UNIVERSITI SAINS MALAYSIA

First Semester Examination
2007/2008 Academic Session

October / November 2007

# EAS 661/4 – Advanced Structural Mechanics

Duration: 3 hours

Please check that this examination paper consists of <u>**SEVEN**</u> pages of printed material before you begin the examination.

<u>**Instructions**</u>: Answer **FIVE (5)** questions only. All questions carry the same marks.

You may answer the question either in Bahasa Malaysia or English.

All questions <u>**MUST BE**</u> answered on a new sheet.

Write the answered question numbers on the cover sheet of the answer script.

1. (a) Figure 1 shows an infinitesimal volume in a three dimensional body under stressed condition. Using the notation of:

$$\boldsymbol{\sigma} = [\ \sigma_x\ \sigma_y\ \sigma_z\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}\ ]^T$$

and

$$\boldsymbol{\varepsilon} = [\ \varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx}\ ]^T$$

for the Cartesian components of stress and the corresponding strain, respectively, derive the constitutive equation $\boldsymbol{\varepsilon} = \boldsymbol{D\sigma}$ for the case of a homogeneous isotropic body. State clearly the meaning of all symbols used in the derivation.

(6 marks)

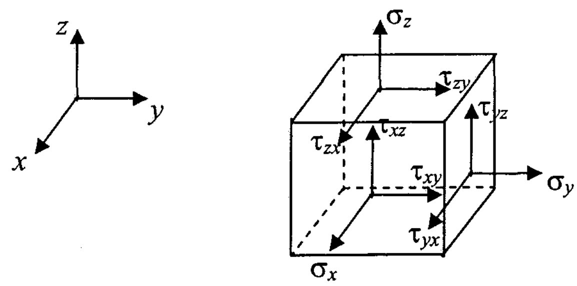


**Figure 1**

(b) Figure 2 shows a thin wall structure subjected to a distributed load $w$ in the plane of the wall. Justify why this problem can be classified as a plane stress problem. Derive the set of equilibrium equations for an infinitesimal volume of the wall.

(14 marks)

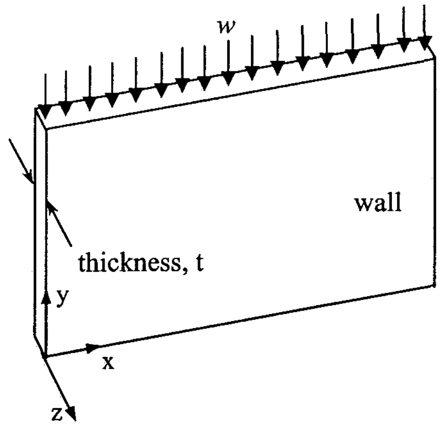


**Figure 2**

2. (a) Define the Principle of Virtual Displacement (PvD). Prove that the statement of PvD can be expressed as follows for a conservative problem:

$\delta W_e = \delta U_p$

where $\delta W_e$ : variation in external work and $\delta U_p$ : variation in strain energy.

By making use of the above statement of PvD for conservative problem, derive the equation of equilibrium for the linearly elastic spring in Figure 3 where $k$ : spring constant, $u$ : elongation of spring and $F$ : force acting on the spring.

(8 marks)


**Figure 3**

(b) Figure 4 shows a simply supported beam with an elastic spring prop at point B. The beam is subjected to a point load $P$ acting at the mid-span and a uniformly distributed load $w$. The following expression for lateral displacement field $v$ has been suggested:

$v = A_o \sin(\pi x / L)$

where $A_o$ is a constant and $x$ is the coordinate representing the axis of the beam. Show that the above displacement field $v$ is admissible. Next, solve for the constant $A_o$ by applying the principle of minimum potential energy. Flexural rigidity of beam is $EI$ and spring constant for the elastic spring is $k$.

(12 marks)

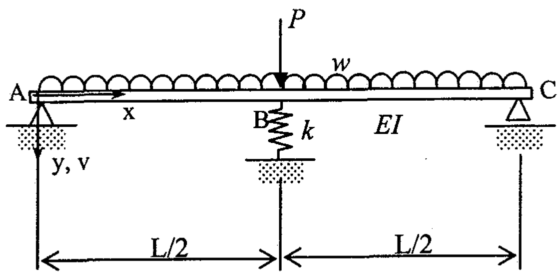


**Figure 4**

3. (a) **"For a small-displacement linearly elastic behaviour, the strain energy of a body can be expressed as a quadratic function of the strain."**

Prove the validity of the above statement for the case of an axially loaded bar as shown in Figure 5, where e is the elongation due to force F.

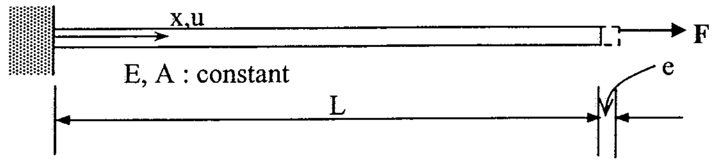


**Figure 5**

(6 marks)

(b) Figure 6 shows a stepped bar subjected to concentrated loads at points 2 and 3. Both points 1 and 4 are fixed. Cross-sectional areas for segments 1-2, 2-3 and 3-4 are 3A, 2A and A, respectively. Show clearly the process of obtaining the solution for the axial displacement *u* of the bar by the use of piecewise Rayleigh-Ritz method.

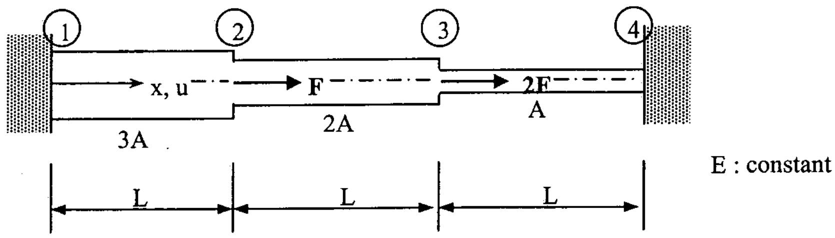


**Figure 6**

Use linear function for the axial displacement field *u*. You are required to show all principles and related equations used. Your answer must show the contents of all equations used in the process. Solution for the axial displacement is not required.

(14 marks)

4. (a) Briefly describe the difference between plane stress and plane strain in the discretization of the finite element problem for planar elasticity.

(5 marks)

(b) Show clearly in a step by step manner the development process of a stiffness matrix, $[K]^e$, for a triangular element in a state of plane stress as shown in Figure 7. Assume $E = 200\ GN/m^2$, $\nu = 0.3$ and $t = 1$ cm. Details of $[K]^e$, is not required.

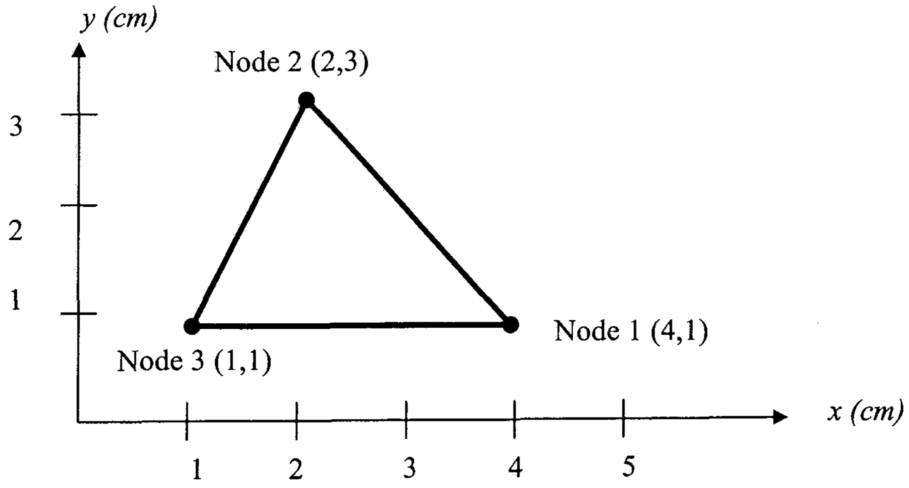


**Figure 7**

(15 marks)

5. (a) Explain the assumptions made in the modeling procedures for materials properties and loading conditions in Finite Element Method.

(5 marks)

(b) Two plates shown in Figure 8(a) and 8(b) shall be analysed as a plane strain problem. Both plates are divided into 9 elements. Each nodes has been labelled accordingly. Calculate the bandwidth, B = (R+1) NDOF for the plate assuming two degrees of freedom at each node.

(5 marks)

5. (c) Rearrange the node labeling in such a way that a minimum value of R is obtained.

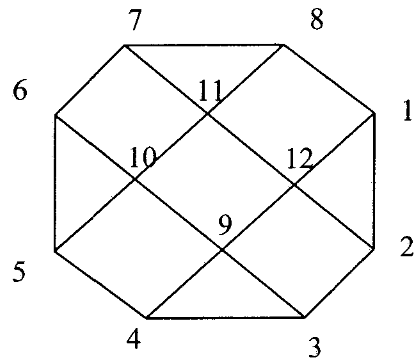


**Figure 8(a)**

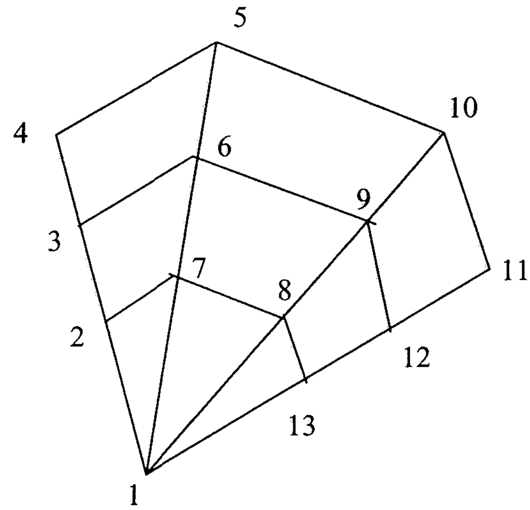


**Figure 8(b)**

(5 marks)

(d) Figure 9 shown a triangular element and its stiffness matrix $[k]^e$ is shown in Table 3. Given E = 200 GN/m$^2$, $\nu$ = 0.3 and t = 1 cm, calculate the value of u3 and v3 if horizontal force of 10 kN is applied at node 3, while node 1 and 2 are pinned.

(5 marks)

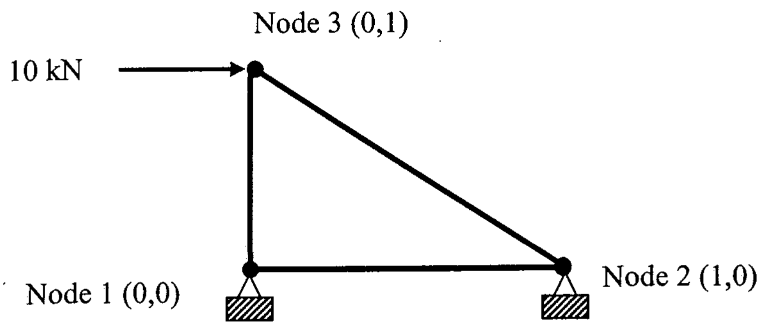


**Figure 9**

**Table 3**

$[K]^e$ =

| | | | | | |
|---|---|---|---|---|---|
| 1.35 | 0.65 | -1 | -0.35 | -0.35 | -0.3 |
| 0.65 | 1.35 | -0.3 | -0.35 | -0.35 | -1 |
| -1 | -0.3 | 1 | 0 | 0 | 0.3 |
| -0.35 | -0.35 | 0 | 0.35 | 0.35 | 0 |
| -0.35 | -0.35 | 0 | 0.35 | 0.35 | 0 |
| -0.3 | -1 | 0.3 | 0 | 0 | 1 |

6. (a) Explain the importance of model validity and accuracy of the following factors in the modeling procedures of Finite Element Method.

(8 marks)

[i] geometry
[ii] material properties
[iii] loading conditions
[iv] constraint conditions

(b) A plate with a hole is subjected to tension, p = 25.0 N/mm$^2$ as shown in Figure 10. It is a 2D plane stress problem. The plate is modeled with linear four-noded rectangular elements and three-noded triangular element. Assume Elastic Modulus E = 7.0 x 10$^4$ N/mm$^2$, thickness, t = 1 mm and Poisson ratio, $\nu = 0.25$.

[i] Explain and perform simplification through symmetry in modeling the plate using Finite Element Method.
[ii] State the boundary conditions applicable to the model in [i].
[iii] Sketch a suitable meshing for the model with rectangular and triangular elements.
[iv] Sketch the stress in z direction near the hole.
[v] Indicate point of maximum tension and maximum deflection.

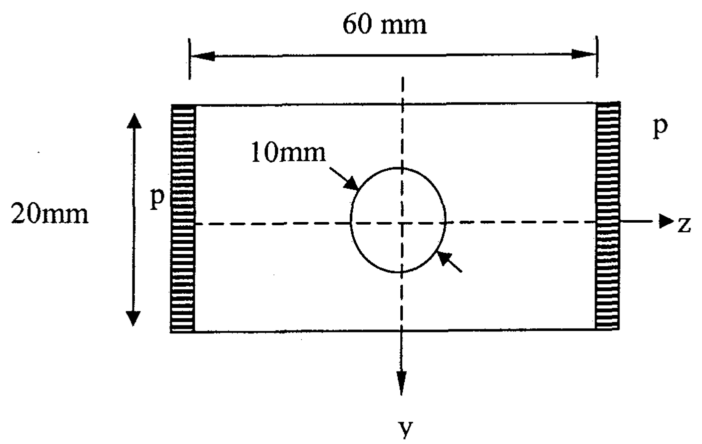


**Figure 10**

(10 marks)

(c) What will happen to the value of maximum stress and maximum displacement in z direction if the problem in b[v] is assumed as plane strain instead of plane stress problem?

(2 marks)

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