UNIVERSITI SAINS MALAYSIA

First Semester Examination
2007/2008 Academic Session

October / November 2007

EAS 663/4 – Dynamics and Stability of Structures

Duration: 3 hours

Please check that this examination paper consists of **NINE** pages of printed material including appendices before you begin the examination.

Instructions: Answer **ALL FOUR (4)** questions in **PART A** and **ONE (1)** question in **PART B**. All questions carry the same marks.

You may answer the question either in Bahasa Malaysia or English.

All questions **MUST BE** answered on a new sheet.

Write the answered question numbers on the cover sheet of the answer script.

PART A : Answer ALL FOUR (4) questions.

1. (a) List **TWO (2)** characteristics that distinguish structural dynamic problems from static ones.

(4 marks)

(b) Define viscous damping. Sketch the displacement response, (v) versus (t) of undamped and damped SDOF systems for free vibration. Does the natural period of vibration, T, change with the presence of damping?

(6 marks)

(c) Figure 1 shows a model of spring-mass SDOF system subjected to harmonic excitation, p(t) = 40 cos 10t N. The weight of the mass block is 200 kN and the spring stiffness, k = 6000 N/m. Assume the damping of the system is equal to 5% of critical damping. Determine the total displacement response of the system which is given by the following equation:

$$v(t) = V\cos(\Omega t - \alpha) + e^{-\zeta\omega t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$V = \frac{v_0}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

where ω : natural circular frequency of the system and v_0 : static displacement due to p_o.

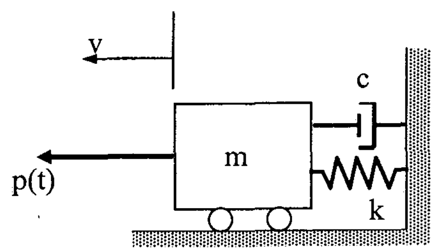

Figure 1

(10 marks)

2. (c) The water tower as shown in Figure 2(b) weighs 700kN when filled with water is subjected to step force with rise time Figure 2(c). It is observed that a horizontal jack force of 30kN is required to displace the tower top by a distance of 20mm. Estimate the maximum lateral displacement response due to dynamic forces. The constant phase is given by the following equation:

$$v(t) = v_0 \left\{ 1 + \frac{1}{\omega t_r} \left[A \sin\langle \omega (t - t_r) + \alpha \rangle \right] \right\}$$

$$A = \sqrt{(1 - \cos \omega t_r)^2 + (\sin \omega t_r)^2} \quad , \quad \tan \alpha = -\frac{\sin \omega t_r}{(1 - \cos \omega t_r)}$$

where where ω : natural circular frequency of the system , v_0 : static displacement due to p_o , v_{max} : maximum response and T_n : natural period of vibration. A plot of R_d (= v_{max} / v_0) versus t_r / T_n is shown in Figure 2(d). Comment on the effect of ratio t_r / T_n on R_d , without carrying out any "exact" dynamic analysis.

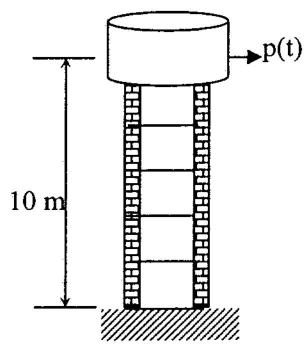

Figure 2(b)

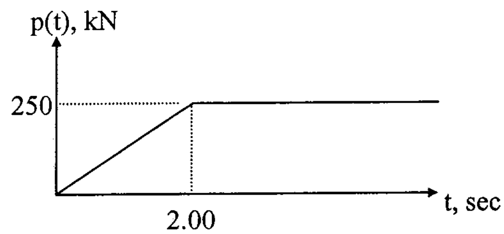

Figure 2(c)

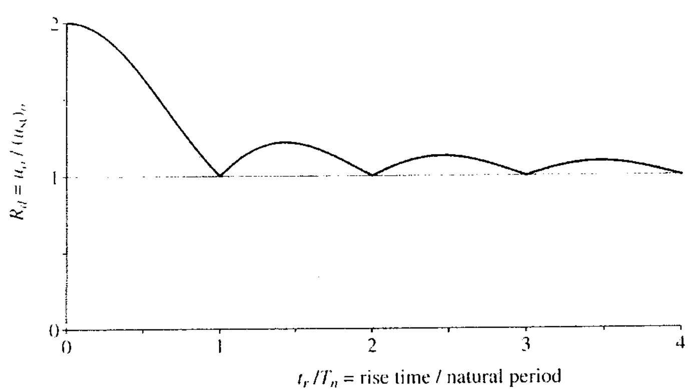

Figure 2(d)

(10 marks)

3. (a) Explain with the aid of suitable sketches the meaning of P-δ and P-Δ effect and their influence on the load carrying capacity of a beam-column.

(6 marks)

(b) Figure 3 shows an initial straight column subjected to an axial load P which acts at an eccentricity e from the centroidal axis of the column. Derive the following relation:

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right) - 1\right]$$

where δ : mid-height deflection and P_E : Euler buckling load (= $\pi^2 EI/L^2$).

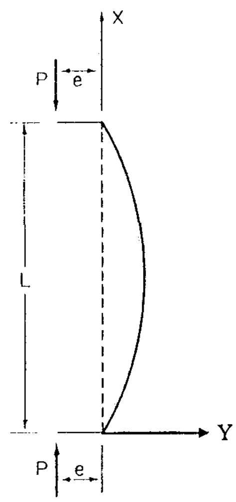

Figure 3

Sketch a plot of P/P_E versus δ for three different values of e. Sketch also on the same plot the graph representing the behaviour of an initially straight column with e = 0. Based on the graph, discuss the effect of imperfection of load on the behaviour of an axially loaded column.

(14 marks)

4. (a) By using the fourth order differential equation for a beam-column and its corresponding solution, obtain the eigenvalue problem which can be used to solve for the critical load of the column in Figure 4. Given that end A of the column is pinned and end B of the column is restrained against rotation but not restrained against translation in y-direction.

(8 marks)

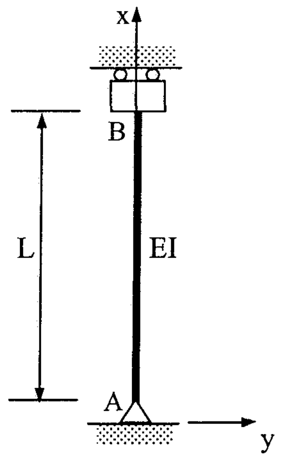

Figure 4

(b) A simple two-bar frame is shown in Figure 5. A load P acts at end B of vertical member AB. Both supports A and C are fixed. Obtain the effective length L_e for the frame by using the following equation for an elastically retrained column:

$$(1-\lambda_1-\lambda_2-\lambda_1\lambda_2\Phi^2)\Phi\sin\Phi+(2+\lambda_1\Phi^2+\lambda_2\Phi^2)\cos\Phi-2=0$$

where $\lambda_1=EI/(\alpha_1 L)$, $\lambda_2=EI/(\alpha_2 L)$, $\Phi=kL$, $k^2=P/EI$, EI : flexural rigidity , L : length of column, α_1, α_2 : rotational stiffness of end 1 and 2 of column being studied, respectively. If the flexural rigidity of beam is changed from 2EI to EI, comment on the change in value of L_e with proper justification without any extra calculation.

(12 marks)

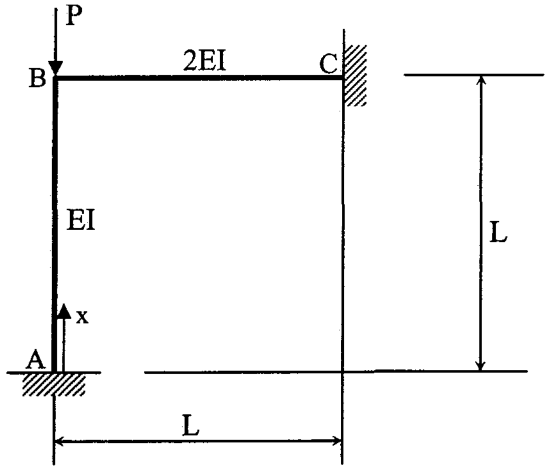

Figure 5

PART B : Answer ONE (1) question.

5. (a) Figure 5 shows a graph of critical stress σ_{cr} that a column can carry versus L_e/r for a particular material, where L_e :effective length of a column and r radius of gyration of the section of the column. Explain the meaning of the graph in terms of failure of a column under compression.

(6 marks)

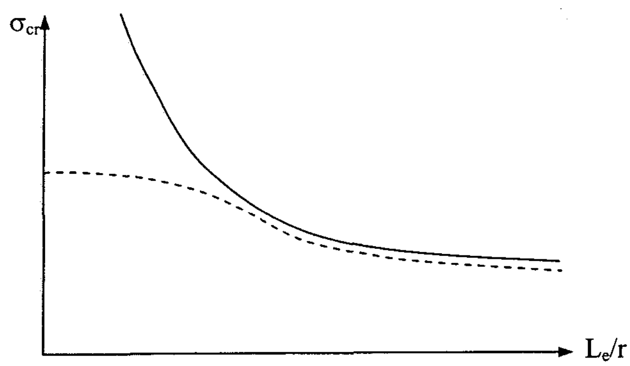

Figure 5

(b) Figure 6 shows a frame subjected to a vertical load at B. Points A, C and D are fixed. Using slope deflection equation approach with stability functions, determine the critical load for the compression member BD. Neglect the effect of axial force in beam member and assume that shortening in compression member BD is negligible prior to buckling.

(14 marks)

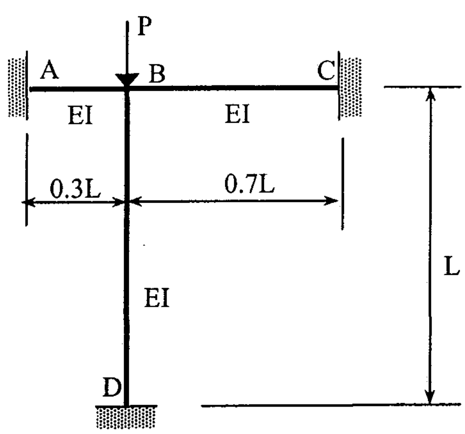

Figure 6

6. (a) The following relation is proposed under Rankine theory:

$$\frac{1}{P_C} = \frac{1}{P_S} + \frac{1}{P_{CR}}$$

where P_C : failure load of a column of a given material and of any length, P_S : failure load in compression of a short column of the same material and P_{CR} : buckling load of a long slender column of the same material. Starting from the above relation, derive the following relation for failure stress σ_C in a column:

$$\sigma_C = \frac{\sigma_S}{1 + k(L_e/r)^2}$$

where σ_S : yield stress in compression of the material of the column, L_e: effective length of the column, r: radius of gyration of the section of the column and $k=\sigma_S/(\pi^2 E)$.

(14 marks)

(b) In an experimental evaluation of the buckling loads for 12.5mm diameter, mild steel, pin-ended columns, the following two sets of results were obtained:

- length 500mm, buckling load 9800N
- length 200mm, buckling load 26400N

Assuming that both results are in agreement with Rankine formula, find the constants σ_S and k. Take E=200000 N/mm^2.

(6 marks)