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*Research Article*

# A Generalized and Improved $(G'/G)$-Expansion Method for Nonlinear Evolution Equations

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A generalized and improved $(G'/G)$-expansion method is proposed for finding more general type and new travelling wave solutions of nonlinear evolution equations. To illustrate the novelty and advantage of the proposed method, we solve the KdV equation, the Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZKBBM) equation and the strain wave equation in microstructured solids. Abundant exact travelling wave solutions of these equations are obtained, which include the soliton, the hyperbolic function, the trigonometric function, and the rational functions. Also it is shown that the proposed method is efficient for solving nonlinear evolution equations in mathematical physics and in engineering.

## 1. Introduction

The world around us is inherently nonlinear, and nonlinear evolution equations (NLEEs) are widely used as models to describe the complex physical phenomena. The exact solutions of NLEEs play a vital role in nonlinear science and engineering. One of the fundamental problems for these models is to obtain their travelling wave solutions. The interest of finding travelling wave solution of NLEEs is increasing and has now become a hot topic to researchers. In recent years, many researchers who are interested in the nonlinear physical phenomena investigated exact solutions of NLEEs. They established many powerful and direct methods. For instance, the inverse scattering method [1], the Backlund transform method [2, 3], the Hirota's bilinear transformation method [4], the truncated Painleve expansion method [5, 6], the Exp-function method [7–11], the tanh-function method [12–15], the Weierstrass elliptic function method [16], the Jacobi elliptic function expansion method [17–23], and so on.

Recently, Wang et al. [24] introduced a widely used straightforward method called the $(G'/G)$-expansion method for obtaining the travelling wave solutions of various NLEEs, where $G(\xi)$ satisfies the second-order linear ordinary differential equation (ODE) $G'' + \lambda G' + \mu G = 0$, and $\lambda$ and $\mu$ are arbitrary constants. Applications of the $(G'/G)$-expansion method, to NLEEs can be found in the articles [25–29] for better understanding.

To show the effectiveness and reliability of the $(G'/G)$-expansion method and to expand the range of its applicability, further research has been carried out by several researchers. Such as, Guo and Zhou [30] proposed the extended $(G'/G)$-expansion method in which the solutions are presented in the form $u = a_0 + \sum_{i=1}^{n}\{a_i(G'/G)^i + b_i\,(G'/G)^{i-1}\sqrt{\sigma(1+(1/\mu)(G'/G)^2}\}$ and obtained new travelling wave solutions of the Whitham-Broer-Kaup-like equation and couple Hirota-Satsuma KdV equations. Applying this extended method Zayed and Al-Joudi [31] constructed the traveling wave solutions of some nonlinear evolution equations. Zayed and El-Malky [32] also applied the extended $(G'/G)$-expansion method to higher-dimensional evolution equations. Zhang et al. [33] presented an improved $(G'/G)$-expansion method to seek general travelling wave solutions. In the original method the solution is presented as nonnegative power of $(G'/G)$, but in [33] Zhang et al. proposed that the power may be any integral number. Zayed and Gepreel [34] employed the improved $(G'/G)$-expansion method to Konopelchenko-Dubrovsky equation, Karsten-Krasil' Shchik equation, Whitham-Broer-Kaup equation, and the fifth-order KdV equations to construct traveling wave solutions. Zayed [35] presented a new approach of the $(G'/G)$-expansion method where $G(\xi)$ satisfies the Jacobi elliptical equation $[G'(\xi)]^2 = e_2G^4(\xi) + e_1G^2(\xi) + e_0$, $e_2, e_1, e_0$ that are arbitrary constants and obtained new exact solutions of some NLEEs. Zayed [36] again presented a further alternative approach of this method in which $G(\xi)$ satisfies the Riccati equation $G'(\xi) = A + B\,G^2(\xi)$, where $A$ and $B$ are arbitrary constants.

Still substantial work has to be done in order for the $(G'/G)$ -expansion method to be well established, since every nonlinear equation has its own physically significant rich structure. In this paper, we propose a generalized and improved $(G'/G)$-expansion method for solving NLEEs in mathematical physics and engineering. To show the reliability and advantages of the proposed method, the KdV equation, the ZKBBM equation, and the strain wave equation in microstructured solids are solved, and further new families of exact solutions are found.

## 2. Description of the Generalized and Improved $(G'/G)$-Expansion Method

Let us consider the nonlinear partial differential equation of the form

$$P(u,\ u_t,\ u_x,\ u_{tt},\ u_{tx},\ u_{xx}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, $P$ is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of the method are as follows.

*Step 1.* Combining the real variables $x$ and $t$ by a complex variable $\xi$, we suppose that

$$u(x,t) \ = u(\xi), \quad \xi = x \pm Vt, \tag{2.2}$$

where $V$ is the speed of the travelling wave. The transformation (2.2) transforms (2.1) into an ODE for $u = u(\xi)$

$$Q(u,\ u',\ u'',\ u''',\ldots) = 0, \tag{2.3}$$

where $Q$ is a function of $u(\xi)$ and its total derivatives.

*Step 2.* Suppose the solution of the ODE (2.3) can be expressed by a polynomial in $(d+(G'/G))$ as follows:

$$u(\xi) = \sum_{n=-m}^{m} \frac{e_{-n}}{(d+(G'/G))^{n}}, \tag{2.4}$$

where either $a_{-m}$ or $a_m$ may be zero, but both $a_{-m}$ and $a_m$ can not be zero simultaneously, $a_n(n = 0, \pm 1, \pm 2, \ldots, \pm m)$ and $d$ are constants to be determined later, and $G = G(\xi)$ satisfies the following second order linear ODE:

$$G'' + \lambda G' + \mu G = 0. \tag{2.5}$$

*Step 3.* The value of the positive integer $m$ can be determined by considering the homogeneous balance between the highest-order derivatives and highest-order nonlinear terms appearing in (2.3). If the degree of $u(\xi)$ is $D[u(\xi)] = m$, then the degree of the other expressions will be as follows:

$$D\left[\frac{d^{p}u(\xi)}{d\xi^{p}}\right] = m + p, \qquad D\left[u^{p}\left(\frac{d^{q}u(\xi)}{d\xi^{q}}\right)^{s}\right] = mp + s(m+q). \tag{2.6}$$

*Step 4.* Substituting (2.4) along with (2.5) into (2.3), we obtain polynomials in $(d + G'/G)^{m}$ and $(d + G'/G)^{-m}$, $(m = 0, 1, 2, 3, \ldots)$. Collecting each coefficient of the resulted polynomials to zero yields a set of algebraic equations for $a_n(n = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm m)$, $d$ and $V$.

*Step 5.* Suppose that the value of the constants $a_n(n = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm m)$, $d$, and $V$ can be obtained by solving the algebraic equations obtained in Step 4. Since the general solution of (2.5) is well known for us, substituting the values of $a_n(n = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm m)$, $d$ and $V$ into (2.4), we obtain more general type and new exact traveling wave solutions of the nonlinear evolution equation (2.1).

## 3. Applications of the Proposed Method

In this section, we employ the proposed method to obtain some new and more general exact travelling wave solutions of the celebrated KdV equation, the ZKBBM equation, and the strain wave equation in microstructured solids.

### *3.1. The KdV Equation*

Let us consider the KdV equation,

$$u_t + uu_x + \delta u_{xxx} = 0. \tag{3.1}$$

Making use of the travelling wave transformation $\xi = x - V\ t$, (3.1) is converted into the following ODE:

$$-Vu' + uu' + \delta u''' = 0. \tag{3.2}$$

Equation (3.2) is integrable, therefore, integrating we obtain that

$$C - Vu + \frac{1}{2}u^2 + \delta u'' = 0, \tag{3.3}$$

where $C$ is an integral constant. Substituting (2.4) into (3.3) and considering the homogeneous balance between the highest-order derivative $u''$ and the nonlinear term $u^2$, we obtain $m = 2$.

Therefore, the solution of (3.3) is of the form

$$u(\xi) = e_2\left(d + \frac{G'}{G}\right)^2 + e_1\left(d + \frac{G'}{G}\right) + e_0 + \frac{e_{-1}}{(d + (G'/G))} + \frac{e_{-2}}{(d + (G'/G))^2}. \tag{3.4}$$

Substituting (3.4) into (3.3), the left hand side is transformed into polynomials in $(d + (G'/G))^m$ and $(d + (G'/G))^{-m}$, $(m = 0, 1, 2, 3, \ldots)$. Equating each coefficient of these polynomials to zero, we obtain a set of simultaneous algebraic equations (we will omit to display them for simplicity) for $e_0$, $e_1$, $e_2$, $e_{-1}$, $e_{-2}$, $d$, $C$, and $V$. Solving the set of simultaneous algebraic equations by using the symbolic computation systems, such as Maple 13, we obtain the following.

*Case 1.* $e_2 = -12\delta$, $e_1 = 12\delta(2d - \lambda)$, $e_0 = e_0$, $e_{-1} = 0$, $e_{-2} = 0$, $d = d$,

$$\begin{aligned} V &= \delta\lambda^2 + 8\delta\mu + e_0 + 12\delta d(d - \lambda), \\ C &= e_0^2/2 + 8\delta\mu e_0 + \delta\lambda^2 e_0 + 24\delta^2\mu^2 + 12\delta^2\lambda^2\mu - 144\delta^2\lambda d^3 + 72\delta^2 d^4 - 96\delta^2\lambda\mu d \\ &\quad - 12\delta\lambda e_0 d + 12\delta e_0 d^2 + 84\delta^2\lambda^2 d^2 + 96\delta^2\mu d^2 - 12\delta^2\lambda^3 d, \end{aligned} \tag{3.5}$$

where $e_0$, $d$, $\lambda$, and $\mu$ are free parameters.

*Case 2.* $e_2 = 0$, $e_1 = 0$, $e_0 = e_0$, $e_{-1} = -12\delta\lambda\mu + d(24\delta d^2 - 36\delta\lambda d + 24\delta\mu + 12\delta\lambda^2)$, $e_{-2} = -12\delta\mu^2 + d(24\delta\lambda\mu - 12\delta d^3 - 12\delta\lambda^2 d - 24\delta\mu d + 24\delta\lambda d^2)$, $d = d$,

$$\begin{aligned} V &= 8\delta\mu + \lambda^2\delta + e_0 + 12\delta d(d - \lambda), \\ C &= \frac{e_0^2}{2} + 8\delta e_0\mu + \delta\lambda^2 e_0 + 24\delta^2\mu^2 + 12\delta^2\lambda^2\mu - 144\delta^2\lambda d^3 + 72\delta^2 d^4 \\ &\quad - 96\delta^2\lambda\mu d - 12\delta\lambda e_0 d + 12\delta e_0 d^2 + 84\delta^2\lambda^2 d^2 + 96\ \delta^2\mu d^2 - 12\delta^2\lambda^3 d, \end{aligned} \tag{3.6}$$

where $e_0$, $d$, $\lambda$, and $\mu$ are free parameters.

*Case 3.* $e_2 = -12\delta$, $e_1 = 0$, $e_0 = e_0$, $e_{-1} = 0$, $e_{-2} = -(3/4)\delta\lambda^4 + 6\delta\lambda^2\mu - 12\delta\mu^2$, $d = \lambda/2$,

$$V = -2\delta\lambda^2 + 8\delta\mu + e_0,$$
$$C = -6\delta^2\lambda^4 + 48\delta^2\lambda^2\mu - 96\delta^2\mu^2 - 2\delta\lambda^2 e_0 + 8\delta\mu e_0 + \left(\frac{1}{2}\right)e_0^2, \tag{3.7}$$

where $e_0$, $\lambda$, and $\mu$ are free parameters.

For Case 1, substituting (3.5) into (3.4) and simplifying, we obtain the following.
When $\lambda^2 - 4\mu > 0$,

$$u_{1_1}(x,t) = -3\delta\left(\lambda^2 - 4\mu\right)\left(\frac{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^2 + 3\delta\lambda^2 + 12\delta d(d-\lambda) + e_0, \tag{3.8}$$

where $\xi = x - \{(\delta\lambda^2 + 8\delta\mu + e_0) + 12\delta d\ (d-\lambda)\}t$, and $A$, $B$ are arbitrary constants. If $A$, $B$, $e_0$, $d$, $\lambda$, and $\mu$ take special values, various known results in the literature can be rediscovered.

Suppose that $A > 0$ and $A^2 > B^2$, then solution (3.8) reduces to

$$u_{1_1}(x,t) = 3\delta\left(\lambda^2 - 4\mu\right)\ \text{sech}^2\left(\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\xi + \xi_0\right) + 12\left(\delta\mu + \delta d(d-\lambda)\right) + e_0, \tag{3.9}$$

where $\xi_0 = \tan\text{h}^{-1}(B/A)$.

When $\lambda^2 - 4\mu < 0$,

$$u_{1_1}(x,t) = -3\delta\left(4\mu - \lambda^2\right)\left(\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\ \xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2 + 3\ \delta\ \lambda^2 + 12\delta d(d-\lambda) + e_0. \tag{3.10}$$

When $\lambda^2 - 4\mu = 0$,

$$u_{1_3}(x,t) = -12\delta\frac{B^2}{(A+B\xi)^2} + 3\delta\lambda^2 + 12\delta d(d-\lambda) + e_0. \tag{3.11}$$

Again for Case 2, substituting (3.6) into (3.4) and simplifying, we obtain the following.

When $\lambda^2 - 4\mu > 0$,

$$u_{2_1}(x,t) = \frac{-12\delta\mu^2 + d(24\delta\lambda\mu - 12\delta d^3 - 12\delta\lambda^2 d - 24\delta\mu d + 24\delta\lambda d^2)}{\left(d - \frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \cosh\left((\sqrt{(\lambda^2-4\mu)}/2)\xi\right) + B\ \sinh\left((\sqrt{(\lambda^2-4\mu)}/2)\xi\right)}\right)^2}$$
$$+ \frac{-12\delta\lambda\mu + d(24\delta d^2 - 36\delta\lambda d + 24\delta\mu + 12\delta\lambda^2)}{d - \frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}} + e_0, \tag{3.12}$$

where $\xi = x - \{(8\delta\mu + \lambda^2\delta + e_0) + 12d\delta(d - \lambda)\}t$ and $A$, $B$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$,

$$u_{2_2}(x,t) = \frac{-12\delta\mu^2 + 12d(2\delta\lambda\mu - \delta d^3 - \delta\lambda^2 d - 2\delta\mu d + 2\delta\lambda d^2)}{\left(d - \frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2}$$
$$+ \frac{-12\delta\lambda\mu + 12d(2\delta d^2 - 3\delta\lambda d + 2\delta\mu + \delta\lambda^2)}{d - \frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-A\ \sin\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right)}} + e_0, \tag{3.13}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$u_{2_3}(x,t) = \frac{-12\delta\mu^2 + d(24\delta\lambda\mu - 12\delta d^3 - 12\delta\lambda^2 d - 24\delta\mu d + 24\delta\lambda d^2)}{(d - (\lambda/2) + (B/(A+B\xi)))^2} + \frac{-12\delta\lambda\mu + d(24\delta d^2 - 36\delta\lambda d + 24\delta\mu + 12\delta\lambda^2)}{(d - (\lambda/2) + (B/(A+B\xi)))} + e_0, \tag{3.14}$$

where $A$ and $B$ are arbitrary constants.

Finally for Case 3, substituting (3.7) into (3.4) and simplifying, we obtain the following.

When $\lambda^2 - 4\mu > 0$,

$$u_{3_1}(x,t) = -3\delta(\lambda^2 - 4\mu)\left(\frac{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^2 + \frac{-3\delta\lambda^4 + 24\delta\lambda^2\mu - 48\delta\mu^2}{(\lambda^2-4\mu)\left(\dfrac{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^2} + e_0, \tag{3.15}$$

where $\xi = x - (-2\delta\lambda^2 + 8\delta\mu + e_0)t$, $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$,

$$u_{3_2}(x,t) = -3\delta\left(4\mu - \lambda^2\right)\left(\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2 + \frac{-3\delta\lambda^4 + 24\delta\lambda^2\mu - 48\delta\mu^2}{(4\mu-\lambda^2)\left(\dfrac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2} + e_0. \tag{3.16}$$

When $\lambda^2 - 4\mu = 0$,

$$u_{3_3}(x,t) = -12\delta\frac{B^2}{(A+B\xi)^2} + \frac{(-3\delta\lambda^4 + 24\delta\lambda^2\mu - 48\delta\mu^2)(A+B\xi)^2}{4B^2} + e_0. \tag{3.17}$$

Wang et al. [24] investigated solutions of the KdV equation by the basic $(G'/G)$-expansion method and obtained only four solutions. But by our proposed method, we obtain ten distinct solutions of the KdV equation with additional free parameter $d$. It is noted that if $d = 0$ and/or $d = \lambda$, then our solutions $u_{1_1}, u_{1_2}$, and $u_{1_3}$(solutions (3.8)–(3.11)) are identical to the solutions $u_1$, $u_2$ and $u_3$ obtained by Wang et al. [24] (see Section A). But if $d \neq 0$ and $d \neq \lambda$, the solutions $u_{1_1}, u_{1_2}$, and $u_{1_3}$ are unlike to Wang et al. [24] solutions. Besides, we obtain additional solutions (3.12)–(3.17) which were not obtained by Wang et al. Therefore, we may assert that the basic $(G'/G)$-expansion method is a particular case of the proposed generalized and improved $(G'/G)$-expansion method. It is noteworthy to mention that, if we set special values the parameters, then some of the solutions match to some known solutions obtained by other methods, and some new solutions of the KdV equation are constructed. Since every nonlinear equation has its own physically significant rich structure; therefore, these new solutions will help us to understand the internal mechanism of the complex physical phenomena. Thus, the proposed generalized and improved $(G'/G)$-expansion method is promising in the discipline of mathematical physics and engineering.

### *3.2. The ZKBBM Equation*

Now we construct the traveling wave solutions of the ZKBBM equation by the proposed method. Let us consider the ZKBBM equation

$$u_t + u_x - 2auu_x - bu_{xxt} = 0. \tag{3.18}$$

The travelling wave variable $\xi = x + Vt$ permits us to reduce (3.18) into the following ODE:

$$(1 + V)u' - 2auu' - bVu''' = 0, \tag{3.19}$$

where prime denotes the derivatives with respect to $\xi$. Equation (3.19) is integrable, therefore, integrating we obtain

$$C + (1 + V)u - au^2 - bVu'' = 0, \tag{3.20}$$

where $C$ is an integral constant.

Considering the homogeneous balanced between the nonlinear term $u^2$ and the highest-order derivative $u''$ in (3.20), we get $m = 2$. Therefore, the formation of solution of (3.20) is same of (3.4).

Substituting (3.4) into (3.20), and collecting all the terms of the same power, the left hand side of (3.20) is converted into polynomials in $(d + G'/G)^m$ and $(d + G'/G)^{-m}$,$(m = 0, 1, 2, 3, \ldots)$. Setting each coefficient of these polynomials to zero, we obtain an overdetermined set of algebraic equations (we will omit them to display for simplicity) for $e_0$, $e_1$, $e_2$, $e_{-1}$, $e_{-2}$, $d$, $C$, and $V$. Solving this overdetermined set of algebraic equations, we obtain the following.

*Case 1.* $e_2 = 0$, $e_1 = 0$, $e_0 = -(bV\lambda^2 - V - 1 + 8bV\mu + 12bVd^2 - 12bV\lambda d)/2a$, $e_{-1} = 6bV(-\lambda\mu + \lambda^2 d + 2d^3 - 3\lambda d^2 + 2d\mu)/a$, $e_{-2} = -6bV(\mu^2 + \lambda^2 d^2 + d^4 - 2\lambda d^3 + 2d^2\mu - 2\lambda\mu d)/a$, $d = d$, $V = V$,

$$C = \frac{(-1 - V^2 - 2V + 16b^2V^2\mu^2 - 8b^2V^2\mu\lambda^2 + b^2V^2\lambda^4)}{4a}, \tag{3.21}$$

where $d$, $V$, $\lambda$ and $\mu$ are free parameters.

*Case 2.* $d = d$, $V = V$, $e_{-2} = 0$, $e_{-1} = 0$, $e_2 = -6\,bV/a$, $e_1 = 6bV(-\lambda + 2d)/a$, $e_0 = -(8bV\mu - V - 1 + bV\lambda^2 + 12bVd^2 - 12bV\lambda d)/2a$,

$$C = \frac{(-8b^2V^2\mu\lambda^2 - 2V - 1 + b^2V^2\lambda^4 + 16b^2V^2\mu^2 - V^2)}{4a}, \tag{3.22}$$

where $d$, $V$, $\lambda$, and $\mu$ are free parameters.

*Case 3.* $d = \lambda/2$, $V = V$, $e_{-1} = 0$, $e_{-2} = 3bV(8\lambda^2\mu - \lambda^4 - 16\mu^2)/8a$, $e_2 = -6\,bV/a$, $e_1 = 0$, $e_0 = (V+1 + 2bV\lambda^2 - 8bV\mu V)/2a$,

$$C = \frac{\left(-128b^2V^2\lambda^2\mu - 2V - 1 + 16b^2V^2\lambda^4 + 256b^2V^2\mu^2 - V^2\right)}{4a}, \tag{3.23}$$

where $V$, $\lambda$, and $\mu$ are free parameters.

Now substituting (3.21)–(3.23) into (3.4), we obtain the following solutions of (3.20):

$$\begin{aligned} u_1(\xi) =& \frac{-\left\{\left(bV\lambda^2 - V - 1 + 8bV\mu\right) + 12d(bVd - bV\lambda)\right\}}{2a} \\ &- \frac{6bV\left\{\lambda\mu - d\left(\lambda^2 + 2d^2 - 3\lambda d + 2\mu\right)\right\}}{a(d + (G'/G))} \\ &- \frac{6bV\left\{\mu^2 + d\left(\lambda^2 d + d^3 - 2\lambda d^2 + 2d\mu - 2\lambda\mu\right)\right\}}{a(d + (G'/G))^2}, \end{aligned} \tag{3.24}$$

$$\begin{aligned} u_2(\xi) =& -\frac{6bV}{a}\left(d + \frac{G'}{G}\right)^2 + \frac{6\,bV(-\lambda + 2\,d)}{a}\left(d + \frac{G'}{G}\right) \\ &- \frac{\left\{\left(8bV\mu - V - 1 + bV\lambda^2\right) + 12d(bVd - bV\lambda)\right\}}{2a}, \end{aligned} \tag{3.25}$$

$$u_3(\xi) = -\frac{6bV}{a}\left(\frac{\lambda}{2} + \frac{G'}{G}\right)^2 + \frac{3bV\left(8\lambda^2\mu - \lambda^4 - 16\mu^2\right)}{8a((\lambda/2) + (G'/G))^2} + \frac{\left(V + 1 + 2bV\lambda^2 - 8bV\mu V\right)}{2a}, \tag{3.26}$$

where $\xi = x + Vt$.

Substituting the general solutions of (2.5) into (3.24), we obtain three types of travelling wave solutions of the ZKBBM equation as the following.

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_{1_1}(x,t) \\ =& \frac{-6bV\left\{\mu^2 + d\left(\lambda^2 d + d^3 - 2\lambda d^2 + 2d\mu - 2\lambda\mu\right)\right\}}{a\left(d - \dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\dfrac{A\ \cosh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right)}{A\ \sinh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right)}\right)^2} \\ &- \frac{6bV\left\{\lambda\mu - d\left(\lambda^2 + 2d^2 - 2\lambda d + 2\mu\right)\right\}}{a\left(d - \dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\dfrac{A\cosh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right) + B\sinh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right)}{A\ \sinh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right) + B\cosh\left(\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi\right)}\right)} \\ &- \frac{\left\{\left(bV\lambda^2 + 8bV\mu - V - 1\right) + 12bVd(d - \lambda)\right\}}{2a}, \end{aligned} \tag{3.27}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$,

$$u_{1_2}(x,t) = \frac{-6bV\{\mu^2 + d(\lambda^2 d + d^3 - 2\lambda d^2 + 2d\mu - 2\lambda\mu)\}}{a\left(d - \dfrac{\lambda}{2} + \dfrac{\sqrt{4\mu-\lambda^2}}{2}\dfrac{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) - B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2}$$
$$- \frac{6bV\{\lambda\mu - d(\lambda^2 + 2d^2 - 2\lambda d + 2\mu)\}}{a\left(d - \dfrac{\lambda}{2} + \dfrac{\sqrt{4\mu-\lambda^2}}{2}\dfrac{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) - B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)}$$
$$- \frac{\{(bV\lambda^2 + 8bV\mu - V - 1) + 12bVd(d-\lambda)\}}{2a}, \tag{3.28}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$u_{1_3}(x,t) = \frac{-6bV\{\mu^2 + d(\lambda^2 d + d^3 - 2\lambda d^2 + 2d\mu - 2\lambda\mu)\}}{a(d - (\lambda/2) + (B/(A+B\xi)))^2}$$
$$- \frac{6\ bV(\lambda\mu - d(\lambda^2 + 2d^2 - 2\lambda d + 2\mu)\}}{a(d - (\lambda/2) + (B/(A+B\xi)))}$$
$$- \frac{\{bV\lambda^2 + 8bV\mu - V - 1 + 12bVd(d-\lambda)\}}{2a}, \tag{3.29}$$

where $A$ and $B$ are arbitrary constants.

Again substituting the general solutions of (2.5) into (3.25), we obtain three types of travelling wave solutions of the ZKBBM equation of the following.

When $\lambda^2 - 4\mu > 0$,

$$u_{2_1}(x,t) = -\frac{3bV(\lambda^2 - 4\mu)}{2a}\left(\frac{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^2$$
$$+ \frac{2bV\lambda^2 - 8bV\mu + V + 1}{2a}, \tag{3.30}$$

where $A$ and $B$ are arbitrary constants.

If $B > 0$ and $A^2 < B^2$, then we can obtain soliton solutions

$$u_{2_1}(x,t) = \frac{3bV(\lambda^2-4\mu)}{2a}\operatorname{sech}^2\left(\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\xi+\xi_0\right) - \frac{bV\lambda^2-4bV\mu-V-1}{2a}, \quad (3.31)$$

where $\xi_0 = \tan\mathrm{h}^{-1}(A/B)$.

When $\lambda^2-4\mu<0$,

$$u_{2_2}(x,t) = -\frac{3bV}{2a}\left(4\mu-\lambda^2\right)\left(\frac{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) - B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2 + \frac{2bV\lambda^2-8bV\mu+V+1}{2a}, \quad (3.32)$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2-4\mu=0$,

$$u_{2_3}(x,t) = \frac{-6bV}{a}\left(\frac{B}{A+B\xi}\right)^2 + \frac{(2bV\lambda^2-8bV\mu+V+1)}{2a}, \quad (3.33)$$

where $A$ and $B$ are arbitrary constants.

Finally substituting the general solutions of (2.5) into (3.26), we obtain the travelling wave solutions of the ZKBBM equation as the following.

When $\lambda^2-4\mu>0$,

$$u_{3_1}(x,t) = \frac{-3bV(\lambda^2-4\mu)^2}{8a\left(d+-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^2} - \frac{6bV}{a}\left(d+-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\ \sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\ \cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^2 + \frac{(V+1+2bV\lambda^2-8bV\mu V)}{2a}, \quad (3.34)$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$,

$$u_{3_2}(x,t) = -\frac{6bV}{a}\left(d - \frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2}\frac{A\ \cos\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right) - B\ \sin\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right)}{A\ \sin\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right)}\right)^2$$
$$- \frac{3bV(\lambda^2 - 4\mu)^2}{8a\left(d - \frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2}\frac{A\ \cos\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right) - B\ \sin\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right)}{A\ \sin\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi\right)}\right)^2}$$
$$+ \frac{(V + 1 + 2bV\lambda^2 - 8bV\mu V)}{2a}, \tag{3.35}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$u_{3_3}(x,t) = -\frac{6bV}{a}\left(d - \frac{\lambda}{2} + \frac{B}{A + B\xi}\right)^2 - \frac{3bV(\lambda^2 - 4\mu)^2}{8a(d - (\lambda/2) + (B/(A + B\xi)))^2}$$
$$+ \frac{(V + 1 + 2bV\lambda^2 - 8bV\mu V)}{2a}, \tag{3.36}$$

where $A$ and $B$ are arbitrary constants.

It is observed that if $d = 0$, then our solutions (3.27)–(3.33) are identical to Zhang et al. [33] solutions obtained by the improved $(G'/G)$-expansion method (see Section B). On the other hand, if $d \neq 0$, then the solutions (3.27)–(3.33) are dissimilar to Zhang et al. [33] solutions. Moreover, we obtain solutions (3.34)–(3.36) which were not obtained by Zhang et al. [33]. Therefore, we may stress that the improved $(G'/G)$-expansion method is also a particular case of our proposed method. It is noticed that the proposed generalized and improved $(G'/G)$-expansion method performs as a viable tool for finding generalized traveling wave solutions. The performance of the proposed method is trustworthy and efficient and gives more new solutions of nonlinear partial differential equations.

### *3.3. The Strain Waves Equation in Microstructured Solids*

Let us consider an engineering application problem of nonlinear bell-shaped and kink-shaped strain waves in microstructured solids as discussed by Porubov and Pastrone [37]. The governing equation for the strain waves in microstructured solids is given by

$$v_{tt} - v_{xx} - \varepsilon\alpha_1(v^2)_{xx} - \gamma\alpha_2 v_{xxt} + \delta\alpha_3 v_{xxxx}$$
$$-\left(\delta\alpha_4 - \gamma^2\alpha_7\right)v_{xxtt} + \gamma\delta(\alpha_5 v_{xxxxt} + \alpha_6 v_{xxttt}) = 0. \tag{3.37}$$

If $\gamma = 0$, we have the nondissipative case, and governed by the double dispersive equation (see [38] for details),

$$v_{tt} - v_{xx} - \varepsilon\alpha_1\left(v^2\right)_{xx} + \delta\alpha_3 v_{xxxx} - \delta\alpha_4 v_{xxtt} = 0. \tag{3.38}$$

The balance between nonlinearity and dispersion takes place when $\delta = O(\varepsilon)$. Therefore, (3.38) becomes,

$$v_{tt} - v_{xx} - \varepsilon\left\{\alpha_1\left(v^2\right)_{xx} - \alpha_3 v_{xxxx} + \alpha_4 v_{xxtt}\right\} = 0. \tag{3.39}$$

The travelling wave transformation $\xi = x - Vt$ allows us to convert (3.39) into the following ODE:

$$\left(V^2 - 1\right)v'' - \varepsilon\alpha_1\left(v^2\right)'' + \varepsilon\left(\alpha_3 - V^2\alpha_4\right)v^{(4)} = 0, \tag{3.40}$$

where $v''$ denotes the second derivative, and $v^{(4)}$ denote the fourth derivative with respect to $\xi$. Equation(3.40) is integrable, therefore, integrating we obtain

$$\left(V^2 - 1\right)v - \varepsilon\alpha_1\left(v^2\right) + \varepsilon\left(\alpha_3 - V^2\alpha_4\right)v'' + C = 0, \tag{3.41}$$

where $C$ is an integral constant.

Balancing the nonlinear term $v^2$ with the highest-order derivative term $v''$, from (3.41), we obtain that $m = 2$. Therefore, the shape of solution of (3.41) is also same as (3.4).

Substituting (3.4) into (3.41), and collecting all the terms of the same power, the left hand side of (3.41) is changed into polynomials in $(d + G'/G)^m$ and $(d + G'/G)^{-m}$,$(m = 0, 1, 2, 3, \ldots)$. Setting each coefficient of these polynomials to zero, we obtain a set of simultaneous algebraic equations (we will omit them for simplicity) for $e_0$, $e_1$, $e_2$, $e_{-1}$, $e_{-2}$, $d$, $C$, and $V$. Solving this overdetermined set of algebraic equations, we obtain the following.

*Case 1.* $V = V$, $e_{-1} = 0$, $e_{-2} = 0$, $e_2 = -6\ (V^2\alpha_4 - \alpha_3)/\alpha_1$, $e_1 = 6(2d - \lambda)(V^2\alpha_4 - \alpha_3)/\alpha_1$, $e_0 = ((-1)/(2\varepsilon\alpha_1))\{\varepsilon(\lambda^2 + 8\mu)(V^2\alpha_4 - \alpha_3) + 12\varepsilon d(d - \lambda)(V^2\alpha_4 - \alpha_3) + 1 - V^2\}$,

$$C = \frac{1}{4\varepsilon\alpha_1}\left\{\varepsilon^2\left(V^2\alpha_4 - \alpha_3\right)\left(\lambda^2 - 4\mu\right)^2 - \left(V^2 - 1\right)^2\right\}, \tag{3.42}$$

where $d$, $V$, $\lambda$, and $\mu$ are free parameters.

*Case 2.* $V = V$, $e_2 = 0$, $e_1 = 0$, $e_{-2} = -6(V^2\alpha_4 - \alpha_3)(d^2 - d\lambda + \mu)^2/\alpha_1$, $e_{-1} = 6(2d - \lambda)(d^2 - d\lambda + \mu)(V^2\alpha_4 - \alpha_3)/\alpha_1$, $e_0 = -\{\varepsilon(\lambda^2 + 8\mu)(V^2\alpha_4 - \alpha_3) + 12\varepsilon d(d - \lambda)(V^2\alpha_4 - \alpha_3) + 1 - V^2\}/2\varepsilon\alpha_1$,

$$C = \frac{\left\{\varepsilon^2(V^2\alpha_4 - \alpha_3)(\lambda^2 - 4\mu)^2 - (V^2 - 1)^2\right\}}{4\varepsilon\alpha_1}, \tag{3.43}$$

where $d$, $V$, $\lambda$, and $\mu$ are free parameters.

Now substituting (3.42)–(3.43) into (3.4), we obtain the following solutions of (3.41):

$$v_1(\xi) = \frac{-6}{\alpha_1}\left(V^2\alpha_4 - \alpha_3\right)\left(d + \frac{G'}{G}\right)^2 + \frac{6}{\alpha_1}(2d-\lambda)\left(V^2\alpha_4 - \alpha_3\right)\left(d + \frac{G'}{G}\right) - \frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2 + 8\mu\right)\left(V^2\alpha_4 - \alpha_3\right) + 12\varepsilon d(d-\lambda)\left(V^2\alpha_4 - \alpha_3\right) + 1 - V^2\right\}, \tag{3.44}$$

$$\begin{aligned} v_2(\xi) = &-\frac{6}{\alpha_1}\left(V^2\alpha_4 - \alpha_3\right)\left(d^2 - d\lambda + \mu\right)^2\left(d + \frac{G'}{G}\right)^{-2} \\ &+ \frac{6}{\alpha_1}(2d-\lambda)\left(d^2 - d\lambda + \mu\right)\left(V^2\alpha_4 - \alpha_3\right)\left(d + \frac{G'}{G}\right)^{-1} \\ &- \frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2 + 8\mu\right)\left(V^2\alpha_4 - \alpha_3\right) + 12\varepsilon d(d-\lambda)\left(V^2\alpha_4 - \alpha_3\right) + 1 - V^2\right\}, \end{aligned} \tag{3.45}$$

where $\xi = x - Vt$.

Substituting the general solutions of (2.5) into (3.44), we obtain three types of travelling wave solutions of the strain wave equation in microstructured solids as follows.

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} v_{1_1}(x,t) = &-\frac{6}{\alpha_1}\left(V^2\alpha_4 - \alpha_3\right) \\ &\times\left(d - \frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\,\frac{A\ \sinh((\sqrt{\lambda^2-4\mu}/2)\ \xi) + B\ \cosh((\sqrt{\lambda^2-4\mu}/2)\ \xi)}{A\ \cosh((\sqrt{\lambda^2-4\mu}/2)\ \xi) + B\ \sinh((\sqrt{\lambda^2-4\mu}/2)\ \xi)}\right)^2 \\ &+ \frac{6}{\alpha_1}(2d-\lambda)\left(V^2\alpha_4 - \alpha_3\right) \\ &\times\left(d - \frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\,\frac{A\ \sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right) + B\ \cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{A\ \cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right) + B\ \sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}\right) \\ &- \frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2 + 8\mu\right)\left(V^2\alpha_4 - \alpha_3\right) + 12\varepsilon d(d-\lambda)\left(V^2\alpha_4 - \alpha_3\right) + 1 - V^2\right\}, \end{aligned} \tag{3.46}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$,

$$\begin{aligned} v_{1_2}(x,t) = &-\frac{6}{\alpha_1}\left(V^2\alpha_4 - \alpha_3\right) \\ &\times\left(d - \frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\,\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^2 \end{aligned}$$

$$+\frac{6}{\alpha_1}(2d-\lambda)\left(V^2\alpha_4-\alpha_3\right)$$
$$\times\left(d-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)+B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)+B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)$$
$$-\frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2+8\mu\right)\left(V^2\alpha_4-\alpha_3\right)+12\varepsilon d(d-\lambda)\left(V^2\alpha_4-\alpha_3\right)+1-V^2\right\}, \tag{3.47}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2-4\mu=0$,

$$v_{1_3}(x,t)=-\frac{6}{\alpha_1}\left(V^2\alpha_4-\alpha_3\right)\left(d-\frac{\lambda}{2}+\frac{B}{A+B\xi}\right)^2$$
$$+\frac{6}{\alpha_1}(2d-\lambda)\left(V^2\alpha_4-\alpha_3\right)\left(d-\frac{\lambda}{2}+\frac{B}{A+B\xi}\right) \tag{3.48}$$
$$-\frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2+8\mu\right)\left(V^2\alpha_4-\alpha_3\right)+12\varepsilon d(d-\lambda)\left(V^2\alpha_4-\alpha_3\right)+1-V^2\right\},$$

where $A$ and $B$ are arbitrary constants.

Again substituting the general solutions of (2.5) into (3.45), we obtain the solutions of the stain wave in microstructured solids as follows.

When $\lambda^2-4\mu>0$,

$$v_{2_1}(x,t)=-\frac{6}{\alpha_1}\left(V^2\alpha_4-\alpha_3\right)\left(d^2-d\lambda+\mu\right)^2$$
$$\times\left(d-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A\sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^{-2}$$
$$+\frac{6}{\alpha_1}(2d-\lambda)\left(d^2-d\lambda+\mu\right)\left(V^2\alpha_4-\alpha_3\right)$$
$$\times\left(d-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A\sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}{A\cosh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)+B\sinh\left(\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi\right)}\right)^{-1}$$
$$-\frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2+8\mu\right)\left(V^2\alpha_4-\alpha_3\right)+12\varepsilon d(d-\lambda)\left(V^2\alpha_4-\alpha_3\right)+1-V^2\right\}, \tag{3.49}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu < 0$,

$$v_{2_2}(x,t) = -\frac{6}{\alpha_1}\left(V^2\alpha_4 - \alpha_3\right)\left(d^2 - d\lambda + \mu\right)^2 \times \left(d - \frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\,\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^{-2} + \frac{6}{\alpha_1}\ (2d-\lambda)\left(d^2 - d\lambda + \mu\right)\left(V^2\alpha_4 - \alpha_3\right) \times \left(d - \frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\,\frac{-A\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}{A\ \cos\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right) + B\ \sin\left(\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi\right)}\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2 + 8\mu\right)\left(V^2\alpha_4 - \alpha_3\right) + 12\varepsilon d(d-\lambda)\left(V^2\alpha_4 - \alpha_3\right) + 1 - V^2\right\}, \tag{3.50}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$v_{2_3}(x,t) = -\frac{6}{\alpha_1}\left(V^2\alpha_4 - \alpha_3\right)\left(d^2 - d\lambda + \mu\right)^2\left(d - \frac{\lambda}{2} + \frac{B}{A + B\xi}\right)^{-2} + \frac{6}{\alpha_1}(2d-\lambda)\left(d^2 - d\lambda + \mu\right)\left(V^2\alpha_4 - \alpha_3\right)\left(d - \frac{\lambda}{2} + \frac{B}{A + B\xi}\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\left\{\varepsilon\left(\lambda^2 + 8\mu\right)\left(V^2\alpha_4 - \alpha_3\right) + 12\varepsilon d(d-\lambda)\left(V^2\alpha_4 - \alpha_3\right) + 1 - V^2\right\}, \tag{3.51}$$

where $A$ and $B$ are arbitrary constants.

Porubov and Pastrone [37] investigated solutions of strain waves equation in microstructured solids, and they found a solution for (3.39) as follows:

$$v = 6k^2\left(V^2\alpha_4 - \alpha_3\right)\cosh^{-2}(k\ (x - Vt)), \tag{3.52}$$

where $k^2 = (V^2 - 1)/(4\varepsilon(V^2\alpha_4 - \alpha_3))$.

In this paper we obtain six solutions with more free parameters by our proposed expansion method. As a result, the proposed method might be an advance and efficient tool in solving nonlinear equations that arise in the field of engineering problems. If $\gamma \neq 0$, (3.37) can also be solved by the proposed expansion method.

## 4. Discussions

The advantages and limitations of the proposed expansion method over the basic $(G'/G)$-expansion method and the improved $(G'/G)$-expansion method are discussed below.

Advantages: The main advantage of the proposed expansion method over the basic $(G'/G)$-expansion method is that it provides more new exact traveling wave solutions along with additional free parameters. All the solutions obtained by basic $(G'/G)$-expansion method are through the proposed expansion method as a particular case, and in addition we obtain some new solutions. To clarify the fact, we apply the method in three equations: two are important in mathematical physics and another one is related to engineering problem, and, in all cases, we obtain some additional new exact solutions. The exact solutions have its great importance to reveal the internal mechanism of the physical phenomena. Apart from the physical relevance, the close-form solutions of nonlinear evolution equations facilitate the numerical solvers to compare the accuracy of their results and help them in the stability analysis.

In the basic $(G'/G)$-expansion method, if the order of the reduced ODE (the ODE obtained from the PDE by using traveling wave variable) is equal or less than three, with the help of symbolic computation software, such as Maple 13, it is mostly possible to find out a useful solution of the algebraic equations resulted in Step 4 of Section 2. Otherwise, it is generally unable to guarantee the existence of a solution of the resulted algebraic equations; this is because the number of the equations included in the set of algebraic equations is generally greater than the number of unknowns. But the proposed generalized and improved $(G'/G)$-expansion method might be used less than or equal to fourth-order-reduced ODE, since it contains more arbitrary constants compared to the basic $(G'/G)$-expansion method.

Algebraic manipulation of the proposed expansion method with the help of Maple is much easier than the other methods.

Limitations: Sometimes the method gives solutions in disguised versions of known solutions that may be found by other methods, and if the order of the reduced ordinary differential equation is large enough, then the method unable to guarantee the existence of solutions of the resulted algebraic equations.

In [24], Wang et al. studied exact solutions of the KdV equation by using the basic $(G'/G)$-expansion method and obtained only four solutions (solutions (A.1)–(A.4), see Section A for lucidity). On the other hand by using the proposed expansion, we obtain ten solutions (solutions (3.8)–(3.17)). It is important to point out that, if we set $d = 0$ and/or $d = \lambda$, then our solutions $u_{1_1}, u_{1_2}$, and $u_{1_3}$(solutions (3.8)–(3.11)) are identical to the solutions $u_1, u_2$, and $u_3$ attained by Wang et al. [24]. But if $d \neq 0$ and $d \neq \lambda$, our solutions $u_{1_1}, u_{1_2}$, and $u_{1_3}$ are different from Wang et al. [24] solutions. Moreover, we obtain additional solutions (3.12)–(3.17). These solutions are new and were not obtained by Wang et al. [24].

By the improved $(G'/G)$-expansion method, Zhang et al. [33] obtained seven solutions (solutions (B.3)–(B.9), see Section B for details) of the ZKBBM equations, but by means of the proposed expansion method we obtained ten solutions (solutions (3.27)–(3.36)). It is noteworthy to observe that if $d = 0$, then our solutions (3.27)–(3.33) are identical to Zhang et al. [33] solutions (B.3)–(B.9). On the other hand if $d \neq 0$, then the solutions (3.27)–(3.33) are dissimilar to Zhang et al. [33] solutions. Furthermore, we obtain solutions (3.34)–(3.36). These solutions are new and were not obtained by Zhang et al. [33].

Not only for these equations, the proposed expansion provides more new exact solutions, and it also provides more new solutions for the Broer-Kaup equation, the Sharma-Tasso-Olver equation, the Gardner equation, the Burgers equation, the KdV Burgers equation, the approximate long-wave equation, the Boussinesq equation, and so on. We have prepared another articles in which the above equations are considered and found much more new exact solutions than the basic $(G'/G)$-expansion method and the improved $(G'/G)$-expansion method.

Therefore, the proposed expansion method is promising for solving nonlinear partial differential equations in mathematical physical and engineering problems.

## 5. Conclusion

A generalized and improved $(G'/G)$-expansion method has been proposed and applied in three equations, such as the KdV equation, the ZKBBM equation, and the strain wave equation in microstructured solids. The obtained solutions are more general, and many known solutions are only a special case of them. Further, this study shows that the proposed method is quite efficient and practically well suited to be used in finding exact solutions of NLEEs. Although the method is applied to only a small number (three) of nonlinear equations, it can be applied to many other equations, and this is our task in the future.

## Appendices

## A. Wang et al.'s Solutions [24]

Wang et al. [24] investigated solutions of the KdV equation by the $(G'/G)$-expansion method, and they obtained the following solutions.

When $\lambda^2 - 4\mu > 0$,

$$u_1 = -3\delta\left(\lambda^2 - 4\mu\right)\left(\frac{A\ \sinh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi + B\ \cosh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi}{A\ \cosh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi + B\ \sinh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi}\right)^2 + 3\delta\lambda^2 + e_0, \tag{A.1}$$

where $\xi = x - (\delta\lambda^2 + 8\delta\mu + e_0)t$, and $A$, $B$ are arbitrary constants.

For $A > 0$ and $A^2 > B^2$, solution (A.1) reduces to

$$u_1 = 3\delta\left(\lambda^2 - 4\mu\right)\operatorname{sech}^2\left(\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\right)\xi + \xi_0\right) + 12\delta\mu + e_0, \tag{A.2}$$

where $\xi_0 = \tan\mathrm{h}^{-1}(B/A)$.

When $\lambda^2 - 4\mu < 0$,

$$u_2 = -3\delta\left(4\mu - \lambda^2\right)\left(\frac{-A\ \sin\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi + B\ \cos\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi}{A\ \cos\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi + B\ \sin\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi}\right)^2 + 3\delta\lambda^2 + e_0. \tag{A.3}$$

When $\lambda^2 - 4\mu = 0$,

$$u_3 = -12\delta\frac{B^2}{(A + B\xi)^2} + 3\delta\lambda^2 + e_0. \tag{A.4}$$

## B. Zhang et al.'s Solutions [33]

Zhang et al. [33] proposed an improved $(G'/G)$-expansion method and solved the ZKBBM equation by the proposed method. They obtained the solutions of the overdetermined set of algebraic equation as follows.

*Case 1.* $e_2 = e_1 = 0$, $e_0 = -(bV\lambda^2 - V - 1 + 8bV\mu)/2a$, $e_{-1} = -6bV\lambda\mu/a$, $e_{-2} = -6bV\mu^2/a$, $V = V$,

$$C = \frac{(-1 - V^2 - 2V + 16b^2V^2\mu^2 - 8b^2V^2\mu\lambda^2 + b^2V^2\lambda^4)}{4a}, \tag{B.1}$$

where $V$, $\lambda$, and $\mu$ are arbitrary constants.

*Case 2.* $V = V$, $e_2 = -6bV/a$, $e_1 = -6bV\lambda/a$, $e_0 = -(8bV\mu - V - 1 + bV\lambda^2)/2a$, $e_{-2} = e_{-1} = 0$,

$$C = \frac{(-8b^2V^2\mu\lambda^2 - 2V - V^2 + b^2V^2\lambda^4 + 16b^2V^2\mu^2 - 1)}{4a}, \tag{B.2}$$

where $V$, $\lambda$, and $\mu$ are arbitrary constants.

For Case 1, Zhang et al. [33] obtained the following solutions.
When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_{1_1} = {} & \frac{-6bV\mu^2}{a\left(-\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\dfrac{A\ \cosh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi + B\ \sinh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi}{A\ \sinh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi + B\ \cosh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi}\right)^2} \\ & - \frac{6bV\lambda\mu}{a\left(-\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\dfrac{A\ \cosh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi + B\ \sinh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi}{A\ \sinh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi + B\ \cosh\left(\sqrt{(\lambda^2 - 4\mu)}/2\right)\xi}\right)} \\ & - \frac{bV\lambda^2 + 8bV\mu - V - 1}{2a}, \end{aligned} \tag{B.3}$$

where $A$ and $B$ are arbitrary constants.
When $\lambda^2 - 4\mu < 0$,

$$u_{1_2} = \frac{-6bV\mu^2}{a\left(-\dfrac{\lambda}{2} + \dfrac{\sqrt{4\mu - \lambda^2}}{2}\dfrac{A\ \cos\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi - B\ \sin\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi}{A\ \sin\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi + B\ \cos\left(\sqrt{(4\mu - \lambda^2)}/2\right)\xi}\right)^2}$$

$$-\frac{6bV\lambda\mu}{a\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\ \frac{A\ \cos\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi-B\ \sin\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi}{A\ \sin\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi+B\ \cos\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi}\right)}$$
$$-\frac{bV\lambda^2+8\ bV\mu-V-1}{2a}, \tag{B.4}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2-4\mu=0$,

$$u_{1_3}=\frac{-6bV\mu^2}{a(-(\lambda/2)+(B/(A+B\xi)))^2}-\frac{6bV\lambda\mu}{a(-(\lambda/2)+(B/(A+B\xi)))}-\frac{bV\lambda^2+8bV\mu-V-1}{2a}, \tag{B.5}$$

where $A$ and $B$ are arbitrary constants.

For Case 2, Zhang et al. [33] obtained the following solutions.

When $\lambda^2-4\mu>0$,

$$u_{2_1}=-\frac{3bV(\lambda^2-4\mu)}{2a}\left(\frac{A\ \cosh\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi+B\ \sinh\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi}{A\ \sinh\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi+B\ \cosh\left(\sqrt{(\lambda^2-4\mu)}/2\right)\xi}\right)^2$$
$$+\frac{2bV\lambda^2-8bV\mu+V+1}{2a}, \tag{B.6}$$

where $A$ and $B$ are arbitrary constants.

If $B>0$ and $A^2<B^2$, then they obtained the soliton solutions

$$u_{2_1}=\frac{3bV(\lambda^2-4\mu)}{2a}\mathrm{sech}^2\left(\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\right)\xi+\xi_0\right)-\frac{bV\lambda^2-4bV\mu-V-1}{2a}, \tag{B.7}$$

where $\xi_0=\tan\mathrm{h}^{-1}(A/B)$.

When $\lambda^2-4\mu<0$,

$$u_{2_2}=-\frac{3bV}{2a}\left(4\mu-\lambda^2\right)\left(\frac{A\ \cos\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi-B\ \sin\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi}{A\ \sin\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi+B\ \cos\left(\sqrt{(4\mu-\lambda^2)}/2\right)\xi}\right)^2$$
$$+\frac{2bV\lambda^2-8bV\mu+V+1}{2a}, \tag{B.8}$$

where $A$ and $B$ are arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$u_{2_3} = \frac{-6bV}{a}\left(\frac{B}{A+B\xi}\right)^2 + \frac{\left(2bV\lambda^2 - 8bV\mu + V + 1\right)}{2a}, \tag{B.9}$$

where $A$ and $B$ are arbitrary constants.

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