UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2015/2016 Academic Session

June 2016

MAT 203 – Vector Calculus
[Kalkulus Vektor]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[*Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*]

Instructions: Answer **SIX** (6) questions.

*[**Arahan:** Jawab **ENAM** (6) soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) The vertices of a triangle *P*, *Q* and *R* are given by $(1, 3, -3)$, $(2, 0, 2)$ and $(4, 1, -2)$ respectively.

   (i) Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (ii) Use the vector method to find the area of the triangle *PQR*.

   (iii) Find the angle at *P*.

   (b) Find the point on the plane $x + 2y + z = 0$ which is closest to the point $(3, 2, 5)$. What is the shortest distance from the given point to the plane?

[14 marks]

1. (a) Diberi bucu-bucu segitiga P, Q dan R ialah $(1, 3, -3)$, $(2, 0, 2)$ dan $(4, 1, -2)$.

(i) Dapatkan vektor $\overrightarrow{PQ}$ dan $\overrightarrow{PR}$.

(ii) Dapatkan luas segitiga PQR dengan menggunakan kaedah vektor.

(iii) Dapatkan sudut pada P.

(b) Dapatkan titik pada satah $x + 2y + z = 0$ yang terdekat dengan titik $(3, 2, 5)$. Apakah jarak terdekat dari titik yang diberi ke satah tersebut ?

[14 markah]

2. Two lines with parametric equations are given by

$$x = 1 + 3s,\ y = 6 + 2s,\ z = 12 - 2s \quad \text{and} \quad x = 1 + 3t,\ y = 6 - 4t,\ z = 12 + t.$$

   (a) What is the intersection point of these two lines?

   (b) Find an equation for the plane that contains these two lines.

   (c) In what points does the first line intersect each of the coordinate planes?

[14 marks]

2. *Diberi dua garis lurus dengan persamaan berparameter*

$$x = 1 + 3s,\ \ y = 6 + 2s,\ \ z = 12 - 2s \quad \textit{dan} \quad x = 1 + 3t,\ \ y = 6 - 4t,\ \ z = 12 + t\,.$$

(a) Apakah titik persilangan di antara dua garis lurus tersebut?

(b) Dapatkan persamaan satah yang mengandungi dua garis lurus tersebut.

(c) Apakah titik-titik persilangan di antara garis lurus pertama dengan satah-satah koordinat?

[14 markah]

3. (a) Suppose $u = P(x, y, z),$ where P is a differentiable function and $x = f(t),$ $y = g(t)$ and $z = h(t).$

(i) Use the chain rule to find an expression for $\frac{du}{dt}$ in terms of *P*, *f*, *g*, *h* and their derivatives.

(ii) Show that the expression in part (i) can be written as

$$\frac{du}{dt} = \left(\text{grad}\, P\right) \text{g} \frac{d\mathbf{r}}{dt}$$

where g is the dot product and $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$ is the vector form of the parameterized curve $x = f(t),\ y = g(t)$ and $z = h(t).$

(b) Suppose f is a differentiable function such that $f(1, 3) = 1$, $f_x(1, 3) = 2$ and $f_y(1, 3) = 4$.

(i) Find the gradient of f at the point $(1, 3)$.

(ii) Find a vector in the plane that is perpendicular to the contour line $f(x, y) = 1$ at the point $(1, 3)$.

(iii) What is the rate of change of f in the direction $\mathbf{i} + \mathbf{j}$ at the point $(1, 3)$?

(c) Let $f(x, y) = x^2 - 4x + y^2 - 4y + 16$.

(i) Find and classify all the critical points of f.

(ii) Find the maximum and minimum values of f subject to the constraint $x^2 + y^2 = 18$.

[20 marks]

3. (a) *Andaikan* $u = P(x, y, z)$, *yang mana P ialah satu fungsi terbezakan dan* $x = f(t)$, $y = g(t)$ *dan* $z = h(t)$.

(i) *Dapatkan ungkapan untuk* $\frac{du}{dt}$ *dalam P, f, g, h dan pembezaannya dengan menggunakan petua rantai.*

(ii) *Tunjukkan bahawa ungkapan di bahagian (i) boleh ditulis sebagai*

$$\frac{du}{dt} = (\text{grad}\, P) \cdot \frac{d\mathbf{r}}{dt}$$

yang mana $\cdot$ *ialah hasil darab titik dan* $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$ *ialah lengkung berparameter* $x = f(t)$, $y = g(t)$ *and* $z = h(t)$ *dalam bentuk vektor.*

(b) *Andaikan f ialah fungsi bolehbeza dengan* $f(1, 3) = 1$, $f_x(1, 3) = 2$ *dan* $f_y(1, 3) = 4$.

(i) *Dapatkan gradient f pada titik (1, 3).*

(ii) *Dapatkan vektor pada satah yang berserenjang dengan garis kontur* $f(x, y) = 1$ *pada titik (1, 3).*

(iii) *Apakah kadar perubahan f dalam arah* $\mathbf{i} + \mathbf{j}$ *pada titik (1, 3)?*

(c) *Biar* $f(x, y) = x^2 - 4x + y^2 - 4y + 16$.

(i) *Dapat dan kelaskan titik-titik genting f.*

(ii) *Dapatkan nilai maksimum and minimum f tertakluk kepada kekangan* $x^2 + y^2 = 18$.

[20 markah]

4. (a) Let C be the curve in $\mathbb{R}^3$ described by the spherical coordinates where $\rho = \theta$ and $\phi = \pi/4$ with endpoints $(x, y, z) = (0,0,0)$ and $(x, y, z) = (\pi\sqrt{2}, 0, \pi\sqrt{2})$. Evaluate the following integral:

$$\int_C \frac{x}{z}\,dx + \frac{y}{z}\,dy + z^3\,dz\,.$$

(b) Given $a > 0$, $b > 0$ and $c > 0$, show that the volume of the region enclosed by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{4}{3}\pi abc$.

[12 marks]

4. (a) Biar C suatu lengkung dalam $\mathbb{R}^3$ diberikan oleh koordinat sfera yang mana $\rho = \theta$ dan $\phi = \pi/4$ berserta titik-titik hujung $(x, y, z) = (0,0,0)$ dan $(x, y, z) = (\pi\sqrt{2}, 0, \pi\sqrt{2})$. Nilaikan kamiran berikut:

$$\int_C \frac{x}{z}\,dx + \frac{y}{z}\,dy + z^3\,dz\,.$$

(b) Diberi $a > 0$, $b > 0$ dan $c > 0$, tunjukkan bahawa isipadu bagi rantau yang dibatasi oleh ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ialah $\frac{4}{3}\pi abc$.

[12 markah]

5. (a) Consider the vector field $\mathbf{F} = (2x + y)\,\mathbf{i} + (x + 3y^2)\,\mathbf{j}$.

(i) Show that **F** is a conservative vector field.

(ii) Find a function f such that $\nabla f = \mathbf{F}$.

(iii) Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is the curve $y = \sin(x^2)$ from $(0,0)$ to $(\sqrt{\pi}, 0)$.

(b) Let C be the circle of radius 1, centered at the origin and oriented counter clockwise. Use Green's Theorem to evaluate the following integral:

$$\oint_C (e^y - y^3)\,dx + (xe^y + x^3)\,dy.$$

[20 marks]

5. (a) *Pertimbangkan medan vektor* $\mathbf{F} = (2x + y)\,\mathbf{i} + (x + 3y^2)\,\mathbf{j}$.

   (i) *Tunjukkan* ***F*** *ialah suatu medan vektor konservatif.*

   (ii) *Dapatkan fungsi* f *sedemikian* $\nabla f = \mathbf{F}$.

   (iii) *Nilaikan kamiran* $\int_C \mathbf{F} \cdot ds$ *yang mana* C *ialah lengkung* $y = \sin(x^2)$ *daripada* $(0,0)$ *kepada* $(\sqrt{\pi}, 0)$.

   (b) *Biar C suatu bulatan berjejari 1, berpusat pada asalan dan dalam arah lawan jam. Gunakan Teorem Green untuk menilai kamiran berikut:*

$$\oint_C (e^y - y^3)\,dx + (xe^y + x^3)\,dy.$$

[20 markah]

6. (a) The parameterized surface S given by $\mathbf{r}(u,v) = \langle \cos(u),\, v,\, \sin(u+v) \rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le \pi/3$ has a boundary which consists of two distinct curves C_1 and C_2.

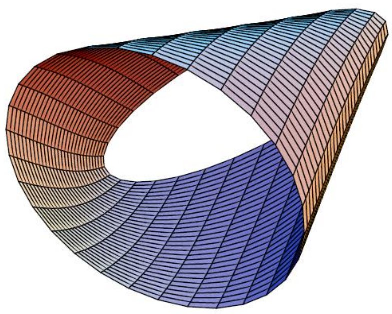


Figure 1. Surface *S*.

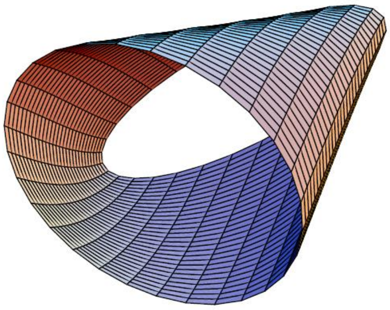
   (i) Set up, but do not evaluate, the integral for the surface area of *S*.

   (ii) Compute the arc length of one of the two boundary curves.

   (b) Let $\mathbf{F}(x,y,z) = \langle x + yz,\, xye^{-xz},\, e^{-xz} \rangle$. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the surface $z = 1 - x^2 - y^2,\, z \ge 0$ oriented so that the normal vector points upwards.

[20 marks]

6. (a) *Permukaan berparameter S yang diberikan oleh* $\mathbf{r}(u,v) = \langle \cos(u), v, \sin(u+v) \rangle$ *untuk* $0 \le u \le 2\pi$ *dan* $0 \le v \le \pi/3$ *mempunyai sempadan yang mengandungi dua lengkung berbeza,* C_1 *dan* C_2 .



Rajah 1. Permukaan S.

(i) *Nyatakan, tanpa menilaikannya, luas permukaan S.*

(ii) *Kirakan panjang lengkung bagi salah satu lengkung sempadan.*

(b) *Biar* $\mathbf{F}(x,y,z) = \left\langle \, x + yz, \; xye^{-xz}, \, e^{-xz} \, \right\rangle$. *Dapatkan* $\iint_S \mathbf{F} \cdot d\mathbf{S}$, *yang mana S ialah permukaaan* $z = 1 - x^2 - y^2, \, z \ge 0$ *berorientasi ke arah atas.*

[20 markah]

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