpha = 0.05$. Assume that the hypothesis of interest is $H_0 : \mu = 100$ against $H_A : \mu > 100$.

(i) What is the power of the hypothesis test if the true population mean is $\mu = 108$?

(ii) If the level of significance changes to $\alpha = 0.01$, does the result change?

(iii) Illustrate the power of the test under $\alpha = 0.01$ versus $\alpha = 0.05$ in a power function plot. Illustrate also the critical regions for (i) and (ii) in a normal distribution plot.

(a) X *adalah bertaburan normal dengan min* μ *yang tidak diketahui dan sisihan piawai 16. Suatu sampel rawak n = 16 dipilih dengan kebarangkalian melakukan ralat Jenis I pada* $\alpha = 0.05$ *. Andaikan bahawa hipotesis yang diingini ialah* $H_0 : \mu = 100$ *lawan* $H_A : \mu > 100$.

(i) *Apakah kuasa bagi ujian hipotesis jika min populasi sebenar ialah* $\mu = 108$ *?*

(ii) *Jika aras keertian bertukar kepada* $\alpha = 0.01$*, adakah keputusannya berubah?*

(iii) *Lakarkan kuasa ujian di bawah* $\alpha = 0.01$ *lawan* $\alpha = 0.05$ *dalam plot fungsi kuasa. Lakarkan juga rantau genting untuk (i) dan (ii) dalam plot taburan normal.*

(b) Let Y have the probability density function given by

$$f_Y(y) = \begin{cases} \dfrac{3(\theta - y)^2}{\theta^3} & 0 < y < \theta \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that $\dfrac{Y}{\theta}$ is a pivotal quantity

(ii) Use the pivotal quantity from (i) to find a 95% lower confidence limit for θ

[20 marks/*markah*]

(b) *Biarkan Y mempunyai fungsi taburan kebarangkalian yang diberi oleh*

$$f_Y(y) = \begin{cases} \dfrac{3(\theta - y)^2}{\theta^3} & 0 < y < \theta \\ 0 & \text{selainnya} \end{cases}$$

(i) *Tunjukkan bahawa* $\dfrac{Y}{\theta}$ *ialah suatu kuantiti pangsian*

(ii) *Gunakan kuantiti pangsian dari (i) untuk mencari 95% had keyakinan bawah bagi* θ *.*

[20 marks/*markah*]

APPENDIX/*LAMPIRAN*

Poisson Distribution Table

λ=	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8
x= 0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.3012	0.2466	0.2019	0.1653
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	0.6626	0.5918	0.5249	0.4628
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197	0.8795	0.8335	0.7834	0.7306
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810	0.9662	0.9463	0.9212	0.8913
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9923	0.9857	0.9763	0.9636
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9985	0.9968	0.9940	0.9896
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9974
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ=	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.5	5.0	5.5
x= 0	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0111	0.0067	0.0041
1	0.4060	0.3546	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074	0.0916	0.0611	0.0404	0.0266
2	0.6767	0.6227	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689	0.2381	0.1736	0.1247	0.0884
3	0.8571	0.8194	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735	0.4335	0.3423	0.2650	0.2017
4	0.9473	0.9275	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678	0.6288	0.5321	0.4405	0.3575
5	0.9834	0.9751	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156	0.7851	0.7029	0.6160	0.5289
6	0.9955	0.9925	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091	0.8893	0.8311	0.7622	0.6860
7	0.9989	0.9980	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599	0.9489	0.9134	0.8666	0.8095
8	0.9998	0.9995	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840	0.9786	0.9597	0.9319	0.8944
9	1.0000	0.9999	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942	0.9919	0.9829	0.9682	0.9462
10	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981	0.9972	0.9933	0.9863	0.9747
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9976	0.9945	0.9890
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9992	0.9980	0.9955
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9983
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ=	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	11.0	10.0	12.0	14.0	15.0
x= 0	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005	0.0002	0.0005	0.0001	0.0000	0.0000
2	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028	0.0012	0.0028	0.0005	0.0001	0.0000
3	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103	0.0049	0.0103	0.0023	0.0005	0.0002
4	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293	0.0151	0.0293	0.0076	0.0018	0.0009
5	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671	0.0375	0.0671	0.0203	0.0055	0.0028
6	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301	0.0786	0.1301	0.0458	0.0142	0.0076
7	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202	0.1432	0.2202	0.0895	0.0316	0.0180
8	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328	0.2320	0.3328	0.1550	0.0621	0.0374
9	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579	0.3405	0.4579	0.2424	0.1094	0.0699
10	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830	0.4599	0.5830	0.3472	0.1757	0.1185
11	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968	0.5793	0.6968	0.4616	0.2600	0.1848
12	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916	0.6887	0.7916	0.5760	0.3585	0.2676
13	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645	0.7813	0.8645	0.6815	0.4644	0.3632
14	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165	0.8540	0.9165	0.7720	0.5704	0.4657
15	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513	0.9074	0.9513	0.8444	0.6694	0.5681
16	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730	0.9441	0.9730	0.8987	0.7559	0.6641
17	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857	0.9678	0.9857	0.9370	0.8272	0.7489
18	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928	0.9823	0.9928	0.9626	0.8826	0.8195
19	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965	0.9907	0.9965	0.9787	0.9235	0.8752
20	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9953	0.9984	0.9884	0.9521	0.9170
21	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9977	0.9993	0.9939	0.9712	0.9469
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9990	0.9997	0.9970	0.9833	0.9673
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9999	0.9985	0.9907	0.9805
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	1.0000	0.9993	0.9950	0.9888
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	0.9997	0.9974	0.9938
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987	0.9967
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9983
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Normal distribution

$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291

Continuous Distributions

Beta $0 < \alpha$ $0 < \beta$	$f(x) = \dfrac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1 - x)^{\beta-1}, \quad 0 < x < 1$ $\mu = \dfrac{\alpha}{\alpha + \beta}, \quad \sigma^2 = \dfrac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$
Chi-square $\chi^2(r)$ $r = 1,2,\ldots$	$f(x) = \dfrac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1}e^{-x/2}, \quad 0 \le x < \infty$ $M(t) = \dfrac{1}{(1 - 2t)^{r/2}}, \quad t < \dfrac{1}{2}$ $\mu = r, \quad \sigma^2 = 2r$
Exponential $0 < \theta$	$f(x) = \dfrac{1}{\theta}e^{-x/\theta}, \quad 0 \le x < \infty$ $M(t) = \dfrac{1}{1 - \theta t}, \quad t < \dfrac{1}{\theta}$ $\mu = \theta, \quad \sigma^2 = \theta^2$
Gamma $0 < \alpha$ $0 < \theta$	$f(x) = \dfrac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1}e^{-x/\theta}, \quad 0 \le x < \infty$ $M(t) = \dfrac{1}{(1 - \theta t)^\alpha}, \quad t < \dfrac{1}{\theta}$ $\mu = \alpha\theta, \quad \sigma^2 = \alpha\theta^2$
Normal $N(\mu, \sigma^2)$ $-\infty < \mu < \infty$ $0 < \sigma$	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$ $M(t) = e^{\mu t+\sigma^2 t^2/2}$ $E(X) = \mu, \quad \mathrm{Var}(X) = \sigma^2$
Uniform $U(a, b)$ $-\infty < a < b < \infty$	$f(x) = \dfrac{1}{b - a}, \quad a \le x \le b$ $M(t) = \dfrac{e^{tb} - e^{ta}}{t(b - a)}, \quad t \ne 0; \quad M(0) = 1$ $\mu = \dfrac{a + b}{2}, \quad \sigma^2 = \dfrac{(b - a)^2}{12}$

Discrete Distributions

Bernoulli $0 < p < 1$	$f(x) = p^x(1 - p)^{1-x}, \quad x = 0,1$ $M(t) = 1 - p + pe^t$ $\mu = p, \quad \sigma^2 = p(1 - p)$
Binomial $b(n, p)$ $0 < p < 1$	$f(x) = \dfrac{n!}{x!(n - x)!} p^x(1 - p)^{n-x}, \quad x = 0,1,2,\ldots,n$ $M(t) = (1 - p + pe^t)^n$ $\mu = np, \quad \sigma^2 = np(1 - p)$
Geometric $0 < p < 1$	$f(x) = (1 - p)^{x-1}p, \quad x = 1,2,3,\ldots$ $M(t) = \dfrac{pe^t}{1 - (1 - p)e^t}, \quad t < -\ln(1 - p)$ $\mu = \dfrac{1}{p}, \quad \sigma^2 = \dfrac{1 - p}{p^2}$
Hypergeometric $N_1 > 0,\ N_2 > 0$ $N = N_1 + N_2$	$f(x) = \dfrac{\binom{N_1}{x}\binom{N_2}{n - x}}{\binom{N}{n}}, \quad x \le n, x \le N_1, n - x \le N_2$ $\mu = n\left(\dfrac{N_1}{N}\right), \quad \sigma^2 = n\left(\dfrac{N_1}{N}\right)\left(\dfrac{N_2}{N}\right)\left(\dfrac{N - n}{N - 1}\right)$
Negative Binomial $0 < p < 1$ $r = 1,2,3,\ldots$	$f(x) = \binom{x - 1}{r - 1}p^r(1 - p)^{x-r}, \quad x = r, r + 1, r + 2,\ldots$ $M(t) = \dfrac{(pe^t)^r}{[1 - (1 - p)e^t]^r}, \quad t < -\ln(1 - p)$ $\mu = r\left(\dfrac{1}{p}\right), \quad \sigma^2 = \dfrac{r(1 - p)}{p^2}$
Poisson $0 < \lambda$	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0,1,2,\ldots$ $M(t) = e^{\lambda(e^t-1)}$ $\mu = \lambda, \quad \sigma^2 = \lambda$
Uniform $m > 0$	$f(x) = \dfrac{1}{m}, \quad x = 1,2,\ldots,m$ $\mu = \dfrac{m + 1}{2}, \quad \sigma^2 = \dfrac{m^2 - 1}{12}$

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