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UNIVERSITI SAINS MALAYSIA

First Semester Examination  
Academic Session 2016/2017

December 2016 / January 2017

**MAA 101 - Calculus for Science Students I**  
***[Kalkulus untuk Pelajar Sains I]***

Duration : 3 hours  
*[Masa : 3 jam]*

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Please check that this examination paper consists of TEN pages of printed materials before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **EIGHT** (8) questions.

**Arahan:** Jawab **LAPAN** (8) soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

1. (a) Solve the inequality:  $\frac{3x}{x^2+2} \geq \frac{1}{x-1}$ .

[ 15 marks ]

(b) Suppose  $f(x) = \sqrt{x-6}$  and  $g(x) = |2x-1|$ .

(i) Find  $f^{-1}(x)$  and its domain.

(ii) Find the domain and range of  $(f \circ g)(x)$ .

(iii) Evaluate the limit  $\lim_{x \rightarrow 0.5} \frac{g(x)}{1-2x}$ .

[ 35 marks ]

1. (a) *Selesaikan ketaksamaan:*  $\frac{3x}{x^2+2} \geq \frac{1}{x-1}$ .

[ 15 markah ]

(b) *Andaikan*  $f(x) = \sqrt{x-6}$  *and*  $g(x) = |2x-1|$ .

(i) *Cari*  $f^{-1}(x)$  *dan domainnya*.

(ii) *Cari domain dan julat bagi*  $(f \circ g)(x)$ .

(iii) *Cari nilai*  $\lim_{x \rightarrow 0.5} \frac{g(x)}{1-2x}$ .

[ 35 markah ]

2. (a) (i) Determine the existence of  $\lim_{x \rightarrow \pi/4} \frac{2 \sin x + \sqrt{2}}{2 \sin x - \sqrt{2}}$ .

(ii) Let  $f(x) = \begin{cases} x^2 & x \leq -2 \\ mx + n & -2 < x < 2 \\ 2x - 6 & x \geq 2 \end{cases}$ .

Find the values  $m$  and  $n$  so that  $f$  is continuous everywhere.

[ 30 marks ]

(b) Consider the function

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ 2x - 2 & 1 < x \leq 2 \\ -x^2 + 4x - 2 & x > 2 \end{cases}$$

For what values of  $x$  is  $f$  differentiable?

[ 20 marks ]

2. (a) (i) Tentukan kewujudan  $\lim_{x \rightarrow \pi/4} \frac{2 \sin x + \sqrt{2}}{2 \sin x - \sqrt{2}}$ .

(ii) Andaikan  $f(x) = \begin{cases} x^2 & x \leq -2 \\ mx + n & -2 < x < 2 \\ 2x - 6 & x \geq 2 \end{cases}$

Cari nilai-nilai  $m$  dan  $n$  supaya  $f$  adalah selanjar pada semua titik.

[ 30 markah ]

(b) Pertimbangkan fungsi

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ 2x - 2 & 1 < x \leq 2 \\ -x^2 + 4x - 2 & x > 2 \end{cases}$$

Untuk nilai  $x$  apakah  $f$  adalah terbezakan?

[ 20 markah ]

3. (a) Let  $f(x) = \frac{6}{x^2 + 1}$ .

Use the definition of the derivative to find  $f'(x)$ .

[ 15 marks ]

(b) Find the derivative of the following functions:

(i)  $f(x) = (x^2 + 1) \tan^{-1}(x^2 + 1)$

(ii)  $f(x) = \frac{1 - \ln x}{1 + \ln x}$

(iii)  $f(x) = \frac{x^9 \cos(5x)}{7^x \sqrt{1+x^2}}$

[ 20 marks ]

(c) Consider the curve with equation  $y^2 = x^2(x+1)$ .

Find the equation of tangent line to curve at the point  $(3, -6)$ .

[ 15 marks ]

3. (a) *Andaikan*  $f(x) = \frac{6}{x^2 + 1}$ .

*Gunakan takrif bagi terbitan untuk mencari  $f'(x)$ .*

[ 15 markah ]

(b) *Cari terbitan bagi fungsi berikut:*

(i)  $f(x) = (x^2 + 1) \tan^{-1}(x^2 + 1)$

(ii)  $f(x) = \frac{1 - \ln x}{1 + \ln x}$

(iii)  $f(x) = \frac{x^9 \cos(5x)}{7^x \sqrt{1+x^2}}$

[ 20 markah ]

(c) *Pertimbangkan lengkung dengan persamaan  $y^2 = x^2(x+1)$ .*

*Dapatkan persamaan garis tangen terhadap lengkung pada titik  $(3, -6)$ .*

[ 15 markah ]

4. (a) Given  $f(x) = \frac{(x+1)^2}{x^2+1}$  with  $f'(x) = \frac{2(1-x^2)}{(x^2+1)^2}$  and  $f''(x) = \frac{4x(x^2-3)}{(x^2+1)^3}$ .

Find:

- (i) all the  $x$ - and  $y$ -intercepts and also the asymptotes;
- (ii) the intervals on which  $f$  is increasing or decreasing;
- (iii) all the local maximum and minimum value of  $f$ , if any;
- (iv) the intervals of concavity and the inflection points, if exist;
- (v) sketch the graph of  $f$ .

[ 25 marks ]

(b) Evaluate the following limits:

(i) 
$$\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$$

(ii) 
$$\lim_{x \rightarrow 3} \left( \frac{1}{x-3} - \frac{6}{x^2-9} \right)$$

(iii) 
$$\lim_{x \rightarrow 0} [1 + \tan(3x)]^{1/x}$$

[ 15 marks ]

(c) Let  $f(x) = e^{x-2} + x^3 - 2$ . Using Newton's method with initial value,  $x_0 = 2$ , find the second approximation,  $x_2$  for the solution of  $f(x) = 0$ .

[ 10 marks ]

4. (a) Diberi  $f(x) = \frac{(x+1)^2}{x^2+1}$  dengan  $f'(x) = \frac{2(1-x^2)}{(x^2+1)^2}$  dan

$$f''(x) = \frac{4x(x^2-3)}{(x^2+1)^3}. \text{ Cari:}$$

- (i) semua pintasan- $x$  dan  $-y$  dan juga asimptot;
- (ii) selang yang mana  $f$  menokok atau menyusut;
- (iii) semua nilai maksimum dan minimum tempatan bagi  $f$ , jika ada;
- (iv) selang kecekungan dan titik lengkung balas, jika wujud;
- (v) lakarkan graf bagi  $f(x)$ .

[ 25 markah ]

(b) *Nilaikan had berikut:*

$$(i) \quad \lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$$

$$(ii) \quad \lim_{x \rightarrow 3} \left( \frac{1}{x-3} - \frac{6}{x^2-9} \right)$$

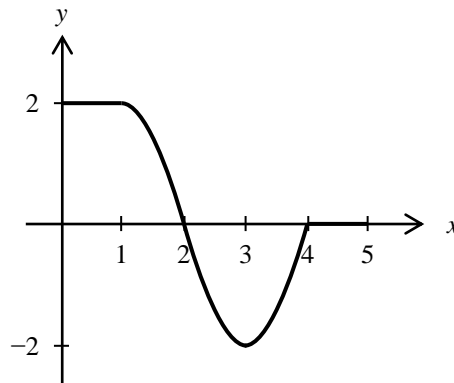
$$(iii) \quad \lim_{x \rightarrow 0} [1 + \tan(3x)]^{1/x}$$

[ 15 markah ]

(c) *Andaikan  $f(x) = e^{x-2} + x^3 - 2$ . Gunakan kaedah Newton dengan nilai awal,  $x_0 = 2$ , cari nilai anggaran kedua,  $x_2$  bagi penyelesaian  $f(x) = 0$ .*

[ 10 markah ]

5. (a) Figure below shows the graph of a function  $f$ . Sketch the graph of an antiderivative of  $f$ , given that the antiderivative function starts at the origin of coordinate.

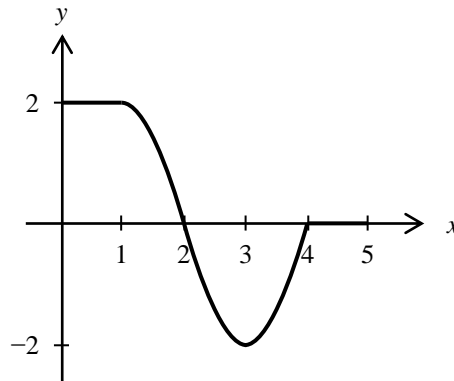


[ 25 marks ]

- (b) Find the average value of the function  $f(x) = \frac{\ln x}{2x}$  on the interval  $[1, e]$ .

[ 25 marks ]

5. (a) *Rajah di bawah menunjukkan graf suatu fungsi  $f$ . Lakarkan graf anti-terbitan bagi  $f$ , diberikan bahwa fungsi anti-terbitan bermula pada asalan.*



[ 25 markah ]

- (b) *Cari nilai purata fungsi  $f(x) = \frac{\ln x}{2x}$  pada selang  $[1, e]$ .*

[ 25 markah ]

6. (a) Evaluate the indefinite integrals

(i)  $\int \ln(x^2 + 2) dx$

[ 15 marks ]

(ii)  $\int \frac{x^2 - x}{\sqrt{1 - x^2}} dx.$

[ 15 marks ]

(b) Use the midpoint approximation to calculate the integral  $\int_0^4 \sqrt{x} \sin x dx$  with 5 subintervals. Round your answer to 4 decimal places.

[ 20 marks ]

6. (a) *Nilaikan kamiran tak tentu*

(i)  $\int \ln(x^2 + 2) dx$

[ 15 markah ]

(ii)  $\int \frac{x^2 - x}{\sqrt{1 - x^2}} dx.$

[ 15 markah ]

(b) *Gunakan penghampiran titik tengah untuk mengira kamiran  $\int_0^4 \sqrt{x} \sin x dx$  dengan 5 subselang. Bulatkan jawapan anda ke 4 tempat perpuluhan.*

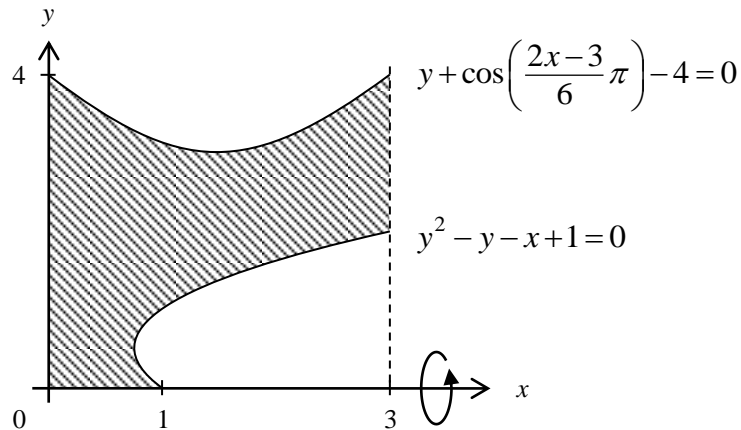
[ 20 markah ]



7. (a) Sketch the region enclosed by the curves  $y = 1 + 4x - x^2$  and  $y = |x - 2|$ . Find its area.

[ 25 marks ]

- (b) Find the volume generated by rotating the region shown in the figure below about the  $x$ -axis.

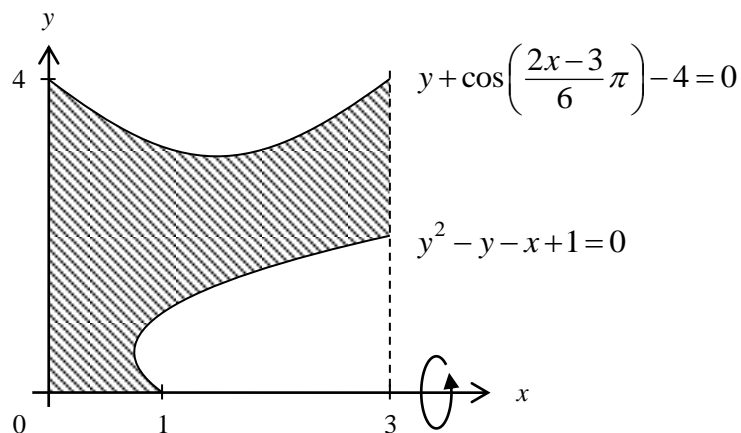


[ 25 marks ]

7. (a) *Lakarkan rantau yang dibatasi oleh lengkung-lengkung  $y = 1 + 4x - x^2$  dan  $y = |x - 2|$ . Cari luas rantau itu.*

[ 25 markah ]

- (b) *Cari isipadu yang terjana dengan memutarakan rantau yang ditunjukkan dalam rajah di bawah sekitar paksi- $x$ .*



[ 25 markah ]

8. (a) Find the coordinates of the leftmost point on the curve  $x = \sin t + 2 \sin 2t$ ,  $y = \cos t + 2 \cos 2t$ , where  $0 \leq t \leq 2\pi$ .

[ 25 marks ]

- (b) Find the length of the curve  $x = 2 - t$ ,  $y = e^{\frac{t}{2}} + e^{-\frac{t}{2}}$ ,  $-2 \leq t \leq 2$ .

[ 25 marks ]

8. (a) Cari koordinat titik paling kiri pada lengkung  $x = \sin t + 2 \sin 2t$ ,  $y = \cos t + 2 \cos 2t$ , yang mana  $0 \leq t \leq 2\pi$ .

[ 25 markah ]

- (b) Cari panjang lengkok untuk lengkung  $x = 2 - t$ ,  $y = e^{\frac{t}{2}} + e^{-\frac{t}{2}}$ ,  $-2 \leq t \leq 2$ .

[ 25 markah ]