UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2015/2016 Academic Year

June 2016

MAT 111 - Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **FOUR** (4) questions.

*[**Arahan**: Jawab **EMPAT** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Suppose A is a square matrix. Show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric. Then show that A can always be expressed as the sum of a symmetric matrix and a skew symmetric matrix.

(b) Consider the following system of equations.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 0 \\ 2x_1 + 4x_2 - 3x_3 &= 0 \\ x_1 + 2x_2 + x_3 + 5x_4 &= 0. \end{aligned}$$

(i) Write the coefficient matrix, A of the system.

(ii) Solve the system using the Gauss-Jordan elimination.

(iii) State the bases for the column space of A.

(iv) State the bases for the row space of A and its row rank.

(v) State the bases for the null space of A and its nullity.

(vi) State the relationship between the rank and the nullity of A.

(c) Let A and B be any 2×2 invertible matrices and

$$C = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}.$$

Compute the following matrix :

$$D^{-1}CBA(BA)^{-1}C^{-1}(C^{-1}D)^{-1}.$$

[100 marks]

1. (a) *Andaikan A satu matriks segiempat sama. Tunjukkan bahawa $A+A^T$ ialah simetri dan $A-A^T$ ialah simetri pencong. Kemudian, tunjukkan bahawa A sentiasa boleh diungkapkan sebagai suatu hasil tambah matriks simetri dan matriks simetri pencong.*

(b) *Pertimbangkan sistem persamaan linear berikut.*

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 0 \\ 2x_1 + 4x_2 - 3x_3 &= 0 \\ x_1 + 2x_2 + x_3 + 5x_4 &= 0. \end{aligned}$$

(i) *Tuliskan matriks pekali A untuk sistem ini.*

(ii) *Selesaikan sistem ini menggunakan kaedah Gauss-Jordan.*

(iii) *Nyatakan asas bagi ruang lajur A.*

(iv) *Nyatakan asas bagi ruang baris A dan pangkatnya.*

(v) *Nyatakan asas bagi ruang nol A dan kenolannya.*

(vi) *Nyatakan hubungan antara pangkat dan kenolan A.*

(c) *Biar A dan B matriks 2×2 tersongsangkan dan*

$$C = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{dan} \qquad D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}.$$

Kira matriks berikut :

$$D^{-1}CBA(BA)^{-1}C^{-1}(C^{-1}D)^{-1}.$$

[100 markah]

2. (a) Let

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

and A be any 3×3 matrix.

(i) Find $\det(B)$.

(ii) Hence, by using the properties of determinants and the Equivalence Statements, show that the matrix A is invertible if and only if the matrix AB is invertible.

(b) Explain what it means for a set of finite vectors to be linearly independent. Then show that a nonempty set of vectors that contains $\mathbf{0}$ is linearly dependent.

(c) Let W be spanned by the vectors

$$\mathbf{v}_1 = (1, 1, 1, 1), \quad \mathbf{v}_2 = (1, 1, 2, 4), \quad \mathbf{v}_3 = (1, 2, -4, -3).$$

(i) Find an orthogonal basis of W.

(ii) Find an orthonormal basis of W.

(iii) Find a basis of the orthogonal complement of W.

[100 marks]

2. (a) *Biar*

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

dan A *suatu matriks* 3×3.

(i) *Dapatkan* $\det(B)$.

(ii) *Seterusnya, tunjukkan bahawa matriks* A *adalah tersonsangkan jika dan hanya jika matriks* AB *tersonsangkan menggunakan sifat penentu dan Pernyataan Setara.*

(b) *Terangkan makna untuk suatu set vektor terhingga adalah tak bersandar linear. Kemudian, tunjukkan bahawa suatu set vektor tak kosong yang mengandungi* $\mathbf{0}$ *adalah bersandar linear.*

(c) *Biar* W *direntang oleh vektor-vektor*

$$\mathbf{v}_1 = (1,1,1,1), \quad \mathbf{v}_2 = (1,1,2,4), \quad \mathbf{v}_3 = (1,2,-4,-3).$$

(i) *Cari asas ortogon untuk* W.

(ii) *Cari asas ortonormal untuk* W.

(iii) *Cari pelengkap berortogon untuk* W.

[100 markah]

3. (a) Let V be the set of 2×2 matrices with real entries, with the standard matrix addition operation. The scalar multiplication is defined as

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & b \\ kc & d \end{bmatrix}$$

for any nonzero real number k. Determine whether V is a vector space.

(b) For each of the following, determine whether W is a subspace of V. Prove your claim.

(i) Let $V = \mathbb{R}^3$ and $W = \{(a, b, c) : a = b\}$.

(ii) Let V be the vector space of real polynomials and W consists of all polynomials with degree more than six and the zero polynomial.

(c) Find the coordinate vector of $\mathbf{v} = (a, b, c)$ in $\mathbb{R}^3$ relative to:

(i) the standard basis,

(ii) the basis $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.

(d) State the conditions for which a function $T : V \to W$ (from vector space V to vector space W) can be called as a linear transformation.

(e) Consider the following linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x - y + z \\ y + 2z \\ 3y + 6z \end{bmatrix}.$$

(i) Find the standard matrix for T.

(ii) Find a basis for the kernel of T.

(iii) Find a basis for the image of T.

[100 marks]

3. (a) *Andaikan* V *suatu set yang mengandungi matriks* 2×2 *dengan unsur nyata serta operasi penambahan matriks yang lazim. Operasi pendaraban skalar diberi sebagai*

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & b \\ kc & d \end{bmatrix}$$

untuk sebarang nombor nyata k, $k \neq 0$. *Tentukan sama ada* V *ialah ruang vektor.*

(b) *Untuk setiap yang berikut, tentukan sama ada* W *adalah subruang* V. *Buktikan tuntutan anda.*

(i) *Andaikan* $V = \mathbb{R}^3$ *dan* $W = \{(a, b, c) : a = b\}$.

(ii) *Andaikan* V *ruang vektor polinomial nyata dan* W *mengandungi semua polinomial berdarjah lebih daripada enam dan polinomial sifar.*

(c) *Cari vektor koordinat bagi* $\mathbf{v} = (a, b, c)$ *dalam* $\mathbb{R}^3$ *relatif kepada:*

(i) *asas piawai,*

(ii) *asas* $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.

(d) *Nyatakan syarat untuk suatu fungsi* $T : V \to W$ *(dari ruang vektor* V *ke ruang vektor* W*) dikatakan sebagai suatu tranformasi linear.*

(e) *Pertimbangkan transformasi linear yang diberi oleh*

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x - y + z \\ y + 2z \\ 3y + 6z \end{bmatrix}.$$

(i) *Cari matriks piawai untuk* T.

(ii) *Cari asas untuk inti* T.

(iii) *Cari asas untuk imej* T.

[100 markah]

4. (a) A matrix A is idempotent if $A^2 = A$. Show that the only possible eigenvalues of an idempotent matrix are $\lambda = 0$ and $\lambda = 1$.

(b) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

(i) Find the eigenvalues and bases for the corresponding eigenspaces of the matrix A.

(ii) Find the invertible matrix P and the diagonal matrix D such that $P^{-1}AP = D$.

(iii) Is P in part (ii) unique? Justify your answer.

(iv) Find the eigenvalues and associated eigenvectors of A^5.

(v) Hence find A^5.

[100 marks]

4. (a) Suatu matriks A dikatakan idempoten jika $A^2 = A$. Tunjukkan bahawa hanya $\lambda = 0$ and $\lambda = 1$ merupakan nilai-nilai eigen yang mungkin untuk suatu matriks idempoten.

(b) Andaikan $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

(i) Dapatkan semua nilai eigen dan asas bagi bagi ruang eigen matriks A.

(ii) Dapatkan matriks tersongsangkan P dan matriks pepenjuru D yang memenuhi $P^{-1}AP = D$.

(iii) Adakah P dalam bahagian (ii) unik? Beri justifikasi untuk jawapan anda.

(iv) Dapatkan semua nilai eigen dan asas bagi bagi ruang eigen matriks A^5.

(v) Kemudian, cari A^5.

[100 markah]

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