UNIVERSITI SAINS MALAYSIA

First Semester Examination
2016/2017 Academic Session

December 2016 / January 2017

MST 564 - Statistical Reliability
[Kebolehpercayaan Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOURTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

(a) Innovation contests. Reliability workshops. Failure conferences. Some companies/universities send their employees/academics/students to attend such contests, workshops and conferences. Why do you think they adopt this approach?

(b) Reliability analysis lowers failure rate over the long term, reduces producer's warranty costs, and makes the customer want to come back. What do you think are the problems involved in reliability analysis?

(c) Given a set of data, how do we infer the properties of the underlying distribution from which the data have been drawn? For example, we have recorded the times to failure of a number of devices of the same design and manufacture. What can we surmise about the probability distribution of times-to-failure that would emerge if a very large population of all such devices was to be tested to failure?

(d) The behaviour of failure rates with time is quite revealing. Comparisons of human mortality and engineering failures add insight into the three broad classes of failures that give rise to the bathtub curve. Does the bathtub curve apply to our era of disruptions and changes? Provide compelling justification for your answer.

[100 marks]

Soalan 1

(a) Pertandingan Inovasi. Bengkel kebolehpercayaan. Persidangan kegagalan. Beberapa syarikat/universiti menghantar pekerja/ahli akademik/pelajar mereka untuk menghadiri pertandingan, bengkel dan persidangan tersebut. Kenapa anda fikir mereka mengamalkan pendekatan ini?

(b) Analisis kebolehpercayaan mengurangkan kadar kegagalan dalam jangka masa panjang, mengurangkan kos jaminan pengeluar, dan membuat pelanggan mahu kembali. Apakah yang anda fikir adalah masalah yang terlibat dalam analisis kebolehpercayaan?

(c) Diberi satu set data, bagaimanakah kita membuat kesimpulan tentang sifat-sifat taburan asas dari mana data telah diambil? Sebagai contoh, kami telah mencatatkan masa kegagalan beberapa peranti reka bentuk dan pembuatan yang sama. Apakah yang boleh kita menyimpulkan tentang taburan kebarangkalian bagi masa ke-kegagalan yang akan diperolehi jika semua peranti dari sebuah populasi yang sangat besar diuji hingga kegagalan?

(d) Kelakuan kadar kegagalan dengan masa agak mendedahkan. Perbandingan kematian manusia dan kegagalan kejuruteraan menambah wawasan ke dalam tiga kelas yang luas kegagalan yang menimbulkan lengkung tab mandi. Adakah lengkung tab mandi diaplikasi kepada zaman kita yang terdiri daripada gangguan dan perubahan? Berikan justifikasi yang menarik untuk jawapan anda.

[100 markah]

Question 2

(a) Ten devices were tested. The test was stopped at the occurrence of the fourth failure. The hours to failure were 16, 40, 180 and 300. Six devices ran 300 hours without failure. Assume these lifetime data follow an exponential distribution.

(i) Calculate the MTBF.
(*Hint*: Estimate of MTBF = total operating hours / number of devices failed)

(ii) If the sample MTBF is 584 hours, should the manufacturer guarantee repairs for failures up to 584 hours in the field?
(*Hint*: Estimated reliability is $e^{-t/MTBF}$)

(b) The fatigue lives of 20 specimens, measured in thousands of stress cycles are found to be

3.1	15.5	25.3	35.4	40.1
6.1	20.9	30.5	35.9	65.5
7.3	21.7	31.4	38.9	70.9
10.4	21.9	32.7	39.6	98.7

(i) Fit a probability plot for these data.

(ii) Estimate the parameters in the fitted probability model.

(iii) Use the fitted model to estimate the probability that a specimen will fail in less than 24.2 thousand cycles.

[100 marks]

Soalan 2

(a) Sepuluh peranti telah diuji. Ujian ini telah dihentikan pada kegagalan keempat. Waktu kegagalan adalah 16, 40, 180 dan 300 jam. Enam peranti berlari 300 jam tanpa kegagalan. Andaikan data jangka hayat ini mengikuti taburan eksponen.

(i) Kirakan MTBF.
(Petunjuk: Anggaran MTBF = jumlah jam operasi / bilangan peranti gagal)

(ii) Jika sampel MTBF adalah 584 jam, perlukah pengeluar menjamin pembaikan untuk kegagalan sehingga 584 jam di lapangan?
(Petunjuk: Anggaran kebolehpercayaan adalah $e^{-t/MTBF}$)

(b) Kehidupan keletihan 20 spesimen diukur dalam ribuan kitaran tekanan didapati

3.1	*15.5*	*25.3*	*35.4*	*40.1*
6.1	*20.9*	*30.5*	*35.9*	*65.5*
7.3	*21.7*	*31.4*	*38.9*	*70.9*
10.4	*21.9*	*32.7*	*39.6*	*98.7*

(i) Suaikan plot kebarangkalian untuk data ini.

(ii) Anggarkan parameter dalam model kebarangkalian yang disuaikan.

(iii) Gunakan model yang disuaikan untuk menganggarkan kebarangkalian bahawa spesimen akan gagal dalam masa kurang dari 24.2 kitaran.

[100 markah]

Question 3

(a) Life Data for 206 turbine disks at 100 hour intervals are given in the following table.

hours	**failures**	**censor**	**hours**	**failures**	**censor**
0-200	0	4	1000-1200	0	18
200-300	1	2	1200-1300	2	5
300-400	1	11	1300-1400	1	13
400-500	3	10	1400-1500	0	14
500-700	0	32	1500-1600	1	14
700-800	1	10	1600-1700	1	14
800-900	0	11	1700-2000	0	5
900-1000	1	9	2000-2100	1	2

Since the censoring takes place randomly, use a suitable method to plot the reliability and cumulative hazard function versus time. Discuss your results.

(b) In an experiment to gain information on the strength, Y , in coded units, of a certain type of braided cord after testing, the strengths of 48 pieces of cord that had been tested for a specified length of time were investigated (see **Table 1**). The intention was to obtain the strengths of all 48 pieces of cord. However, seven pieces were damaged during the course of the experiment, so that their actual strengths would be greater than the values actually recorded.

Table 1. Cord Strength

Not damaged									
36.3	41.7	43.9	49.9	50.1	50.8	51.9	52.1	52.3	52.3
52.4	52.6	52.7	53.1	53.6	53.6	53.9	53.9	54.1	54.6
54.8	54.8	55.1	55.4	55.9	56.0	56.1	56.5	56.9	57.1
57.1	57.3	57.7	57.8	58.1	58.9	59.0	59.1	59.6	60.4
60.7									
Damaged									
26.8	29.6	33.4	35.0	40.0	41.9	42.5			

(i) Discuss whether the data in Table 1 have been left- or right-censored.

(ii) The experimenter thought that, even after testing, the cord strength should be above 53 in coded units. Determine the Kaplan-Meier estimate of $S(53)$ where S is the survival function of Y. What is your conclusion?

[100 marks]

Soalan 3

(a) Data hayat untuk 206 cakera turbin pada selang 100 jam diberikan dalam jadual berikut:

jam	***kegagalan***	***tapis***	***jam***	***kegagalan***	***jam***
0-200	*0*	*4*	*1000-1200*	*0*	*18*
200-300	*1*	*2*	*1200-1300*	*2*	*5*
300-400	*1*	*11*	*1300-1400*	*1*	*13*
400-500	*3*	*10*	*1400-1500*	*0*	*14*
500-700	*0*	*32*	*1500-1600*	*1*	*14*
700-800	*1*	*10*	*1600-1700*	*1*	*14*
800-900	*0*	*11*	*1700-2000*	*0*	*5*
900-1000	*1*	*9*	*2000-2100*	*1*	*2*

Sejak penapisan itu berlaku secara rawak, gunakan kaedah yang sesuai untuk memplot kebolehpercayaan dan fungsi bahaya kumulatif melawan masa. Bincangkan keputusan anda.

(b) Dalam satu eksperimen untuk mendapatkan maklumat mengenai kekuatan, dalam unit pengkodan, jenis tertentu tali jalinan selepas ujian, kekuatan 48 keping tali yang telah diuji untuk tempoh masa yang tertentu telah disiasat (lihat Jadual 1). Tujuannya adalah untuk mendapatkan kekuatan semua 48 keping tali. Walau bagaimanapun, tujuh keping rosak semasa eksperimen, supaya kekuatan sebenar mereka akan lebih besar daripada nilai sebenarnya direkodkan.

***Jadual 1**. Kekuatan tali.*

Tidak rosak									
36.3	*41.7*	*43.9*	*49.9*	*50.1*	*50.8*	*51.9*	*52.1*	*52.3*	*52.3*
52.4	*52.6*	*52.7*	*53.1*	*53.6*	*53.6*	*53.9*	*53.9*	*54.1*	*54.6*
54.8	*54.8*	*55.1*	*55.4*	*55.9*	*56.0*	*56.1*	*56.5*	*56.9*	*57.1*
57.1	*57.3*	*57.7*	*57.8*	*58.1*	*58.9*	*59.0*	*59.1*	*59.6*	*60.4*
60.7									
Rosak									
26.8	*29.6*	*33.4*	*35.0*	*40.0*	*41.9*	*42.5*			

(i) Bincangkan sama ada data dalam Jadual 1 adalah ditapis kiri atau kanan.

(ii) Penguji kaji berfikir bahawa, walaupun selepas ujian, kekuatan tali perlu lebih daripada 53 dalam unit berkod. Tentukan anggaran Kaplan-Meier bagi $S(53)$, di mana S ialah fungsi hayat. Apakah kesimpulan anda?

[100 markah]

Question 4

(a) Consider the severe viral hepatitis data from a clinical trial in which a steroid (Group 1) and control group (Group 2) were compared. Lifetime in weeks were measured as given in **Table 2**. Here, the notation 4+ indicates an observed right-censored time at 4 weeks and frequencies are given in parentheses. Group 1 has 14 data points and Group 2 has 15 data points.

Table 2. Severe viral hepatitis data.

Group 1	Group 2
1(3)	1+
1+	2+
4+	3(2)
5	3+
7	5+(2)
8	16+(8)
10	
10+	
12+	
16+(3)	

(i) Fit a Cox proportional hazards model to the severe viral hepatitis data in **Table 2**.

(ii) Use the statistical package you fancy to obtain the estimates of the fitted survival functions for each group.

(iii) What is your overall conclusion?

(b) **Table 3** shows the data collected from 500 subjects of the Worcester Heart Attack Study. The main goal of this study is to describe factors associated with trends over time in the incidence and survival rates following hospital admission for acute myocardial infarction.

(i) What kind of questions would be of interest for the data analysis of this heart attack study?

(ii) Perform an appropriate analysis of the data in **Table 3** (use the data file provided).

(iii) Interpret the results obtained in (ii). What can you conclude?

Table 3. Data of 500 subjects (first 5 cases displayed).

Patient	Lenfol (days)	Status	Age (yrs)	Gender	HR	BMI
1	2178	0	83	0	89	26
2	2172	0	49	0	84	24
3	2190	0	70	1	83	22
4	297	1	70	0	65	27
5	2131	0	57	0	73	39

Note that:

Lenfol (Length of follow up, terminated due to death or lost to follow up)
Status (0 = lost to follow up, 1 = dead)
Age (Age during hospitalisation)
Gender (0 = Male, 1 = Female)
HR (Initial heart rate)
BMI (Body Mass Index)

[100 marks]

Soalan 4

(a) Pertimbangkan data virus hepatitis yang teruk dari percubaan klinikal di mana steroid (Kumpulan 1) dan kumpulan kawalan (Kumpulan 2) dibandingkan. Umur hidup dalam beberapa minggu diukur seperti yang diberikan dalam Jadual 2. Di sini, notasi 4+ menunjukkan masa kanan ditapis yang diperhatikan pada 4 minggu dan frekuensi diberikan dalam kurungan. Kumpulan 1 mempunyai 14 titik data dan Kumpulan 2 mempunyai 15 titik data.

***Jadual 2**. Data virus hepatitis yang teruk.*

Kumpulan 1	***Kumpulan 2***
1(3)	*1+*
1+	*2+*
4+	*3(2)*
5	*3+*
7	*5+(2)*
8	*16+(8)*
10	
10+	
12+	
16+(3)	

*(i) Suaikan model bahaya berkadaran Cox kepada data virus hepatitis yang teruk dalam **Jadual 2**.*

(ii) Gunakan pakej statistik yang anda gemar untuk mendapatkan anggaran fungsi hayat yang disuaikan bagi setiap kumpulan.

(iii) Apakah kesimpulan anda secara keseluruhan?

*(b) **Jadual 3** menunjukkan data yang diperoleh daripada 500 pesakit dari Kajian Serangan Jantung Worcester. Matlamat utama kajian ini adalah untuk menjelaskan faktor-faktor yang berkaitan dengan trend masa ke masa dalam insiden dan kadar hayat berikutan kemasukan hospital untuk sakit jantung.*

(i) Apakah jenis soalan yang akan menarik minat untuk analisis data ini dalam kajian serangan jantung?

(ii) Laksanakan analisis yang sesuai untuk data dalam Jadual 3 (menggunakan fail data yang disediakan).

(iii) Tafsirkan keputusan yang diperolehi dalam (ii). Apakah kesimpulan?

Jadual 3. Data 500 pesakit (5 kes pertama ditunjukkan).

Pesakit	***Lenfol (hari)***	***Status***	***Umur (tahun)***	***Jantina***	***HR***	***BMI***
1	*2178*	*0*	*83*	*0*	*89*	*26*
2	*2172*	*0*	*49*	*0*	*84*	*24*
3	*2190*	*0*	*70*	*1*	*83*	*22*
4	*297*	*1*	*70*	*0*	*65*	*27*
5	*2131*	*0*	*57*	*0*	*73*	*39*

Perhatikan bahawa:

Lenfol (panjang susulan, ditamatkan disebabkan oleh kematian atau hilang susulan)
Status (0 = hilang dari susulan, 1 = mati)
Umur (umur ketika di hospital)
Jantina (0 = lelaki, 1 = perempuan)
HR (kadar jantung awal)
BMI (Body Mass Index)

[100 markah]

APPENDIX

Summary of Reliability Formulae

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(t)dt$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

APPENDIKS

Ringkasan Rumus-Rumus Kebolehpercayaan

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(t)dt$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following an **Exponential Distribution:**

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \lambda$$
$$H(t) = \lambda t$$
$$MTTF = \frac{1}{\lambda}$$

Lifetime following a **Weibull Distribution:**

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha} \right)^{\beta} \right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

$$H(t) = \left(\frac{t}{\alpha} \right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta} \right)$$

$$\text{Design Life} = t_R = \alpha(-\ln R)^{1/\beta}$$

APPENDIKS (sambung)

Ringkasan Rumus-Rumus Kebolehpercayaan (sambung)

Masahayat mengikut ***Taburan Eksponen:***

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \lambda$$
$$H(t) = \lambda t$$
$$MTTF = \frac{1}{\lambda}$$

Masahayat mengikut ***Taburan Weibull:***

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha} \right)^{\beta} \right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

$$H(t) = \left(\frac{t}{\alpha} \right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta} \right)$$

$$\text{Design Life} = t_R = \alpha(-\ln R)^{1/\beta}$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following a **Normal Distribution:**

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2} \right]$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$h(t) = \frac{f(t)}{1 - \Phi\left(\frac{t-\mu}{\sigma} \right)}$$

Lifetime following a **Lognormal Distribution:**

$$f(t) = \frac{1}{\sqrt{2\pi}st} \exp\left[-\frac{1}{2s^2} \left(\ln \frac{t}{t_{median}} \right)^2 \right]$$

$$F(t) = \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$MTTF = t_{median} \exp\left(\frac{s^2}{2} \right)$$

$$t_R = t_{median} \exp\left(s z_{1-R} \right)$$

APPENDIKS (sambung)

Ringkasan Rumus-Rumus Kebolehpercayaan (sambung)

Masahayat mengikut ***Taburan Normal:***

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2} \right]$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$h(t) = \frac{f(t)}{1 - \Phi\left(\frac{t-\mu}{\sigma} \right)}$$

Masahayat mengikut ***Taburan Lognormal:***

$$f(t) = \frac{1}{\sqrt{2\pi}st} \exp\left[-\frac{1}{2s^2} \left(\ln \frac{t}{t_{median}} \right)^2 \right]$$

$$F(t) = \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$MTTF = t_{median} \exp\left(\frac{s^2}{2} \right)$$

$$t_R = t_{median} \exp\left(s z_{1-R} \right)$$

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