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UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2015/2016 Academic Session

June 2016

**CPT112 – Discrete Structures**  
*[Struktur Diskret]*

Duration : 3 hours  
*[Masa : 3 jam]*

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**INSTRUCTIONS TO CANDIDATE:**

***[ARAHAN KEPADA CALON:]***

- Please ensure that this examination paper contains **FOUR** questions in **EIGHT** printed pages before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT** soalan di dalam **LAPAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

- Answer **ALL** questions.

*[Jawab **SEMUA** soalan.]*

- You may answer the questions either in English or in Bahasa Malaysia.

*[Anda dibenarkan menjawab soalan sama ada dalam bahasa Inggeris atau bahasa Malaysia.]*

- In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi bahasa Inggeris hendaklah diguna pakai.]*

1. (a) Find the Cartesian product of  $A = \{\text{Ahmad, Chong}\}$  and  $B = \{a, b, c\}$ .

*Cari hasil darab Cartesian  $A = \{\text{Ahmad, Chong}\}$  dan  $B = \{a, b, c\}$ .*

(20/100)

- (b) Let  $A = \{a, b, c, d\}$  and  $B = \{a, b, c, d, e, f, g, h\}$ . Find

*Biarkan  $A = \{a, b, c, d\}$  dan  $B = \{a, b, c, d, e, f, g, h\}$ . Cari*

(i)  $A \cup B$

(4/100)

(ii)  $A \cap B$

(4/100)

(iii)  $A - B$

(4/100)

(iv)  $B - A$

(4/100)

(v)  $(A \cup (B - A))$

(12/100)

- (c) Let  $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ . Find

*Biarkan  $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  dan  $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ . Cari*

(i)  $A \vee B$

(8/100)

(i)  $A \wedge B$

(8/100)

(iii)  $A \odot B$

(12/100)

- (d) Find the Boolean product of A and B where

*Cari Boolean bagi hasil darab A dan B di mana*

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(24/100)

2. (a) Given  $S_n = 0 + 5 + 10 + 15 + \dots + 5n = 5n(n+1)/2$ , where  $n \in \mathbb{N}$ .

*Diberi  $S_n = 0 + 5 + 10 + 15 + \dots + 5n = 5n(n+1)/2$ , yang mana  $n \in \mathbb{N}$ .*

- (i) By using mathematical induction, prove:

*Dengan menggunakan matematik aruhan, buktikan:*

$$0 + 5 + 10 + 15 + \dots + 5n = 5n(n+1)/2, n \in \mathbb{N}.$$

(20/100)

- (ii) Find an implicit formula for  $S_n$ .

*Dapatkan formula tersirat untuk  $S_n$ .*

(10/100)

- (iii) Write a recursive pseudocode that can find the  $n$ -th term of  $S_n$ .

*Tuliskan satu pseudokod rekursi yang dapat mencari nilai ke- $n$  untuk  $S_n$ .*

(20/100)

- (b) A snail has been climbing a wall for many days. Within a day, its climb (in cm) is equal to: 3 times the climb of the previous day minus 2 times the climb of the previous, previous day (two days before) – this happen because it was slipping down the wall everytime it fell asleep at night. Note: Its yield for its first day climb is 1 cm and its yield for the second day climb is 2 cm.

*Seekor siput telah memanjat dinding untuk beberapa hari. Dalam masa sehari, pendakian (dalam cm) ia adalah sama dengan: 3 kali pendakian hari sebelumnya, ditolak dengan 2 kali pendakian dua hari sebelumnya - ini berlaku kerana siput itu tergelongsor ke bawah setiap kali ia tertidur pada sebelah malam. Nota: hasil pendakian hari pertama ia adalah 1 cm dan hasil pendakian hari kedua ia adalah 2 cm.*

- (i) Find an implicit formula to model the above situation.

*Cari satu rumus tersirat untuk memodelkan situasi di atas.*

(10/100)

- (ii) Use Linear Homogeneous Recurrence Relation (LHRR) to change the implicit formula found in 2(b)(i) to the equivalent explicit formula.

*Gunakan LHRR untuk menukarkan rumus tersirat di 2(b)(i) kepada rumus nyata/tersurat.*

(30/100)

- (iii) On what day the snail will reach the top of the wall if the wall is 1 meter high.

*Pada hari keberapakah siput tersebut akan sampai ke puncak dinding jika tinggi dinding itu ialah 1 meter.*

(10/100)

3. (a) Consider a relation  $R = \{(a, b) \mid a \text{ divides } b\}$  on the set  $\{1, 2, 3, 4, 5, 6\}$ . List down all the ordered pairs in  $R$ .

*Pertimbangkan hubungan  $R = \{(a, b) \mid a \text{ yang membahagikan } b\}$  pada set  $\{1, 2, 3, 4, 5, 6\}$ . Senaraikan semua pasangan tertib di  $R$ .*

(13/100)

- (b) For each of the relations below, which represent relations based on a set of School of Computer Sciences students

$R = \{(b, c) \mid b \text{ is taller than } c\}$ .

$R = \{(b, c) \mid b \text{ was born on the same day as } c\}$ .

$R = \{(b, c) \mid b \text{ has same name as } c\}$ .

*Bagi setiap hubungan di bawah, yang berhasrat untuk mewakili hubungan antara satu set pelajar Pusat Pengajian Sains Komputer*

$R = \{(b, c) \mid b \text{ adalah lebih tinggi daripada } c\}$ .

$R = \{(b, c) \mid b \text{ dilahirkan pada hari yang sama dengan } c\}$ .

$R = \{(b, c) \mid b \text{ mempunyai nama sama seperti } c\}$ .

which relations on the set of School of Computer Sciences students is

*nyatakan keadaan yang mana hubungan di set pelajar Pusat Pengajian Sains Komputer adalah*

- (i) transitive.

*transitif.*

(9/100)

(ii) reflexive.

*refleksif.*

(9/100)

(iii) symmetric.

*simetri.*

(9/100)

- (c) In Malaysia, JPJ plans to have car registration numbers with two vowels letter and the last 4 are even digits. In that case, what are total of car registration number plates could be issued?

*Di Malaysia, JPJ merancang untuk menggunakan nombor pendaftaran kereta dengan dua huruf vokal dan 4 digit terakhir menggunakan nombor genap. Dalam kes ini, berapakah jumlah plat nombor pendaftaran kereta boleh dikeluarkan?*

(20/100)

- (d) How many poker hands of five cards can be dealt from a standard deck of 52 cards?

*Berapa banyak kombinasi poker yang menggunakan lima kad boleh diatur daripada 52 dek kad yang standard?*

(20/100)

- (e) Suppose that there are 47 academic staff at the School of Computer Sciences, USM. How many ways are there to select 5 academic staff as exco members?

*Andaikan terdapat 47 kakitangan akademik di Pusat Pengajian Sains Komputer, USM. Berapa banyak cara yang anda ada untuk memilih 5 staf akademik sebagai ahli exco?*

(20/100)

4. (a) Use the Dijkstra's algorithm to find the shortest path and the length of the path between the vertices **A** and **H** based on the following weighted graph shown in Figure 1:

*Gunakan algoritma Dijkstra untuk mencari laluan terdekat dan panjang laluan di antara nod A dengan H berdasarkan graf wajaran berikut ditunjukkan dalam Rajah 1:*

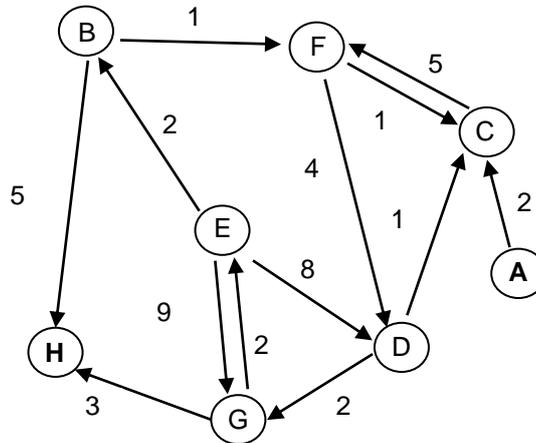


Figure 1  
Rajah 1

Produce a tracing as shown in Table 1 as your answer.

*Hasilkan satu jejakan seperti Jadual 1 sebagai jawapan anda.*

Nodes \ Iterations	Iterations			
	1	2	...	8
A	(0,A)			
B	$\infty$			
...	$\infty$			
$\infty$	$\infty$			

Table 1  
Jadual 1

(20/100)

- (b) Given the following tree (graph) as shown in Figure 2:

*Diberi pepohon (graf) seperti yang ditunjukkan dalam Rajah 2:*

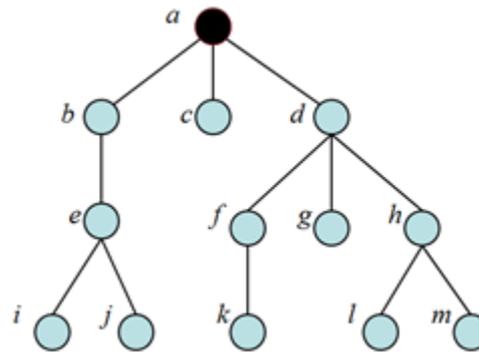


Figure 2

Rajah 2

- (i) Find the pre-order traversal of the tree.

*Cari penyusunan pepohon "pre-order".*

(8/100)

- (ii) Find the post-order traversal of the tree.

*Cari penyusunan pepohon "port-order".*

(8/100)

- (iii) Using the ordering  $c, d, e, f, g, h, i, j, k, l, m, a, b, c$  and the depth-first search algorithm, find a spanning tree for the graph (note: the graph is already a tree which is rooted at node  $a$ ).

*Dengan menggunakan susunan  $c, d, e, f, g, h, i, j, k, l, m, a, b, c$  dan algoritma gelintaran dalam-dahulu, cari pepohon rentang untuk graf (nota: graf ialah pepohon di mana berakar pada nod  $a$ ).*

(8/100)

- (c) Given  $G = (V, T, S, P)$  is the phrase-structure grammar with  $V = \{0, 1, A, B, S\}$ ,  $T = \{0, 1\}$ , and a set of productions  $P$  consisting of

*Diberi  $G = (V, T, S, P)$  ialah tatabahasa frasa-struktur dengan  $V = \{0, 1, A, B, S\}$ ,  $T = \{0, 1\}$ , dan set produksi  $P$  yang terdiri daripada*

$$\begin{aligned} S &\rightarrow 0A, \\ S &\rightarrow 1A, \\ A &\rightarrow 0B, \\ B &\rightarrow 1A, \\ B &\rightarrow 1 \end{aligned}$$

- (i) Show that 10101 belongs to the language generated by  $G$ .

*Tunjukkan bahawa 10101 tergolong dalam bahasa yang dihasilkan oleh  $G$ .*

(12/100)

- (ii) Show that 10110 does not belong to the language generated by  $G$ .

*Tunjukkan bahawa 10110 bukan tergolong dalam bahasa yang dihasilkan oleh  $G$ .*

(12/100)

- (iii) What is the language generated by  $G$ ?

*Apakah bahasa yang dihasilkan oleh  $G$ ?*

(12/100)

- (d) Draw the state diagram for the finite-state machine which is represented using the following state table.

*Lukiskan gambar rajah keadaan bagi mesin keadaan terhingga yang diwakilkan dengan menggunakan jadual keadaan berikut.*

State	$f$		$g$	
	Input		Output	
	$0$	$1$	$0$	$1$
$S_0$	$S_0$	$S_4$	$1$	$1$
$S_1$	$S_0$	$S_3$	$0$	$1$
$S_2$	$S_0$	$S_2$	$0$	$0$
$S_3$	$S_1$	$S_1$	$1$	$1$
$S_4$	$S_1$	$S_0$	$1$	$0$

(20/100)