APPROPRIATING THE FIBONACCI SERIES IN ORIGINAL MUSIC COMPOSITIONS

by

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MENGGUNAKAN SIRI FIBONACCI DALAM KARYA MUZIK ASAL

ABSTRAK

Semenjak zaman dahulu hingga ke zaman kini, terdapat banyak hubungan diantara muzik dan matematik, dan kedua-dua perkara ini terletak pada teras idea dan konsep manusia. Penggunaan siri Fibonacci sering dipertikaikan, khususnya dalam muzik sebelum abad ke-20, tetapi wujudnya beberapa penyokong yang terdiri daripada penganalisis muzikologi yang menggunakan siri ini dengan cara "superimpose" pada repertoir yang sedia ada. Ini termasuk para komposer moden dan pasca-moden yang menggunakan siri ini secara sengaja pada tahap minat yang berlainan. Walaupun terdapat banyak salah faham dan fenomena mistik (mysticism) yang wujud didalam penggunaan siri ini,ia telah dikaitkan dan diperhatikan dari mulanya secara langsung atau tidak langsung dalam karya "renaissance". Aplikasi utamanya dalam muzik adalah menjadi domain masa muzik (domain of musical time), khususnya dalam reka bentuk struktur muzik. Tesis ini mempertimbangkan kesesuaian umum model matematik sebagai pengawal parameter muzik, menggunakan siri Fibonacci dan sifat matematik dan perkadarannya sebagai pengawal. Ia juga alamat di peringkat menengah masalah ontologi muzik dari dalam bidang komposisi muzik. Ini dilakukan secara priori supaya dapat membangunkan bahasa estetik yang berpusat pada siri Fibonacci. Tesis ini secara keseluruhan meneroka aplikasi selanjutnya dari siri Fibonacci kepada karya muzik di luar domain reka bentuk formal, khususnya kepada struktur melodi dan rentak antara parameter muzik lain. Langkah pertama adalah mendefinisikan siri ini melalui ontologi muzik dan kemudian mengenal pasti kemungkinan artikulasi antara bidang matematik dan struktur muzik. Memandangkan saluran lain seperti "cross-pollination" di antara kedua-dua bidang ini dengan

memaklumkan penyelidikan seterusnya dalam bidang-bidang ini dan selainnya yang berkaitan. Ia juga menyumbang kepada pemahaman dari segi ontologi mengenai bidang seni, dan juga menimbulkan persoalan yang sangat menarik dalam bidang falsafah. Hasil kajian ini dibentangkan dalam bentuk portfolio muzik yang terdiri daripada ciptaan karya asal dalam beberapa gaya yang berlainan, tetapi hanya bertujuan sebagai jawapan sampel yang boleh dikajiselidik, dan bukan sebagai ringkasan koleksi musikal. Muzik yang termasuk dalam portfolio ini direka untuk meneroka parameter matematik yang berbeza, serta gaya muzik yang pelbagai format dalam tradisi estetika klasik barat. Walaupun parameter tertentu dalam bidang teori muzik dan masa yang abstrak dapat disampaikan melalui siri ini tanpa kehilangan konsistensi atau fungsi ontologi, sesetengah yang lain tidak dapat digunakan dalam konteks yang sama, khususnya, bidang warnanada lebih sukar untuk dinyatakan secara matematik dan merupakan bidang usaha masa depan yang mungkin, terutamanya melalui sumber bunyi bukan akustik yang parameternya dapat ditentukan melalui kaedah matematik yang boleh diukur.

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ABSTRACT

Relationships between music and mathematics are many and lie at the very core of our ideas and conceptions about music since antiquity until contemporary times. Usage of the Fibonacci series in particular in music before the twentieth century is disputed, but several proponents exist of musicological analyses that superimpose the series over existing repertoire, as well as some degree of interest in this series from modern and post-modern composers that use it intentionally. Much misconception and mysticism exists around usage of the series, and while occurrences in music have been both correctly and incorrectly attributed and observed in works from the renaissance period onwards, its main application in music has been into the domain of musical time, and more specifically in the design of musical structures. This thesis considers in the first place the general suitability of mathematical models as controllers of musical parameters, using the Fibonacci series and its mathematical properties and proportions as controllers. It also addresses on a secondary level the problem of musical ontology from within the field of musical composition; this is done *a priori* in order to be able to develop an aesthetical language that is centered on Fibonacci. This thesis as a whole explores further applications of the Fibonacci series to musical composition beyond the domain of formal design, in particular to melodic and rhythmic structures among other musical parameters. It does this by defining first an ontology of music and then identifying possible points of articulation between the mathematical and the musical structures. Considering other avenues of cross-

pollination between these two areas informs future research in these and other related areas, as well as contributing to our ontological understanding of the arts, a question of great interest in the field of philosophy. The output of this research is presented in the form of a musical portfolio comprised of original compositions in a few different styles, but intended merely as sample answers of the possibilities being explored, and not a summative musical collection. The music included in the portfolio was designed to explore different mathematical parameters as well as different musical genres with various formats within the western classical aesthetic tradition, and while certain parameters within the domain of abstract musical theory and time were able to be relayed through the series without losing their consistency nor ontological function, some others were not. In particular, the domain of timbre was more difficult to articulate mathematically and is an area of possible future efforts, especially through non-acoustic sound sources whose parameters can be dictated through quantifiable mathematical means.

CHAPTER I: INTRODUCTION

"Art is always more abstract than we imagine. The form and the colour speak to us about form and colour, and that's the end of it"

Oscar Wilde in "The Picture of Dorian Gray"

1.1 Introductory Statement

Music is an abstract art based on many different mathematical patterns and ideas. One of those such patterns, the mutually related Fibonacci series and/or golden mean, exists in mathematics and geometry, and its precursors have been known since antiquity. Modern interest in the series arises from the second half of the nineteenth century onwards and encompasses both reputable scientific works in fields as varied as biology, computer sciences, philosophy and arts, as well as pseudo-scientific claims in multiple interdisciplinary areas. Regardless of the veracity of all these claims, interest in the series has remain steady and alive well into the twenty-first century, with a constant stream of expanding research, interdisciplinary and multidisciplinary works related to the Fibonacci series and its related concepts; this is probably best exemplified by the existence of the *Fibonacci Quarterly,* an academic journal dedicated exclusively to this mathematical series and currently in its fifty-fourth edition. The series has been attributed, observed, forced upon, identified and applied in architecture, sculpture, painting, design, cinema, biology, astronomy, computer sciences, stock market predictions, and music, among many other seemingly dissimilar disciplines, which has led to significant research interest throughout a considerable span of time into the applications of the numerical series and its several mathematical and geometrical ramifications to many practical, vocational, scientific, sociological, technological and philosophical areas. Indeed, the series and its related concepts and applications have generated as much genuine scientific interest, as they have done to attract pseudo-scientific, in some cases even bordering on a mystical, almost cult-like fascination with the numbers. It is not the place for this thesis to interject in past nor current debates over multi-disciplinary usage of the series, nor to make arbitrary judgments over its apparent ubiquity; these are questions better suited to other areas of expertise. It suffices that there is enough scholar interest in the series and its related applications to undertake this study, which will look at usage of the series from the field of music, and more specifically, music composition.

This thesis takes as a premise the idea that musical patterns can be mathematical; something already inherent and taken for granted in the majority of discussions related to use of the golden mean in music and arts. These patterns are applied into musical composition through an exploration of the different points of convergence between the mathematical series and musical language, and attempting to formulate the musical aesthetics built around the series itself and its properties, rather than using the series tangentially as a tool to govern proportion in the structural design. This exploration, and the processes described here, can theoretically open the door to other possible cross-pollinations between arts and mathematics in a more general and broad view, and without Fibonacci or the golden ratio as input parameters necessarily. Applications of the Fibonacci series in the past to musical compositions seem to have dealt mainly with superimpositions of the series over an already pre-existing musical style and aesthetical language, something that has been both applauded and rejected by scholars from both sides of the musicological spectrum. Regardless of the outcome of this debate, the argument seems to center on a very narrow application of the mathematical concepts underlying the series into music, one in which the chief interest has been to apply the series to proportional values of time within the duration of a piece of music, disregarding the many other parameters within a musical and ontological framework. This research explores this gap by theorizing applications of the series to other aspects of musical composition besides the formal layout and structure of a piece of music (but without excluding it), like the rhythm and its relationships, melody and its construction, among other layers which will be outlined soon by defining the constituent parts of a musical work. These building elements and their role in this particular musical ontology need to be defined and deconstructed *a priori* before subjecting each of these individual layers to a Fibonacci growth process. These musical-mathematical growth processes, fundamental to their application to other fields and inherent in the series and its related proportions, are explained in detail in other subsequent chapters that are devoted almost entirely to this topic.

This thesis narrows down the very large field of study that is musical composition into a discussion of abstract music compositional practices within the domain of the western classical music tradition, and applies and extrapolates the equally abstract mathematical proportions inherent in the Fibonacci series and its related golden mean proportion to the different component aspects of the musical architecture. It is precisely because of these abstract properties in both sciences that such cross-pollination between music and mathematics is at all possible, and the reasons why it has been suggested by many recent musicologists and philosophers as well as discovered by a handful of previous composers and artists. The aesthetical and the generative applications of the Fibonacci series to music are explored in this thesis in a variety of fundamental concepts from the design of the musical compositions, in the structures, themes, motifs or rhythms from which the music spawns, in order to study the effect of such a cross-pollination in fairly well established musical aesthetical concepts. This is not done in order to achieve a certain perceived sense of balance, value or significance, in the musical output, as these concepts are purely subjective, arbitrary and dependent on specific aesthetical frameworks (Gortais 2). These concepts of balance and unity, at least when applied to arts and other aesthetical fields, are not quantifiable nor absolute, but they are subjected to the specific aesthetics of each individual work and time period. Within those limitations and ontological boundaries, the concepts of balance, unity, tension, hierarchical centers (tonality), color, movement and pace, among other musical parameters or its analog concepts, do acquire certain specific meanings and, while are still not quantifiable, do exist and can be defined in a specific musical way. For this reason, it becomes necessary to try to identify, conceptualize and define those musical parameters first, in order to manipulate and cross-pollinate them. As an example, it can be considered how the musical parameters outlined within harmonic design, melodic resolution, orchestration and color (timbre), among others in a specific piece of music, can play an important part in our psychological perception of the non-musical concepts of tension and relaxation in that piece; or how our sensorial perception of pulse, pace and movement is directly tied in to the music's rhythmic design and rhythmic recurrence, metric grouping and hierarchical organization of beats, speed of the music and tempo fluctuations, among other parameters. These ontological parameters are specific to each art and discipline and work within a clear semantic framework that is unique to each field; furthermore, within each field different time periods and different geographical settings account for great variation in aesthetical perceptions and judgments. Therefore, in order to look at the effects of the mathematical concepts on musical parameters, we need first to define which parameters are these and how they are classified and related to one another. For this thesis we will deal mainly within the ontological domain of music, and to be more precise, Western classical music will be the main focus of study and application, though some small ventures and experiments are presented in other styles and genres, and in the subsequent discussions popular commercial music and folk music (also presented sometimes as world music, traditional music, or ethnic music) are briefly discussed, acknowledged and at least conceptualized to some extent, albeit not being the main focus area.

Through a look at musical aesthetics and musical semantics within the context of musical composition, the idea of proportion is reframed through the Fibonacci series and its inherent proportions and mathematical connections are explored together with the formal elements in the musical construction to produce a portfolio of sample music consisting of works, sections of works, examples and sketches, in different formats which are built by layering one or more concurrent Fibonacci musical elements in the composition in order to explore each of the musical parameters from a mathematical perspective. In other words, to create a musical style that is characterized by common elements of the classical, traditional and popular repertoire, but using the Fibonacci series and the golden mean as the input material for each of the designing parameters of the music. In music, this idea of design and architecture, or the reaching and arriving at a series of climaxes or culmination points that articulate sections of time and terracing of structural elements, is achieved and effected by manipulating several aspects of the harmonic design, the tonal setting, the treatment of timbre and acoustic orchestration, the design of tempi, structural and formal considerations, among others; these in turn can also be subjected, just as well as the construction aspect of the architectural proportion of the music, to a generation by parameters from the Fibonacci series or the golden mean proportion, without losing any part of the syntax and meaning of the music, be it traditional, contemporary or classical, for the sake of a purely abstract mathematical construct.

1.2 Statement of Problem

This thesis addresses two different but ultimately related problems: firstly, are musical structures and parameters susceptible to a generation from mathematical parameters? In particular, through the Fibonacci series? Secondly, which are those specific musical parameters? The first problem deals with the applicability of mathematical concepts, in this case in particular what is understood as the Fibonacci series and the golden mean proportion, as models for a cross-pollination of these mathematical abstract constructions into different equally abstract parameters of musical construction. This problem in itself is multi-disciplinary, and is a question that has generated considerable debate and research interest in diverse disciplines and not confined to music and arts only, as will be seen in later chapters. In the field of music it includes specific interest in its applications to analysis of musical works, as shown in the analytical works of Erno Lendvai (Lendvai) and Roy Howat (R. Howat) as well as study of its applications to musical composition (Kis Zuvela), among many other possible uses. This is an interdisciplinary question by nature, and one that will generate multiple observations directly related to music, more specifically to the field of musical composition, and hence the main focus of interest throughout. The second problem is the definition of a workable ontology of music, one which is applicable to not only western classical music (as most musical ontologies are) but to general musical composition within any aesthetic stream, in order to apply the concepts and parameters observed in the Fibonacci series itself in the first place. This is of secondary interest, as it is not the main area of research and has to do more directly with the field of philosophy, ontology and aesthetics. Nevertheless, this step is necessary and it entails defining an analytical framework that will guide the methodological approach in order to develop an aesthetical framework for the specific elements and parameters needed to create the musical works, and needs to be done *a priori* in order to address the first question.

The first problem is a very specific one, and deals with the applicability of the Fibonacci series and the golden mean proportion to musical structures and parameters. This is a very exploratory question in its nature and it is addressed through a qualitative study of the mathematical properties and its application to different musical parameters. Previous research into the Fibonacci series and its applicability to music deals chiefly with the appropriation of the series as a superimposition over a pre-existing aesthetical language proportional model to determine the parameters of formal structure (Mongoven 1-2) or to determine time durations as metric proportions (Berg 390-391), something that is paralleled in other artistic areas in an analogous way by research into the applications of the series into ordered hierarchies of sizes in architectural proportions (Salingaros 1-4) as well as research into structural architectural proportions (Frings). By extension this also aims to address on a broader sense the more general question on the suitability of mathematical parameters as controllers to determine musical parameters, and ultimately something that could potentially be extrapolated to other parameters in different art forms. In order to answer this question however, the identification and definition of these parameters within the music is of paramount importance in order to create a working methodology. The definition of a specific entity, of its constituency and categories, its component parts and their relationships, is something that is addressed within the philosophical area of ontology. Various different ontologies of music have been defined and proposed by past philosophers, perhaps the most influential ones in philosophy being those models by Plato, Kant, and Nietzsche. Modern interest in the field centers on the duality between different conflicting types of realists on one hand, such as Julian Dodd (Dodd, "Musical works as eternal types.") and Jerrold Levinson (Levinson) among others, whom influenced by a platonistic view of ontology argue that musical entities do indeed exist, and non-realists philosophers on the other hand like Ross Cameron (Cameron), who argue that such entities do not exist and it is a futile exercise to even discuss musical works and their ontology (Kania). While these discussions in musical ontology are fascinating food for philosophers and metaphysical thought, especially abstract thought, they might have several epistemological connections and hopefully some real repercussions for philosophers, but they do not provide much additional working information to a listener, performer or composer, nor hold any practical qualities of use in arts, abstract musical composition, and applications of extra-musical content, the question at the center of this thesis. As such, looking at current and relevant ethno-musicological models for analysis of musical performances such as those presented by Jeff Todd Titon (Titon 1-33) and Bruno Nettl (Nettl 7-13) for analyses of world and folk music, one can propose a simple ontological and musical framework that focuses in the generative processes at work behind musical composition instead of models that focus on the performance practices behind music making.

A growing number of works and academic publications in musicology, music theory and many other related and non-related scientific disciplines, from the last 150 years referencing to the Fibonacci series seem to show a genuine scientific interest in the interdisciplinary properties of this particularly interesting series and its ability to cross-pollinate at a very abstract level of design, perception of proportion and generation of growth patterns, relationships between the abstract language of music and the equally abstract language of mathematics and a clear awareness of the connections between the Fibonacci series as applied in the arts (Livio 4-6). However, due to the lack of a clear ontology of music and its elemental parameters, most common usage of the series in music and architectural works is applied in proportion to the formal layout of the music or work, leaving susceptible elements at the building block level largely unexplored. In the field of music these are specifically the motivic and thematic design, rhythmical models, textural spacing, tempo considerations and relations, time signatures, form organization, proportion as well as several other miscellaneous elements. This lack of interest in musical ontology from the point of view of composers is countered on the other hand by interest in the ontological definition of music from within the field of philosophy, as shown in the writings of Andrew Kania, Jerold Levinson and others, with relatively heated debates and contrasting views and positions from its different philosophical proponents, but no direct implications for musicians and artists; at least not in a practical nor pragmatic sense. As expressed in advance, the main application of the series to a musical language and aesthetical creed has amounted chiefly to a superimposition of the series over a pre-existing given musical style, without going down to the level of the very building blocks of the music, nor allowing the extra-musical elements to dictate the formulation of an aesthetical style that is based on these mathematical concepts. This gap in the appropriation of the Fibonacci series to music, and more specifically to musical construction, opens the door to explore some further applications of the series and its related concepts, and poses some new questions and challenges to composers or artists of other disciplines looking to work with either this particular series, or with other mathematical elements as input for artistic processes. More specifically still, these processes are built looking to create pieces of music that are infused from its very core elements with the Fibonacci series or the golden mean, hence aesthetically unified by extra-musical elements, in this case the mathematical parameters. While this exploration is not meant to be summative and exhaustive due to the very limitations of working in an artistic field, the work presented here does intend to consider very specific technical problems pertinent to the craft of musical composition, such as harmonic design, rhythmic design, melodic design, construction of musical textures and layers, melodic and motivic development, counterpoint and contrapuntal considerations, among others, and observe and analyze what effects the usage of the mathematical series will have on each of these layers and structures.

1.3 Background of Study

Work on the Fibonacci series is not even by a long stretch a new area of research interest within the academic community, nor is it confined to the fields of mathematical sciences nor music and arts. Fibonacci and golden ratio connections to music are already present in the middle of the nineteenth century in the works of German philosopher Adolf Zeising (1810-1876). Zeising envisioned the golden ratio as a universally unifying aesthetical principle that could encompass proportion and beauty in everything from plants, animals and human anatomy, to musical intervals, architecture and sculpture (Livio 178-179). While Zeising's claims in the nineteenth century seem a bit farfetched in today's standards, and indeed many of his claims have been debunked, there is no denying that his writings sparked much interest in the field and he had ascending influence in a variety of thinkers and intellectuals that spanned across many different disciplines.

Wherever this influence came from, it was already very clear by the beginning of the twentieth century that the idea of usage of the golden ratio in relation to musical composition, and more specifically in musical proportion, had been discussed in music circles for some time, which led the influential Austrian composer Arnold Schoenberg (1874-1951) to remark in his 1911 book 'Theory of Harmony' (published in German under the title *Harmonielehre)* that while he didn't believe the golden mean could be seen as the single unifying theory of aesthetic appeal to our senses, he acknowledged it to be an organizing principle among many other such concepts, and he goes on to state in the same text that '*I do not believe then that a composition must run to just such and such a length, no longer, no shorter, nor that a motive, regarded as the germ from which the composition grew, would have admitted none other than this one, single form of elaboration. Otherwise it would hardly be possible to write two or more different fugues on the same subject, as Bach and others did repeatedly'* (Schoenberg 139).

Perhaps one of the most central figures in all discussions about usage of the golden mean as an application in music is the Hungarian composer Bela Bartok (1881-1945). Although

Bartok himself left no such clues in his manuscripts, letters or sketches, many future musicologist and analysts seem to have found such proportional design in his music, with the work of Hungarian music theorist Erno Lendvai considered to be seminal in understanding Bartok through golden mean proportions. His 1971 analytical work *"Béla Bartók: An Analysis of His Music"* was met with both enthusiasm and skepticism from academic circles and sparked much further interest in analysis of Bartok's works, in applications of Fibonacci and the golden mean to music, as well as in refutations of such aesthetical theories.

Regardless of where one stands in this debate, the fact that in the twentieth century the Fibonacci series generated interest in many fields, not the least in the field of music, is self-evident by the sheer number of academic and amateur publications that attest to this interest. This interest however, amounts to two different spheres of application, the first into analysis of music, the second into musical composition. Proponents of using the golden mean as a tool for musical analysis deconstruct musical works of previous composers and look for correlations between the structural and architectural design of the music and the golden mean and analyze the music according to its proportions. Incidences are plenty and have been observed and attributed as far back as the Renaissance period, in works by Guillaume Dufay (Sandresky 291-306), J.S. Bach (Cruz), Wolfgang A. Mozart (Putz 275-282), Giacomo Puccini (Atlas 269-291), among many other examples which will be outlined later in the following chapter, including a look at the above mentioned works of Bela Bartok. All of these analyses require a leap of blind faith from the reader, justified or not in the mathematical proportions themselves, since nothing in the writings, correspondences, or works of any of these composers can corroborate their explicit use of the series or the ratio, hence the possibility must be considered that out of millions of works of music some will exhibit golden mean proportions by pure coincidence and not by conscious design. Thus, without clear evidence from the composers themselves, applications of the series to musical analysis are unable to surmount this problem of attribution or superimposition of the series *a posteriori*, something that will always raise questions to its legitimacy. It is only through a look at musical composition, a process done *a priori*, that we can find intentional and explicit usage of the series into the process of musical design. Though this is something that we can find only in the work of a handful of other later twentieth century composers such as Joseph Schillinger (1895-1943) Karlheinz Stockhausen (1928-2007), Luigi Nono (1924-1990) and Ernst Krenek (1900-1991) among other works by composers of later generations. (J. Kramer 136-148)

Interest in the series remains steadily well fed into the twenty first century, and along with an unsurprisingly growing body of research on Fibonacci and golden mean in the areas of mathematics and geometry, much other cross-disciplinary research in areas as varied as computer sciences, biology, architecture, design, economics and music, attests to this interest as well. Some recent examples in popular culture also seem to show a clear reflection of this interest in the golden section, with movies like the 2006 international blockbuster *"The Da Vinci Code"* (based on the 2003 novel by Dan Brown bearing the same title) or Darren Aronofsky's 1998 independent black and white film "*Pi",* having important roles for the Fibonacci number in their dramatic plot. In popular music, examples include the Hip-Hop duo Black Star who uses the Fibonacci sequence explicitly in the lyrics of the chorus of their 1998 song "*Astronomy (8th light)"*, and the progressive rock band Tool who explicitly acknowledge that their song "*Lateralus",* from the 2002 album of the same name, uses the Fibonacci sequence to dictate the number of syllables during each verse of the song, as well as the metric organization of the used time signatures (Keenan). It seems therefore, that interest in the Fibonacci series and its related concepts is still current and relevant, justified or not. More importantly, the multidisciplinary qualities of the sequence and proportional ratio seem to keep attracting interest from both academic and amateur sources, as well as some representation in popular culture and sources.

1.4 Definition of Key Terms

The concepts listed below will be used considerably often throughout this thesis; as such it is necessary and appropriate to explain them and define them in order to understand their meaning within the context intended. This is even so more pressing to avoid confusion of terminologies, especially considering that the topics underlying the work presented here are by nature multi-disciplinary and may have multiple and/or different precedents in seemingly unrelated fields of study.

Aesthetics: aesthetics is the branch of philosophy that studies the nature of beauty, art, and our appreciation of this beauty. It has far reaching implications in other fields of philosophy (Budd). In a more general way, and in common daily use, aesthetics can be understood and defined as a set of principles and ideas that governs our sense of beauty and perception. This is something that is subjective in its very nature, but strongly influenced by cultural norms and practices.

Cross-pollination: this concept is originally used in the field of biology and refers to the transfer of pollen from one flower to the stigma of another flower by the action of wind or insects, as explained in the *Merriam-Webster* dictionary. Within the context of this thesis however, cross-pollination refers to the seeding of ideas from one discipline to another, in this case from mathematics and geometry into music. The concept also implies some kind of translation of parameters, or appropriation of specific qualities from one area to the other.

Musical Development: defined in the Concise Oxford Dictionary of Music as 'the treatment of the detailed phrases and motifs of a previously heard theme' (Kennedy 178), development in music is a process that entails the reworking of previously stated musical materials, as well as a manipulation and transformation of those materials to some degree. These transformations can take intervallic, rhythmic, harmonic and textural, timbral or dynamic form, as well as more subtle processes such as melodic and/or rhythmic inversions, retrogrades, augmentations and diminutions. These processes can take place in one of the above mentioned layer, or in a few simultaneous layers, however, some degree of reference is expected to be kept with the original materials at some layer in order to avoid changing the materials from a development into a statement of new material.

Musical architecture: this term is used widely in musical analysis and composition, and it is commonly used to refer to the structure and formal design of a piece of music. Architecture implies a certain organology between the different parts of a musical work, in an analogous way to the different parts of a building or physical structure.

Musical ontology: Musical ontology is "the study of the kinds of musical things there are and the relations that hold between them" (Kania). It is derived from the philosophical study of metaphysics, the field of ontology defines what kinds of things (entities) there are, as well as the different parts that constitute it and their relationships. Musical ontology is then the study of what music is, its constituent elements, including all its purely musical parameters and the relationships between them. Most philosophical work in musical ontology deals with the performance of western classical musical works, though recent works show some interest in the ontology of popular musical styles (Kania).

Unity: defined by the Merriam Webster dictionary as “the quality or state of not being multiple: oneness” (Unity). Admittedly, this concept as it is generally applied in the arts can indeed be perceived as quite vague since its application can be judgmental and subjective, but at the very least it implies some kind of underlying unifying factor in the constructing fabric of an artistic work, so that it is perceived as a whole and not in separate sections. In common musical parlance unity implies that a work of music employs a certain degree of self-reference in its constituent materials and parameters in order to maintain audible connections in between different sections of music, or even different movements of the same work.

Appropriation: in a general sense, to appropriate means to take possession of something which one does not own in the first place. Throughout this thesis the term appropriation refers to the taking of mathematical parameters and converting them into musical parameters. This term will be used in a similar way to the term ‘translation’, as it is essential to effect the cross-pollination intended.

Translation: in common day to day usage, to translate something means to convert it from one language to another. In this thesis, the term translation is used in a similar way to the term ‘appropriation’ described earlier, and it is referred to as a transformation or a conversion of abstract mathematical concepts into equally abstract musical entities.

1.5 Theoretical Framework

This section outlines very briefly what the Fibonacci sequence and the golden mean ratio are in themselves, and discusses some of the salient properties and characteristics of the series. Since this is not a thesis in the field of mathematics, the discussions are limited in their scope to provide just a basic understanding of the sequence and its ramifications and implications in other areas.

1.5.1 The Numerical Fibonacci Series

The Fibonacci sequence was formulated by the Italian mathematician Leonardo Bonacci (also known as Fibonacci, Leonardo of Pisa, Leonardo Pisano Bigollo, Leonardo Fibonacci. ca. 1170 – ca. 1250) in his 1202 book *Liber Abaci* (Sigler). The numerical sequence was the solution to a mathematical problem presented by Fibonacci in the book involving the growth of a population of rabbits from generation to generation, under unrealistic and idealized reproductive conditions, and yields the following sequence of integer numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, etc. Later mathematicians and researchers of Fibonacci numbers such as Edouard Lucas (1842-1891) and Martin Ohm (1792–1872), including our modern understanding of the formula, express it as starting from a value of zero as such: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, etc. Naturally, the sequence can be continued to infinity, but for the sake of brevity and clarity these first 14 number of the series will suffice. It is also important to note that Leonardo Fibonacci himself did not postulate the sequence of numbers, nor named the series after himself, but merely introduced it to the West in *Liber Abaci*.

The sequence is additive in its nature, something that will be reflected in the following chapter when defining the function for the sequence, and this means that while each number is the sum of the previous two members in the sequence, the beginning of the sequence needs just two values in order to fulfill the function. It will be shown later that the sequence can be seeded equally with starting values 0 and 1, as well as 1 and 1, since both sets of values will fulfill the definition of its function. Both of these properties have very interesting musical connections from the very outset of this proposition: the idea that a member is the sum of the previous two members can be immediately correlated in music to time values, as well as the beginning of a musical sequence with values 0 and 1. Even though it is not obvious at a first glance through, a musical representation of value 0 has a very strong candidate in musical rests, which are audible silence.

One more property of the series that is of interest at this point is that each number of the series when divided by the previous number in the series approaches the irrational value known as *Phi*, and stands at 1.618. Obviously this is not true when one divides 3/2 (=1.5), nor 5/3 (=1.666666666666667), nor 8/3 (=1.6), but the higher one goes in the series the closer we approach this value, for example 233/144 (=1.618055555555556). This property is known as convergence, and it is of interest because of its connections to geometry, since it yields the mathematical expression that will be paralleled and corroborated geometrically as the golden mean,

1.5.2 The Geometrical Golden Mean

The concept of the golden proportion in geometry predates the numerical sequence by many centuries. The number sequence is intimately and inextricably connected to what the ancient Greeks called the golden mean in geometry, though this connection was only observed much later in the works of astronomers and mathematicians like the celebrated and influential astronomer Johannes Kepler (1571-1630) or Luca Pacioli (1445-1517), whose 1509 book '*De Divina Proportione'* (on the Divine Proportion) was of considerable influence to future generations (O'Connor and Robertson). Incidentally, the book was illustrated by Pacioli's longtime collaborator Leonardo Da Vinci. The earliest recorded definition of the golden mean is found in Euclid's *Elements* (c. 300 BC) and is described as such under Book VI, definition 2: "A straight line is said to have been *cut in extreme and mean ratio* when, as the whole line is to the greater segment, so is the greater to the less" (Fitzpatrick 156). The connections of the extreme and mean ratio to geometry are best understood through their use in the so called golden triangles, golden rectangles, golden spirals, golden pentagrams, as well as in their relation to two of the five platonic solids: Icosahedron and dodecahedron in particular use the golden mean for the calculation of their geometrical coordinates. (Plato 31-32)

1.5.3 Musical Ontology

For the purpose of this thesis, and the application of the concepts presented into musical compositions, an ontological framework is presented over which the cross-pollination of mathematics and music will occur. This model is explained in detailed analysis in the methodology section, but is summarized here to ease the flow and understanding of the following discussions.

The framework developed here is more directly concerned with the constituent parameters of music, rather than with defining music itself, something which arises out of necessity since it is paramount to define those musical parameters before attempting any kind of music-mathematical translation. The model may be summarized into four different layers of musical construction, each one explained briefly below:

- Theory: of course, more specifically musical theory. It encompasses all the abstract concepts that govern our ideas behind musical tones, tuning systems, intervallic and melodic distances, and tonality (a hierarchy of the musical tones), among others. These interactions can operate on a melodic (horizontal) plane, as well as in a harmonic (vertical) plane. Key parameters in this layer include tuning of pitches, intervals, scales and chord structures. Other parameters such as melody, harmony and counterpoint have their abstract origins here in this layer, but do not arise until they are articulated in time, both musical time and temporal time.
- Time: music is a by definition a temporal art, (J. Kramer 72-73). As much as music lacks an actual physical medium and vehicle, it cannot be conceptualized without time. All the abstract concepts in the first layer of musical construction, music theory, are precisely that, purely abstract concepts without a vehicle to transmit them. Time, musical time, provides that. The key parameters in this layer are pulse, tempo/speed, rhythm and form and structure.
- Timbre: this third musical layer is directly dependent on the sound sources being used, as well as the manner of articulating each particular source. Each sound source produces its own very unique sonic footprint, something tied in directly to the acoustic properties of each instrument and sound source, and the specific harmonic partials it highlights. Furthermore, beyond the instrument chosen to instantiate a particular music, the combination of two or more instruments can