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UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2014/2015 Academic Session

June 2015

**CPT115 – Mathematical Methods for Computer Science**  
*[Kaedah Matematik bagi Sains Komputer]*

Duration : 2 hours  
*[Masa: 2 jam]*

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**INSTRUCTIONS TO CANDIDATE:**

***[ARAHAN KEPADA CALON:]***

- Please ensure that this examination paper contains **THREE** questions in **TEN** printed pages before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TIGA** soalan di dalam **SEPULUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

- Answer **ALL** questions.

*[Jawab **SEMUA** soalan.]*

- You must answer the questions in English.

*[Anda mesti menjawab soalan dalam bahasa Inggeris.]*

- In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi bahasa Inggeris hendaklah diguna pakai.]*

1. (a) Determine the rank of the following matrix,  $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ .

(10/100)

- (b) Determine the consistency of the following system of linear equations and then solve it for the three unknowns:

$$2x + 2y + z = 5$$

$$x - y + z = 1$$

$$3x + y + 2z = 4$$

(10/100)

- (c) If  $z_1 = 2 + i$ ,  $z_2 = 3 - 2i$  and  $z_3 = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$ , determine the conjugate of  $z_1 z_2$  and  $(z_3)^4$ .

(20/100)

- (d) Prove De Moivre's theorem which states that for any rational number  $n$ , the following relationship is true:  $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$ .

(30/100)

- (e) Given that  $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$  and  $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ , the scalar product between two vectors is given by  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ . Derive the expression for the angle between two vectors,  $\theta$ .

(30/100)

2. (a) Write down the mathematical criteria to test whether a given function is even or odd.

(10/100)

- (b) Apply the above criteria (written in 2(a)) to verify whether  $f(x) = x^2 - 1$  is an even or an odd function.

(10/100)

(c) Given  $h(x) = \frac{2x-3}{5x-7}$ , find  $h^{-1}(x)$ .

(10/100)

(d) Assume that the loser of a card game for a betting amount of RM  $x$  is required to pay a penalty which is captured by the following mathematical function:

$$h(x) = \begin{cases} x^3 & \text{if } x \leq 3 \\ 9x & \text{if } x \geq 3 \end{cases}$$

(i) Sketch a graph of  $h(x)$  and shade the area represented by the integral

$$I = \int_0^5 h(x).$$

(ii) Evaluate the above integral  $I$  (given in Question 2(d)(i)).

(20/100)

(e) For the standard trigonometric equation  $y = \tan x$ , we know that  $\frac{dy}{dx} = \sec^2 x$ . Hence find  $\frac{dy}{dx} = \tan(5x - 5)$ .

(10/100)

(f) To fit a second order polynomial with three data points,  $(x_0, y_0)$ ,  $(x_1, y_1)$  and  $(x_2, y_2)$ , the Lagrangian formula takes the following form:

$$y = \frac{\{(x-x_1)(x-x_2)\} \{[(x_0-x_1)(x_0-x_2)]\} y_0 + \{(x-x_0)(x-x_2)\} \{[(x_1-x_0)(x_1-x_2)]\} y_1 + \{(x-x_0)(x-x_1)\} \{[(x_2-x_0)(x_2-x_1)]\} y_2}{\{(x-x_1)(x-x_2)\} \{[(x_0-x_1)(x_0-x_2)]\} + \{(x-x_0)(x-x_2)\} \{[(x_1-x_0)(x_1-x_2)]\} + \{(x-x_0)(x-x_1)\} \{[(x_2-x_0)(x_2-x_1)]\}} \quad (1)$$

(i) By extending equation (1), state the Lagrangian formula to fit a third order polynomial for a given set of four data points.

(10/100)

(ii) Hence find the third degree polynomial  $f(x)$  satisfying the following data:

X	1	3	5	7
Y	24	120	336	720

(20/100)

(g) Justify whether the differential equation  $u^3 \frac{du}{dv} + u = 20$  is linear or non-linear.

(10/100)

3. (a) Below are the final fitnesses of an algorithm from ten repeated simulations:

16.6	13.0	15.2	15.4	10.7
17.6	18.4	13.7	15.1	15.1

Use a stem-and-leaf plot to order the given data. Then find the minimum, maximum, mean, mode and outlier values.

(10/100)

- (b) The following scores were obtained in a statistics exam:

74	80	65	85	95
72	76	72	93	84
75	75	60	74	75
63	78	87	90	70

Calculate the mean, median, variance, and standard deviation for this data set.

(10/100)

- (c) Consider the following data:

2	7	4	4	6	1
8	15	12	7	3	16
1	2	11	5	15	4

Find the 5-number summary of the data (min, Q1, Q2, Q3, max).

(10/100)

- (d) Let  $X$  be a normal random variable with standard deviation = 10. Given  $z$  value = 1.65.

- (i) Find the margin of error for a 90 percent confidence interval for mean corresponding to a sample size of 12.

(3/100)

- (ii) Interpret the result of Question 3(d)(i).

(4/100)

- (iii) With reference to Question 3(d)(i), find an approximate 90 percent confidence interval for mean if the 12 sample values of  $X$  are as follows:

95	103	107	98	90	110
92	108	90	94	105	100

(7/100)

- (iv) Interpret the result of Question 3(d)(iii).

(6/100)

- (e) 25 percent of students failed mathematics, 15 percent failed chemistry, and 10 percent failed both mathematics and chemistry. A student is selected at random.

- (i) If the student failed chemistry, what is the probability that he or she failed mathematics?

(8/100)

- (ii) If the student failed mathematics, what is the probability that he or she failed chemistry?

(8/100)

- (iii) What is the probability that the student failed mathematics or chemistry?

(8/100)

- (iv) What is the probability that the student failed neither mathematics nor chemistry?

(8/100)

- (f) Suppose a student dormitory in a university consists of the following:

- (1) 30 percent are first year students of whom 10 percent own a car.
- (2) 40 percent are second year students of whom 20 percent own a car.
- (3) 20 percent are third year students of whom 40 percent own a car.
- (4) 10 percent are final year students of whom 60 percent own a car.

A student is randomly selected from the dormitory.

- (i) Find the probability that the student owns a car.

(9/100)

- (ii) If the student owns a car, find the probability that the student is a third year student.

(9/100)

1. (a) Tentukan pangkat matriks berikut,  $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ .

(10/100)

- (b) Tentukan konsistensi sistem persamaan-persamaan linear berikut dan selesaikan bagi ketiga-tiga anu:

$$2x + 2y + z = 5$$

$$x - y + z = 1$$

$$3x + y + 2z = 4$$

(10/100)

- (c) Jika  $z_1 = 2 + i, z_2 = 3 - 2i$  dan  $z_3 = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$ , tentukan konjugat bagi  $z_1 z_2$  dan  $(z_3)^4$ .

(20/100)

- (d) Buktikan teorem De Moivre's yang menyatakan bahawa bagi mana-mana nombor rasional  $n$ , hubungan berikut adalah benar:  $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ .

(30/100)

- (e) Diberi  $\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$  dan  $\vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$ , produk skala antara dua vektor adalah  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$ . Terbitkan ungkapan bagi sudut antara dua vektor,  $\theta$ .

(30/100)

2. (a) Tuliskan kriteria matematik untuk menguji sama ada suatu fungsi yang diberikan adalah genap atau ganjil.

(10/100)

- (b) Gunakan kriteria di atas (daripada 2(a)) untuk mengesahkan sama ada  $f(x) = x^2 - 1$  adalah fungsi genap atau ganjil.

(10/100)

- (c) Diberi  $h(x) = \frac{2x-3}{5x-7}$ , cari  $h^{-1}(x)$ .

(10/100)

- (d) Andaikan pemain yang kalah dalam permainan kad untuk jumlah pertaruhan sebanyak RM  $x$  dikehendaki membayar penalti berdasarkan fungsi matematik berikut:

$$h(x) = \begin{cases} x^3 & \text{if } x \leq 3 \\ 9x & \text{if } x \geq 3 \end{cases}$$

- (i) Lakarkan graf bagi  $h(x)$  dan lorekkan kawasan yang diwakili oleh kamiran

$$I = \int_0^5 h(x).$$

- (ii) Cari kamiran  $I$  di atas (daripada Soalan 2(d)(i)).

(20/100)

- (e) Bagi persamaan trigonometri standard  $y = \tan x$ , diberi  $\frac{dy}{dx} = \sec^2 x$ . Cari nilai untuk  $\frac{dy}{dx} = \tan(5x - 5)$ .

0.(10/100)

- (f) Bagi memuatkan polinomial peringkat kedua dengan tiga titik data,  $(x_0, y_0)$ ,  $(x_1, y_1)$  dan  $(x_2, y_2)$ , formula Lagrangian mengambil bentuk berikut:

$$y = \frac{\{(x - x_1)(x - x_2)\} / \{(x_0 - x_1)(x_0 - x_2)\}}{y_0} + \frac{\{(x - x_0)(x - x_2)\} / \{(x_1 - x_0)(x_1 - x_2)\}}{y_1} + \frac{\{(x - x_0)(x - x_1)\} / \{(x_2 - x_0)(x_2 - x_1)\}}{y_2} \quad (1)$$

- (i) Dengan memperluaskan persamaan (1), nyatakan formula Lagrangian untuk memuatkan polinomial peringkat ketiga dengan empat titik data.

(10/100)

- (ii) Cari polinomial peringkat ketiga  $f(x)$  yang memenuhi data berikut:

X	1	3	5	7
Y	24	120	336	720

(20/100)

- (g) Terangkan sama ada persamaan pembezaan  $u^3 \frac{du}{dv} + u = 20$  adalah linear atau tidak linear.

(10/100)

3. (a) Nombor-nombor di bawah menunjukkan nilai kecergasan akhir suatu algoritma daripada sepuluh simulasi berulang:

16.6	13.0	15.2	15.4	10.7
17.6	18.4	13.7	15.1	15.1

Susunkan data dengan menggunakan kaedah '*stem-and-leaf plot*'. Kemudian, cari nilai minimum, maksimum, purata, mod, dan undur luaran.

(10/100)

- (b) Markah berikut diperoleh dalam peperiksaan statistik:

74	80	65	85	95
72	76	72	93	84
75	75	60	74	75
63	78	87	90	70

Kira nilai purata, median, varians, dan sisihan piawai bagi set data di atas.

(10/100)

- (c) Pertimbangkan data berikut:

2	7	4	4	6	1
8	15	12	7	3	16
1	2	11	5	15	4

Kira ringkasan 5 nombor data (min, Q1, Q2, Q3, max).

(10/100)

- (d) Biar  $X$  adalah pemboleh ubah rawak normal dengan sisihan piawai = 10. Diberi nilai  $z = 1.65$ .

- (i) Kira jidar selisih untuk selang keyakinan 90 peratus untuk purata yang sepadan dengan saiz sampel sebanyak 12.

(3/100)

- (ii) Terang dan rumuskan keputusan daripada Soalan 3(d)(i).

(4/100)



- (iii) Merujuk kepada Soalan 3(d)(i), kira anggaran selang keyakinan 90 peratus bagi purata jika nilai 12 sampel X adalah seperti berikut:

95	103	107	98	90	110
92	108	90	94	105	100

(7/100)

- (iv) Terang dan rumuskan keputusan daripada Soalan 3(d)(iii).

(6/100)

- (e) 25 peratus pelajar gagal matematik, 15 peratus gagal kimia, dan 10 peratus gagal kedua-dua matematik dan kimia. Seorang pelajar dipilih secara rawak.

- (i) Sekiranya pelajar gagal kimia, apakah kebarangkalian dia gagal matematik?

(8/100)

- (ii) Sekiranya pelajar gagal matematik, apakah kebarangkalian dia gagal kimia?

(8/100)

- (iii) Apakah kebarangkalian untuk pelajar yang gagal matematik atau kimia?

(8/100)

- (iv) Apakah kebarangkalian untuk pelajar yang tidak gagal matematik atau kimia?

(8/100)

- (f) Andaikan sebuah asrama pelajar di universiti terdiri daripada berikut:

- (1) 30 peratus adalah pelajar tahun pertama yang 10 peratus daripadanya memiliki kereta.
- (2) 40 peratus adalah pelajar tahun kedua yang 20 peratus daripadanya memiliki kereta.
- (3) 20 peratus adalah pelajar tahun ketiga yang 40 peratus daripadanya memiliki kereta.
- (4) 10 peratus adalah pelajar tahun akhir yang 60 peratus daripadanya memiliki kereta.

Seorang pelajar dipilih secara rawak dari asrama.

- (i) Cari kebarangkalian pelajar itu memiliki kereta.

(9/100)

- (ii) Jika pelajar itu memiliki kereta, cari kebarangkalian pelajar itu adalah pelajar tahun ketiga.

(9/100)