UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2015/2016 Academic Session

June 2016

MAT 101 - Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[*Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*]

Instructions: Answer **SIX** (6) questions.

*[**Arahan:** Jawab **ENAM** (6) soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Suppose $f(x)=\begin{cases} x^2 & , \quad x>2 \\ 2+x & , \quad 0\le x\le 2 \\ \dfrac{1}{x} & , \quad x<0 \end{cases}$.

(i) Find $\lim_{x\to 2} f(x)$.

(ii) Why is f continuous at 2?

(iii) What is $\lim_{x\to 0^-} f(x)$?

[40 marks]

(b) Find the following limit if it exists.

(i) $\lim_{t\to 0}\dfrac{\sin 3t}{\sin 2t}$

(ii) $\lim_{x\to 0}\dfrac{3-\sqrt{9+x}}{x}$

[40 marks]

(c) Suppose $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ does not exist. Prove that $\lim_{x\to a}[f(x)+g(x)]$ does not exist.

[20 marks]

1. (a) Andaikan $f(x)=\begin{cases} x^2 & , \quad x>2 \\ 2+x & , \quad 0\le x\le 2 \\ \dfrac{1}{x} & , \quad x<0 \end{cases}$.

(i) Cari $\lim_{x\to 2} f(x)$.

(ii) Kenapa f *selanjar pada* 2 *?*

(iii) Apakah $\lim_{x\to 0^-} f(x)$ *?*

[40 markah]

(b) *Cari had yang berikut jika ia wujud.*

(i) $\lim_{t\to 0}\frac{\sin 3t}{\sin 2t}$

(ii) $\lim_{x\to 0}\frac{3-\sqrt{9+x}}{x}$

[40 markah]

(c) *Andaikan* $\lim_{x\to a} f(x)$ *wujud tetapi* $\lim_{x\to a} g(x)$ *tidak wujud. Buktikan bahawa* $\lim_{x\to a}[f(x)+g(x)]$ *tidak wujud.*

[20 markah]

2. (a) Prove $\lim_{x\to 1}(2x+1)=3$ using the ε - δ -definition.

[30 marks]

(b) Suppose $f(x)=\begin{cases} x^2 & , \quad x\geq 0 \\ x^3 & , \quad x<0 \end{cases}$. Using the definition of derivatives, determine whether $f'(0)$ exists.

[30 marks]

(c) Find the derivative of the function. **Do not simplify your answer.**

(i) $y=x^{\pi}+e^2$

(ii) $y=e^{2x}\sec x$

(iii) $y=\sqrt{\ln(x^2+1)}$

(iv) $y=\frac{\sin x}{x\ln x}$

(v) $y=e^{e^{e^x}}$

[40 marks]

2. (a) *Buktikan bahawa* $\lim_{x\to 1}(2x+1)=3$ *dengan menggunakan takrif* ε - δ .

[30 markah]

(b) *Andaikan bahawa* $f(x)=\begin{cases} x^2 & , \quad x\geq 0 \\ x^3 & , \quad x<0 \end{cases}$. *Dengan menggunakan takrif terbitan, tentukan sama ada* $f'(0)$ *wujud.*

[30 markah]

(c) *Cari terbitan bagi fungsi yang berikut.* ***Jangan permudahkan jawapan anda.***

(i) $y = x^{\pi} + e^2$

(ii) $y = e^{2x}\sec x$

(iii) $y = \sqrt{\ln(x^2+1)}$

(iv) $y = \dfrac{\sin x}{x\ln x}$

(v) $y = e^{e^{e^x}}$

[40 markah]

3. (a) State the Intermediate Value Theorem.

[15 marks]

(b) Show that the equation $x^3 + x = 1$ has a real root between 0 and 1.

[25 marks]

(c) Show that the equation $x^3 + x = 1$ has exactly one real root.

[30 marks]

(d) Find the absolute maximum and absolute minimum of the function $f(x) = x^3 - 3x + 1$ on the closed interval $[0,3]$.

[30 marks]

3. (a) *Nyatakan Teorem Nilai Pertengahan.*

[15 markah]

(b) *Tunjukkan bahawa persamaan* $x^3 + x = 1$ *mempunyai satu punca nyata antara* 0 *dan* 1.

[25 markah]

(c) *Tunjukkan bahawa persamaan* $x^3 + x = 1$ *mempunyai tepat-tepat satu punca nyata.*

[30 markah]

(d) *Cari nilai maksimum mutlak dan minimum mutlak bagi fungsi* $f(x) = x^3 - 3x + 1$ *pada selang tertutup* $[0,3]$.

[30 markah]

4. (a) The region bounded by the graph of $y = x^2$ and $y = x$ is rotated about the y-axis. Compute the volume of the solid obtained using the shell method.

[40 marks]

(b) Find each of the following integrals.

(i) $\int x \sec^2 x \, dx$

(ii) $\int \dfrac{x-4}{x^2 - 5x + 6} \, dx$

[60 marks]

4. (a) *Rantau yang dibatasi oleh graf bagi* $y = x^2$ *dan* $y = x$ *dikisarkan sekitar paksi* y *. Hitungkan isipadu kisaran dengan menggunakan kaedah kerangka.*

[40 markah]

(b) *Cari setiap kamiran yang berikut.*

(i) $\int x \sec^2 x \, dx$

(ii) $\int \dfrac{x-4}{x^2 - 5x + 6} \, dx$

[60 markah]

5. (a) Show that $\tan x \geq x$ whenever $0 \leq x < \frac{\pi}{2}$.

[30 marks]

(b) Find $F'(\frac{\pi}{4})$, where $F(x) = \int_1^{2x} t \sin t \, dt$.

[30 marks]

(c) Does the definite integral $\int_4^7 \frac{x^2}{(x-1)(x-10)} \, dx$ exist? Why?

[15 marks]

(d) Evaluate the telescoping sum $\sum_{i=1}^{99} \left(\frac{1}{i+1} - \frac{1}{i} \right)$.

[25 marks]

5. *(a)* *Tunjukkan bahawa* $\tan x \geq x$ *apabila* $0 \leq x < \frac{\pi}{2}$.

[30 markah]

(b) *Cari* $F'(\frac{\pi}{4})$ *untuk* $F(x) = \int_1^{2x} t \sin t \, dt$.

[30 markah]

(c) *Adakah kamiran tentu* $\int_4^7 \frac{x^2}{(x-1)(x-10)} \, dx$ *wujud? Kenapa?*

[15 markah]

(d) *Hitungkan hasil tambah teleskop* $\sum_{i=1}^{99} \left(\frac{1}{i+1} - \frac{1}{i} \right)$.

[25 markah]

6. (a) Suppose $y = x^x$. Find $\dfrac{dy}{dx}$ using logarithmic differentiation.

[30 marks]

(b) Suppose $f(x) = 2x + \cos x$. Assuming f is one-to-one, find the derivative of f^{-1} at 1.

[20 marks]

(c) Is the following statement true or false? Just write down the correct answer, that is, either "TRUE" or "FALSE".

(i) If f is an even function with domain ¡ , then $f(-x) = f(x)$ for infinitely many real numbers x.

(ii) The limit $\lim\limits_{x \to 0} \sin\left(\dfrac{1}{x}\right)$ does not exist.

(iii) If $\lim\limits_{x \to a} [f(x) + g(x)]$ exists, then both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ must exist.

(iv) If a function f is continuous at a, then f is differentiable at a.

(v) If f is an even function, then $\int_{-3}^{3} f(x)\, dx = 0$.

(vi) There exists $\varepsilon > 0$ such that for every $\delta > 0$, we have $\varepsilon = \delta$.

(vii) If f is differentiable at a, then $\lim\limits_{x \to a} f(x)$ exists.

(viii) If $f(a) = f(b)$ whenever $a = b$, then f is one-to-one.

(ix) If $\lim\limits_{x \to a} f(x) = \infty$, then $\lim\limits_{x \to a} f(x)$ exists.

(x) Continuity of f on $[a, b]$ is sufficient to guarantee the Riemann integrability of f on $[a, b]$.

[50 marks]

6. (a) *Andaikan* $y = x^x$. *Cari* $\frac{dy}{dx}$ *dengan menggunakan cara pembezaan logaritma.*

[30 markah]

(b) *Andaikan* $f(x) = 2x + \cos x$. *Dengan anggapan* f *adalah satu-ke-satu, cari terbitan* f^{-1} *pada 1.*

[20 markah]

(c) *Adakah kenyataan berikut benar atau palsu? Cuma tuliskan jawapan yang betul, iaitu sama ada "BENAR" atau "PALSU".*

(i) *Jika* f *ialah satu fungsi genap dengan domain* $\mathbb{R}$, *maka* $f(-x) = f(x)$ *untuk nombor* x *tak terhingga banyak.*

(ii) *Had* $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ *tidak wujud.*

(iii) *Jika* $\lim_{x \to a}[f(x) + g(x)]$ *wujud, maka kedua-dua* $\lim_{x \to a} f(x)$ *dan* $\lim_{x \to a} g(x)$ *mesti wujud.*

(iv) *Jika suatu fungsi* f *adalah selanjar pada* a , *maka* f *terbezakan pada* a .

(v) *Jika* f *ialah satu fungsi genap, maka* $\int_{-3}^{3} f(x)\, dx = 0$.

(vi) *Wujud* $\varepsilon > 0$ *supaya untuk setiap* $\delta > 0$, $\varepsilon = \delta$.

(vii) *Jika* f *terbezakan pada* a , *maka* $\lim_{x \to a} f(x)$ *wujud.*

(viii) *Jika* $f(a) = f(b)$ *apabila* $a = b$, *maka* f *adalah satu-ke-satu.*

(ix) *Jika* $\lim_{x \to a} f(x) = \infty$, *maka* $\lim_{x \to a} f(x)$ *wujud.*

(x) *Keselanjaran* f *pada* $[a, b]$ *adalah mencukupi untuk memastikan kebolehkamiran Riemann* f *pada* $[a, b]$.

[50 markah]

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