

**FORECASTING STOCK MARKET VOLATILITY  
USING WAVELET TRANSFORMATION  
ALGORITHM OF GARCH MODEL**

by

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## LIST OF ABBREVIATIONS

ARCH	Autoregressive conditional heteroscedasticity;
EGARCH	Exponential generalized autoregressive conditional heteroscedasticity;
GARCH	Generalized autoregressive conditional heteroscedasticity;
DWT	Discrete wavelet transformation;
MODWT	Maximal overlap discrete wavelet transform;
$w_1$	Wavelet coefficient of scale $j = 1$ ;

**PERAMALAN KEMERUAPAN PASARAN SAHAM  
MENGUNAKAN ALGORITMA TRANSFORMASI WAVELET BAGI  
MODEL GARCH**

**ABSTRAK**

Kemeruapan pasaran saham adalah perkara penting terutamanya kepada dua pihak berkepentingan. Pengamal melalui kanta mata sendiri melihat pandangan tentang kesan kelakuan harga aset dan risiko. Sebaliknya, pembuat dasar dibebani dengan insiden cabaran kewangan dan ketidakstabilan makroekonomi yang ditimbulkan oleh fenomena pasaran saham. Kebimbangan tinggi terhadap kesan dwi turun naik kemeruapan pasaran saham berlaku terutamanya bagi negara-negara membangun dengan pasaran saham yang masih muda yang mempunyai ciri-ciri kelemahan. Ini kerana indeks pasaran ekonomi membangun tidak seperti rakan-rakan ekonomi yang lebih maju. Seterusnya, disebabkan oleh jurang yang wujud dalam konteks yang berbeza, penggunaan model tunggal dalam menjelaskan kesan kemeruapan pasaran saham mungkin menjadi pertikaian atau tidak tepat. Oleh yang demikian, objektif kajian ini adalah untuk membangunkan algoritma transformasi yang boleh menerangkan lebih baik tingkah laku pasaran saham negara-negara Afrika berbanding model kemeruapan konvensional yang sedia ada. Kajian ini menggunakan pulangan harian indeks pasaran saham empat negara Afrika bagi tempoh dari 2 Januari 2000 hingga 31 Disember 2014, untuk membandingkan prestasi ramalan bagi model linear GARCH(1,1) dan model wavelet diskret maksimum bertindih(MODWT)-GARCH(1,1) yang baharu dicadangkan. Keputusan menunjukkan bahawa walaupun kedua-dua model menyuai data pulangan dengan baik, ramalan yang dihasilkan oleh model GARCH(1,1) adalah kurang anggaran pulangan yang dicerap sedangkan model cadangan baharu MODWT-GARCH (1,1) menjana nilai ramalan tepat bagi pulangan

yang dicerap. Selain itu, ramai penyelidik menemui bahawa jika siri pulangan pasaran saham adalah terpencong secara bererti, model linear-GARCH(1,1) menghasilkan nilai anggaran ramalan pulangan yang rendah. Walau bagaimanapun, kajian ini menunjukkan bahawa apabila model simetri MODWT-GARCH(1,1) dan model asimetri MODWT-EGARCH (1,1) dibandingkan, keputusan membuktikan bahawa walaupun kedua-dua model menyuai data pulangan dengan baik, ramalan yang dihasilkan oleh model MODWT-EGARCH(1,1) sebenarnya kurang anggaran pulangan yang dicerap manakala model MODWT-GARCH(1,1) menjana nilai ramalan yang pulangan yang lebih tepat. Selain itu, dapatan kajian yang penting adalah dibuktikan bahawa model linear MODWT-GARCH(1,1) memberikan ramalan kemeruapan yang tepat bagi pasaran saham negara-negara Afrika berbanding model linear tradisional GARCH(1,1). Amat penting juga bahawa peramalan model simetri MODWT-GARCH(1,1) turut lebih baik daripada model asimetri MODWT-EGARCH(1,1) bagi peramalan lima hari purata pulangan pasaran saham negara-negara Afrika. Kajian ini mencadangkan bahawa metodologi yang digunakan dalam kajian ini hendaklah digunakan ke negara lain yang mempunyai ciri-ciri pasaran saham sama seperti Amerika Latin, beberapa negara-negara mundur di Eropah Selatan dan Asia di samping diaplikasi kepada konteks pemodelan multivariat kemeruapan di negara-negara masing-masing.

# **FORECASTING STOCK MARKET VOLATILITY USING WAVELET TRANSFORMATION ALGORITHM OF GARCH MODEL**

## **ABSTRACT**

Stock market volatility is of essential concern, particularly to two major stakeholders. While the practitioner looks through his own lenses with the bird's-eye-view, he or she bothers himself or herself about the consequences of this behaviour on asset pricing and risk. Conversely, policy makers are burdened with the incidence of financial challenges and macroeconomic instability posed by the stock market phenomenon. Of optimum concern of these dual effects of stock market volatility, emanates predominantly from developing countries with infant stock markets, characterized by vulnerabilities. This is because, developing economies may parade market indices not explicitly possessed by their superior developed counterparts. Consequently, due to the disparities inherent in different context, the application of a single-most model in unravelling the effects of stock market volatility may be contentious or inaccurate. Therefore, the objective of this study is to develop a hybrid volatility models using the maximal overlap discrete wavelet transform algorithm (MODWT) denoising and decomposition tool that can aptly capture these African countries' stock markets behaviour better than the existing conventional volatility models. The study therefore uses the daily returns of four African countries' stock market indices for the period from January 2, 2000, to December 31, 2014, to compare the forecasting performance of the symmetric GARCH(1,1) model and the forecasting performance of the newly proposed Maximal Overlap Discreet Wavelet Transform-GARCH(1,1) model. The results show that although both models fit the returns data well, the forecast produced by the GARCH(1,1) model underestimates the original returns whereas the newly proposed MODWT-GARCH(1,1) model generates an

accurate forecast value of the original returns. Additionally, many researchers documented that if stock markets' returns series are significantly skewed, linear-GARCH(1,1) grossly underestimates the forecast values of the returns. However, this study showed that when the symmetric MODWT-GARCH(1,1) model and the asymmetric MODWT-EGARCH(1,1) model are exhaustively compared, the results proved that although both models fit the returns data well, the forecast produced by the MODWT-EGARCH(1,1) model actually underestimates the original returns whereas the MODWT-GARCH(1,1) model generates an accurate forecast value of the original returns. There is an overwhelming evidence from the findings in the research that the symmetric MODWT-GARCH(1,1) model gives an accurate forecast volatility of the African countries' stock markets more than the traditional linear GARCH(1,1) model. Of great importance, is that the symmetric MODWT-GARCH(1,1) model exceeds the asymmetric MODWT-EGARCH(1,1) model in given the accurate five days average forecast values of the stock markets of African countries. The research recommends that the methodology employed in the study should be applied to other countries that have similar stock markets' characteristics such as the Latin America, some underdeveloped countries in Southern Europe and Asia in addition to applying the phenomenon on the multivariate context of volatility modelling in these respective countries.

## CHAPTER 1

### INTRODUCTION

#### 1.1 Background of the study

Stock market volatility is of paramount importance to both market practitioners and policy makers, particularly for emerging countries (Levine, 1998 and Yu, 2002). The practitioner is concerned about stock market volatility because it affects asset pricing and risk, whereas the policy maker attempts to curb excessive volatility to ensure financial and macroeconomic stability (Chinzara, 2011). In both cases, an efficient quantitative tool for modeling stock market volatility is needed to minimize the risk of inaccurate measurement. In this regard, researchers continue to search for the best volatility model that is able to capture various stylized facts associated with market volatilities.

The volatility modeling of price returns was first performed by Engle (1982), wherein an autoregressive conditional heteroskedasticity model, the ARCH model, was used to predict UK inflation rate uncertainty. Engle noted that major changes tend to be followed by significant changes in either sign and that small changes tend to be followed by small changes. This phenomenon was designated volatility clustering. The author measured clustering effects based on an assumption of constant conditional return mean value.

However, other stylized volatility features could not be captured by the ARCH model. Bollerslev (1986) generalized the ARCH model by creating the Generalized Conditionally Heteroskedasticity model (GARCH model). The model

considerably extended the capacities of the ARCH model to account for stylized aspects of return volatility, given that it removed the excess kurtosis in returns series. However, the GARCH model, which is actually a linear model, could not address the fat-tailed distributions of financial time series.

The Exponential GARCH (EGARCH) model originated by Nelson (1991), the Quadratic GARCH (QGARCH) model originated by Engle et. al. (1993), and other models such as the Glosten, Jaganathan, and Runkle (GJR) model; Jaganathan and Runkle (2003), are known as non-linear GARCH models, and they address the skewed distributions of financial time series, which are a very common characteristic of financial time series.

Furthermore, stock market returns are practically influenced by agent speculations and investor decisions over different time horizons that range from minutes to years. In such a situation, a useful tool of analysis may be wavelet analysis (Gallegati, 2008).

Wavelets are particular types of function that are localized both in time and frequency domain that are utilized in the decomposition of time series into additional elementary functions containing various information relating to the time series. Within the numerous utilized statistical signal extraction and filtering methods, in addition to denoising methods, wavelets constitute just one tool. The ability to decompose macroeconomic time series into components of their time scale, is a major advantage of wavelet analysis. Haar (Discrete), symmlets and coiflets (symmetric), daiblet (asymmetric), among others, make up the different categories of the available wavelets filters; they differ in their filter transfer function and filter

lengths in terms of characteristics. This study is based on the Maximal Overlap Discrete Wavelet Transform (MODWT) tool. The MODWT represents an improvement on the Discrete Wavelet Transform (DWT). Through the simple modification of the pyramid algorithm utilized in computing DWT coefficient, the MODWT is obtained; and it is perceived as the DWT universal set. The MODWT, among other comparative advantages over the DWT, can accommodate any sample size; in addition, in terms of data filtering starting point of a time series, it is insensitive (Gallegati, 2008).

The MODWT filtering method offers insights into the dynamics of financial time series beyond those revealed through existing methodologies (Gencay et al., 2001). A number of concepts, such as those of nonstationarity, multiresolution and approximate decorrelation, emerge from MODWT filters. Moreover, MODWT filters serve as a straightforward tool for studying the multiresolution properties of a process. They can also decompose a financial time series into different time scales, given that they reveal structural break and volatility clusters and identify the local and global dynamic properties of a process at such time scales. In addition, MODWT filters can conveniently dissolve the correlation structure of a process across time scales.

A book by Gencay et al. (2001) applied the DWT to daily IBM stock return series and found a large group of rapidly fluctuating returns between observations at certain intervals of the wavelet coefficients. They observed that, at the same frequency level, there were significant fluctuations in wavelet coefficient  $w_1$  and a small increase in fluctuations of wavelet coefficient  $w_2$  and that wavelet coefficients  $w_3$  and  $w_4$  were essentially zero. A study by Conejo et al. (2005) employed a time

series analysis, a neural network and wavelet forecasting technique that predicts 24 market-clearing prices of a day-ahead electric energy market by using PJM Interconnection data. They exhaustively compared the forecasting errors generated from the techniques and recommended the study of combined wavelet transform and time series algorithms in future research. A study by Liu et al. (2013) presented two hybrid forecasting frameworks, the Wavelet-Genetic Algorithm (GA)-Multilayer Perceptron (MLP) and the Wavelet-Particle Swarm Optimization (PSO)-Multilayer Perceptron (MLP), for predicting non-stationary wind speeds and for comparing the forecasting performance of the different algorithm combinations of the two hybrid frameworks. Their results, based on three experimental cases, show that among other results, in both of the hybrid frameworks, the contributions of the GA and PSO components to improving the MLP were not significant whereas those of the wavelet component were significant.

A similar study by Tan et al. (2010) proposed a novel price forecasting method based on wavelet transform approaches combined with ARIMA and GARCH models, and it was compared with some of the most recently published price forecasting techniques. The comparative results clearly showed that the proposed forecasting method was far more accurate than the other forecasting method.

## **1.2 Statement of the Problem**

Although several articles on the stock price volatility levels of developed capital markets have been published, scarce research has been conducted on this subject with respect to African markets. According to Gokcan (2000), African stock

markets are some of the important emerging markets that are now attracting global investors. However, despite offering attractive investment avenues, investors are wary of the volatility risks associated with these markets due to the difference between their risk and return characteristics and those of developed markets. The risks of investing in these markets are more severe, and investors tend to lose money due to forecasting errors that arise from a lack of proper telecommunication and transportation resources and due to different accounting system characteristics. Therefore, it is necessary to exhaustively and efficiently seek a methodology that will considerably reduce the amount of errors made in forecasting the returns of these stock markets, thereby increasing investor confidence and efficient portfolio management.

One great advantage of these African emerging economies is that they are dynamic. This is as opposed to their developed countries counterparts that are strongly established for decades with strong institutions that are slow in reacting to economic crisis. Therefore, many invented formulas that these developed economies have been using to handle their unfortunate financial situations are no longer useful for them.

On the contrary, African developing nations are relatively new, embryonic and emerging. They are therefore more flexible and more elastic in their ability to invent new ideas and invent brand new responses to their specific conditions and as well as to the international global economic and financial crisis. Therefore, whenever, there is change in form of global financial crisis like the World is currently experiencing, or a shift from the normal global standards as is the situation with these developing African countries, researchers can be proactive and invent new

ideas that can modify the existing invented formulas that would suit these African countries and take advantage of their financial crisis.

This research therefore seeks to develop a Maximal Overlap Discreet Wavelet Transformation (MODWT) algorithm that would modify the most popular existing symmetric GARCH(1,1) volatility model and the most popular asymmetric EGARCH(1,1) volatility model that would take care of the peculiarities of these African countries' developing economies for forecasting their volatilities effectively and efficiently.

### **1.3 Objectives of the study**

In line with the discussions above, the objective of this study is to develop a new Maximal Overlap Discreet Wavelet Transform symmetric GARCH(1,1) model (MODWT-GARCH(1,1) model) algorithm and to examine the capability of the newly developed symmetric MODWT-GARCH(1,1) model algorithm in modeling and forecasting the volatility of African stock markets. Therefore, the study will compare the forecast error results obtained using the GARCH(1,1) model. In doing so, the study uses the forecast error obtained using the MODWT-GARCH(1,1) model algorithm.

The second objective of this study is to develop a new algorithm Maximal Overlap Discreet Wavelet Transform symmetric and asymmetric models. That is the MODWT-GARCH(1,1) model and the MODWT-EGARCH(1,1) model respectively. Subsequently, the study examines the capability of the newly developed MODWT-GARCH(1,1) model algorithm in forecasting the volatility of African stock markets.

Therefore, the study compares the forecast error results obtained using the symmetric MODWT-GARCH(1,1) model as a base model and the forecast error results obtained using the asymmetric MODWT-EGARCH(1,1) model.

The third objective of this study is to evaluate and to take decision on the usefulness of the asymmetric MODWT-EGARCH(1,1) model. Therefore, the study compares the forecast error results obtained using the asymmetric MODWT-EGARCH(1,1) model as a base model and the forecast error results obtained using the traditional asymmetric EGARCH(1,1) model.

#### **1.4 Significance of the study**

A plethora of stock markets studies seems to have chiefly been focused on developed and near-developed economies utilizing various models to offer empirical investigation to these effects. Consequently, due to the disparities inherent in different context, the application of a single-most model in unravelling the effects of stock market volatility may be contentious or inaccurate. This is because, developing economies may parade market indices not explicitly possessed by their superior developed counterparts. The quandary has been and continue to be how to adopt an efficient quantitative tool that possesses an apt measurement tendency that conscripts all the antecedents, based on a volatility model, universal to studying the volatility of stock markets in developing economies. This is of great importance because developing economies such as the African stock markets offer a wider range of possibilities for foreign investors to make profit.

## **1.5 Limitations of the study**

The study used only the newly developed algorithm Wavelet-GARCH model. The study also used only the daily returns of four African countries' NSE 20 (Kenya); All Share Price Index (Nigeria); FTSE/JSE100 (South Africa) and TUNNIDEX (Tunisia) stock market indices for the period January 2, 2000, to December 31, 2014. The Maximal Overlap Discrete Wavelet Transform-GARCH(1,1) model and the Maximal Overlap Discrete Wavelet Transform-EGARCH(1,1) model are exhaustively compared in different ways with their traditional counterparts of symmetric GARCH(1,1) and the asymmetric EGARCH(1,1). Furthermore, six developed countries' stock markets index viz; Britain (FTSE ALL SHARE); China (SHANGAI); France (FRANCE CAC40); Germany (DAX30); Japan (TOKYO SE); USA (S&P500) were modelled in similar fashion to examine the robustness of the proffered methodology. The data was downloaded from the DataStream.

Furthermore, the study uses only the most popular symmetric GARCH(1,1) volatility model and the most popular asymmetric EGARCH(1,1) volatility model. These traditional volatility models are used as benchmarks for measuring the superiority of the newly developed MODWT-GARCH(1,1) and the MODWT-EGARCH(1,1) volatility models.

## **1.6 Organisation of the thesis**

The thesis is organized in the following manner: Chapter two gives the details of the plethora of literature related to the research thesis. We looked at the contentious issues in the market microstructure literature in volatility and stock

expected returns and the considerable attention that volatility has received both in developed and developing countries. Additionally, this chapter also looked at the important implications of volatility to investors portfolio positioning and liquidity of investment portfolio. Finally, the chapter maintained that the issue of asymmetric impact and arrival of information affecting the volatility of the embryonic African countries stock markets remains elusive.

The methodology adopted in the research thesis is presented in chapter three. The stylized facts of asset return and the emergence of the volatility models are discussed. The properties and features of the model about facts are as well examined. The method employed for estimating parameters of the model is carefully discussed. We also depicted the model design and discussed the statistical tests used in the course of the research.

The data analyses; results and discussion is given in chapter four. We used the daily returns of four African countries' stock market indices and also used the daily returns of six developed countries stock market indices for the period from January 2, 2000, to December 31, 2014. The purpose is to compare the linear GARCH(1,1) model and the newly proposed Maximal Overlap Discrete Wavelet Transform-GARCH(1,1) model among the four African countries' stock market indices. In like manner, we compared the Maximal Overlap Discrete Wavelet Transform symmetric-GARCH(1,1) model with the Maximal Overlap Discrete Wavelet Transformation asymmetric-EGARCH(1,1) model. Subsequently, the Maximal Overlap Discrete Wavelet Transform asymmetric-EGARCH(1,1) model is exhaustively compared with the traditional asymmetric EGARCH(1,1) model. The six developed stock market indices were used for robust checking of the performance

of the Maximal Overlap Discrete Wavelet Transform symmetric-GARCH(1,1) model and the performance of the Maximal Overlap Discrete Wavelet Transform asymmetric-EGARCH(1,1) model.

Finally, in chapter five, the research thesis says that there is a justification that the symmetric MODWT-GARCH(1,1) model gives an accurate forecast volatility of African countries stock market more than the symmetric GARCH(1,1) model. Of great importance, is that the symmetric MODWT-GARCH(1,1) model exceeds the asymmetric MODWT-EGARCH(1,1) model in given the accurate five days average forecast values of the stock markets' returns volatility of the African countries even in the face of asymmetry. A recommendation that the Maximal Overlap Discrete Wavelet algorithm transformation be applied in the multi variate context of volatility modelling was the concluding remarks of the research.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

One of the contentious issues in the market microstructure literature has been the volatility and stock expected returns. Volatility has received considerable attention both in developed and developing countries due to important implication for investors' portfolio positioning and liquidity of their investment portfolio. However, the issue remains elusive with regards to the question of asymmetric impact of good news (market advances) and bad news (market retreats) or information arrival in these emerging markets. It is general phenomena that, the negative shocks raise volatility more than positive shocks in the market. This phenomenon has been attributed to the leverage effect (Nelson, 1991; Engle and Ng, 1993; and Engle and Patton, 2001).

#### **2.2 Volatility models**

The seminal work by Engle (1982) and those of Pindyck (1983) and Bollerslev et al. (1988) and Bollerslev et al (1992) all provide evidence that volatility is time varying and that news tend to be clustered together regarding the size of their impact on stock prices. This is known as volatility clustering and may be related to market dynamics.

Volatility clustering also characterizes the transmission of news from one

market to another. Scholars like Bennett and Kelleher (1988), Von Furstenberg et al. (1989), Susmel and Engle (1994), Hamao et al. (1990), King and Wadhvani (1990), Schwert (1989) and Karolyi (1995) demonstrate this type of transmission of news. In their various analyses, they reported that the transmission of volatility between markets is also time-varying and that lagged spill overs of price changes and price volatility exist between major stock markets. Implying that, when volatility is high, price changes in major stock markets tend to become highly correlated.

The researchers, Engle and Patton (2001), made it known that financial time series such as stock prices often exhibit the phenomena of volatility clustering because of arrival of diverse information from various sources such as economic events and news and other exogenous economic event such as war and other undesirable events that have greater impact on the time series pattern of stock price. Thus, in most cases, financial time series behave in such a way that does not conform to the normality distribution.

In practice, however, researchers have uncovered many characteristics about volatility of financial time series; these are the so called stylized facts. The literature review of the stylized facts has been such that numerous researchers have written on individual facts to support their claims. For instance, Engle and Patton (2001) examining mean reverting as saying there is a normal level of volatility to which volatility will eventually return. Gouriéroux and Jasiak (2007) claimed that because of the foregoing, very long forecast of volatility should all converge to the normal level of volatility.

Another well-known characteristic review of the financial time series that

generates volatility is the fat tail of the unconditional distribution of the series. This is known as asymmetry. Bollerslev (2001) reported that the asymmetry is sometimes ascribed to leverage effect and sometimes to risk premium effect. The earlier researchers in this direction include; Black (1976), Christie (1982), Nelson 1991). These researchers also clarified that bad news (negative shock) is largely responsible for the asymmetry as compared to the good news (positive shock). Glosten, et al. (1993) claimed that the asymmetry structure of volatility generates skewed distributions of forecast prices and simple derivative pricing assumptions.

The seminal works of Mandelbrot (1997) and Fama (1965) stated that the rate of returns (percentage changes in prices) implicit in the time series of stock prices are time dependent. They further said that leptokurtosis, skewness and volatility clustering characterizes the distribution of daily stock prices.

### **2.3 Applications of volatility Models**

The stylized facts observed in financial markets is the natural course for the application of symmetric and asymmetric GARCH(1,1) types models on financial time series data.

The symmetric and asymmetric GARCH(1,1) models imposes an autoregressive structure on the conditional variance of financial time series. This, allows the volatility shocks to persist overtime. Both the symmetric and the asymmetric GARCH(1,1) models are martingale difference, implying that all expectations are unbiased. The symmetric GARCH(1,1) model has been extensively used to study the conditional volatility of stock markets and ascertain the

predictability of future stock returns (Bollerslev et al., 1988). Several recent studies provide evidence that the symmetric GARCH(1,1) methodology can capture the volatility characteristics of financial time series. Examples of such studies include Akgiray (1989), and Baillie and DeGennaro (1990). Furthermore, Lamoureux and Lastrapes (1990) also applied symmetric GARCH(1,1) model to both individual stocks and indexes. The result of their study showed that the GARCH(1,1) processes can describe the data generating processes of the observed financial time series.

An extended version of the symmetric GARCH(1,1) model is the asymmetric EGARCH(1,1) model that was developed by Nelson (1991). The EGARCH(1,1) model method is more advantageous than the symmetric GARCH(1,1) model in modeling the stock returns for the following reasons. One, it allows for the asymmetry in the responsiveness of stock returns to the sign of shocks to returns. Two, unlike GARCH(1,1) specification, the EGARCH(1,1) model, specified in logarithms, does not impose the nonnegative conditions on parameters. Lastly, modeling stock returns and its uncertainty in logarithms hampers the effects of outliers on the estimation results. The EGARCH(1,1) model has been widely used in studying interest rates, interest rate futures markets and it is also used to model foreign exchange rates (Berument et al., 2001).

As stated by Rabemananjara and Zakoian, (1993), another important family of symmetric GARCH(1,1) models is the Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH) model. The model enormously solves the problem of the possibility that the conditional standard deviation of the EGARCH model can be very close to zero or be exactly zero. One other disadvantage of the EGARCH model is that it assumes that the effect of positive and negative volatility

remains fixed overtime and the other limitation of the EGARCH model is that it is a linear model that implies a moving average equation in the natural logarithm of the squares of the conditional standard deviation process. The TGARCH model is superior to the EGARCH model in that it incorporates non-linear effects in the conditional variance while keeping the linearity of the GARCH model when the data do not contain non-linearity.

The researchers McAleer and Da Veiga (2008), said that several empirical studies employing power GARCH (PGARCH) model in modelling high frequency intra-day data offered better return volatility forecasts than those given by the standard GARCH(1,1) model. However, Gokcan (2000) have documented that a plethora of empirical findings suggested that among all other types of symmetric and asymmetric volatility models, the standard GARCH(1,1) model is still useful in modelling the volatility of emerging countries' economies.

#### **2.4 Current related literature on volatility**

Contemporary research findings such as the study by Alom et al. (2012) said understanding the nature of volatility in commodity prices warrants adequate attention because such volatility is likely to lead to increased production and opportunity costs, as well as accelerate uncertainty and risk, contributing to a slowdown of economic activities. The study further examines the asymmetry and persistency in the volatility of a set of petroleum future price returns within the framework of a set of asymmetric generalized autoregressive conditional heteroscedasticity (GARCH) type models.

The study, specifically, employed threshold GARCH, exponential GARCH, asymmetric power ARCH and component GARCH models using daily data over the period 1995–2010. The research unveils that: (1) throughout the time of 1995–2010, all future price returns show persistent and asymmetric effects of shocks to the volatility but the level of persistency and degree of asymmetry differ from product to product; (2) throughout part of the period 1995–2001, persistency and asymmetry are evident for all series with the exception of premium motor spirit future price returns; (3) the recent subsample of 2002–2010 shows mixed evidence and all series show persistent effects of shocks to the volatility while asymmetry is supported in crude oil and propane only.

The conclusion from the study is that based on the forecasting accuracy of the adopted models, and because of the characteristics of the petroleum products, different models should rather be used because no single model can be recommended based on the time periods involved. The findings of their study also recommend that in the presence of asymmetric and persistent volatility and for the reasons of accommodating long lasting effects of shocks to the volatility, strong policy decisions should be considered. Again, because positive shocks have not fully compensated the negative effects of shocks, counter-cyclical policies should be taken to ensure a stable business environment and to counter the optimistic and pessimistic overreactions of businesses.

The researchers Gil-Alana et al. (2015) studied the asymmetry nature in the volatility of USA, Europe and Asian countries stock markets data. The study employed the use of the Jaganathan and Runkle generalized autoregressive conditional heteroscedastic (GJR-GARCH) model to analyse the leverage effects in

these markets due to the frequent occurrences of the so called “bull and bear” in these markets.

The analysis conducted in Yaya et al. (2015) using estimates from the GJR models confirmed the asymmetry in the market phases (bull and bear), with the bear periods having more (significant) impacts on the conditional volatility of the stocks returns. Yaya et al. (2015) finally recommended an extension of their study to looking at the forecasting ability of the volatility series at each of the identified bull and bear phases.

After the 2008 financial crisis, Danielsson et al. (2016) said that forecasting risk has become a major public concern. They further said that measures of statistical risk play a much more useful responsibility in decision and policy making within institutions of finance much more than before the crisis period. Therefore, obtaining an accurate model of forecasting is of essential value. The major research gap filled by their research was the development of internal mechanism for managing financial institutions and prudence in macroeconomic activities. They, however, quickly pointed out that the major problem in measuring risk is the lack of ability of a single economic and statistical model that estimates risk.

The researchers Danielsson et al. (2016) further stated that most proposed methods only cater for periods in which there are no distress financially. And, those models developed do not take care of periods of financial instabilities when they are much desired by policymakers so as to enable them strategies sectors of emergency and direct capital to such sectors and as well help these policymakers to formulate an important financial policy regulation. Finally, their study recommended that a model

incorporating recession period should be examined to see whether it would give a better out-of-sample forecast as compared to the traditional usage of historical averages.

The study by Gupta et al. (2016) asserted that an important economic indicator is the exchange rate in monetary markets, internationally. They said that the volatility of exchange rate is the major reason of global uncertainty in the economy, asserting that this uncertainty affects a wide range of most of macroeconomic variables and it adversely affect corporate organisations and it also affects a vast majority of leverage decisions.

In their research, Gupta et al. (2016) stated that the importance of exchange rate movements and prediction has been very significant in the present times due to growing volatility in the forex market. The movements of exchange rate affect the prices of assets; currency value; current account; reported profits and international reserves and deficit of the country. Gupta et al. (2016) further said that the most important huge task for practitioners, researchers and policy makers is first the decision to hedge exchange rate followed by the method of forecasting exchange rate and finally followed by the decision of how to measure the volatility of exchange rate. Gupta et al. (2016) then finally recommends that fundamental analysis and technical analysis can be used to manage the risk in exchange rate.

## **2.5 Wavelets-volatility models**

The stock market provides an example of a market in which the agents involved consist of heterogeneous investors making decisions over different time

horizons ranging from minutes to years and operating at each moment on different time scales which could be from speculative to investment activity. In such context, where both the time horizons of economic decisions may differ per the time scale of the analysis, a useful analytical tool may be represented by wavelet analysis.

The researchers Kristjanpoller and Minutolo (2016) aptly applied GARCH-ANN (GARCH-Artificial Neural Network) in the study of future market price of crude oil. Their results suggest that the GARCH-ANN model has successfully improved the volatility forecast of market prices of petroleum better than the traditional volatility models of forecasting. Their results show that the GARCH-ANN model has greatly improve the forecast performance of the traditional GARCH(1,1) model by as much as thirty one percent using the horizon of twenty-one days.

Secondly, Kristjanpoller and Minutolo (2016) develops a method of deciding which variables in finance are very useful in determining the volatility of crude oil prices and its prices in the future. This is of great importance as the prices of crude oil keeps fighting with continuous slowdown in the prices of commodities, globally. Studying and forecasting this effect in the commodities market based on understanding of the movement of the independent variables could assist financial and political rulers kill the effect on the overall.

The study by Correa et al. (2016) put forward the Wavelet-Autoregressive Integrated Moving Average-GARCH (WARIMA-GARCH) method, which is a new forecasting method capable of integrating wavelet variables. The variables were gotten from wavelet decomposition of the underlying series taken as exogenous variables, which resulted in a considerable enhancement in forecasting routines when

compared to the ARIMA-GARCH, ANN and Wavelet ANN models.

The research study by Bouoiyour and Selmi (2015) revisited the relationship between real exchange rate uncertainty and exports performance to establish whether there is a significant short run dynamic between them. In their work, they combined wavelet analysis with the best model selected among the existing GARCH extensions, which is basically are the linear versus nonlinear, and symmetrical versus asymmetrical etc.

The research paper by Jothimani et al. (2016) proposed and presented a hybrid forecasting approach by integrating the advantages of decomposition and machine learning models. The research presented two hybrid models, namely, maximal overlap Discrete wavelet transform-artificial neural network (MODWT-ANN) and maximal overlap Discrete wavelet transform-support vector regression (MODWT-SVR). These models were used to predict 1-step ahead forecasts of National Stock Exchange Fifty index. The hybrid approach first used the MODWT to decompose the time series data. Then, it uses the support vector regression (SVR) and the artificial neural network (ANN) models to predict each subseries of the MODWT independently. Finally, the forecasted MODWT-SVR and the forecasted MODWT-ANN sub-series are aggregated to obtain the final forecasts. The presented models involving the MODWT decomposition showed a consistent superior performance in predicting the weekly National Stock Exchange Fifty index, as compared to only applying the traditional ANN model and the traditional SVR model on the entire series.

The study by Barunik, et al. (2016), investigated how the wavelet