UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008
Peperiksaan Semester Kedua
Sidang Akademik 2007/2008

April 2008
April 2008

EMH 331/4 – *Finite Element Method in Mechanical Engineering*
Kaedah Unsur Terhingga dalam Kejuruteraan Mekanik

Duration : 3 hours
Masa : 3 jam

INSTRUCTIONS TO CANDIDATE:
ARAHAN KEPADA CALON :

Please check that this paper contains **EIGHT (8)** printed pages, **TWO (2)** pages appendix and **SIX (6)** questions before you begin the examination.
*Sila pastikan bahawa kertas soalan ini mengandungi **LAPAN (8)** mukasurat bercetak, **DUA (2)** mukasurat lampiran dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan.*

Answer **FIVE (5)** questions.
*Jawab **LIMA (5)** soalan.*

Answer all questions in **English** or **Bahasa Malaysia** or a combination of both.
*Calon boleh menjawab semua soalan dalam **Bahasa Malaysia** atau **Bahasa Inggeris** atau kombinasi kedua-duanya.*

Start answering each question in a new page.
Setiap soalan mestilah dimulakan pada mukasurat yang baru.

Appendix/*Lampiran*:

1. The Given Selected Formula [2 pages/*mukasurat*]

Q1. [a] **For the assemblage shown in Figure Q1[a], determine the nodal displacements, the forces in each element and the nodal reactions.**

Untuk aturan yang ditunjukkan dalam Rajah S1[a], tentukan anjakan nod, daya dalam setiap elemen dan daya tindakbalas nod.

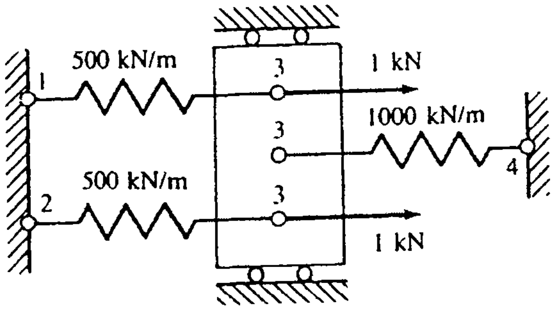

Figure Q1[a]
Rajah S1[a]

(50 marks/*50 markah*)

[b] **The triangular element is shown in Figure Q1[b]. The coordinates are given in units of millimeters. Assume plane stress conditions. Let Young's modulus E = 210 GPa, Poisson ratio, v = 0.25, and thickness t = 10 mm.**

(i) **Evaluate the stiffness matrix**

(ii) **If the nodal displacements are given as u_1 = 0.02 mm; v_1 = 0.01 mm; u_2 = 0.05 mm; v_2 = 0.0 mm; u_3 = 0.03 mm; v_3 = 0.01 mm, determine the element stresses σ_x, σ_y, τ_{xy}, σ_1 and σ_2, and the principal angle θ_p.**

Elemen segitiga adalah seperti yang ditunjukkan dalam Rajah S1[b]. Koordinat yang diberikan adalah dalam unit millimeter. Andaikan keadaan tegasan satah. Ambil modulus Young E = 210 GPa, nisbah Poisson , v = 0.25, dan ketebalan t = 10 mm.

(i) *Kirakan matriks kekakuan*

(ii) *Jika anjakan nod diberikan seperti berikut: u_1 = 0.02 mm; v_1 = 0.01 mm; u_2 = 0.05 mm; v_2 = 0.0 mm; u_3 = 0.03 mm; v_3 = 0.01 mm, kirakan tegasan elemen σ_x, σ_y, τ_{xy}, σ_1 and σ_2, and sudut prinsipal θ_p.*

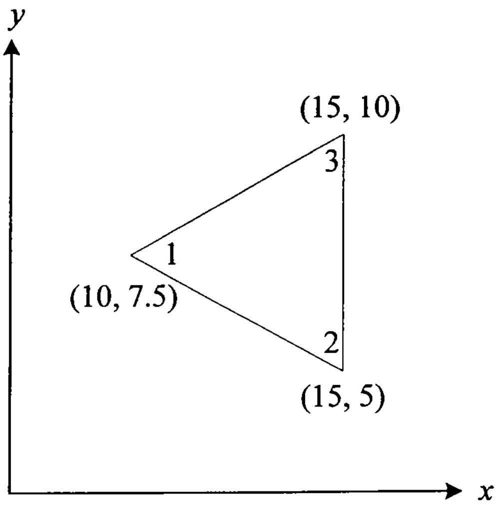

Figure Q1[b]
Rajah S1[b]

(50 marks/*50 markah*)

Q2. [a] List and briefly describe the general steps in the finite element method.

Senaraikan dan terangkan dengan ringkas langkah-langkah umum dalam kaedah unsur terhingga.

(20 marks/*20 markah*)

[b] Use the principle of minimum potential energy to find the equilibrium of displacement of spring system as shown in Figure Q2[b]. Briefly discuss your findings.

Gunakan kaedah prinsip minima tenaga potensi untuk menentukan perseimbangan anjakan pada pegas yang ditunjukkan dalam Rajah S2[b]. Terangkan dengan ringkas penemuan yang dicapai.

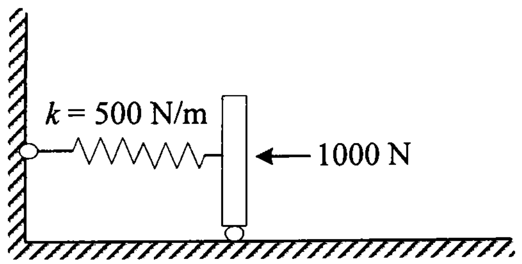

Figure Q2[b]
Rajah S2[b]

(50 marks/*50 markah*)

[c] **The bar shown in Figure Q2[c] has the following parameters: E = 210 GPa, A = 4.0 x 10^{-4}m^2, and L = 2 m, and the angle between x and $\hat{x}$ is 50°. By assuming the global displacements that have been previously determined to be d_{1x} = 0.3 mm, d_{1y} = 0.0 mm and d_{2x} = 0.6 mm, d_{2y} = 0.8 mm respectively, determine the axial stress in the bar.**

Bar yang ditunjukkan di dalam Rajah S2[c] mempunyai parameter-parameter berikut: E = 210 GPa, A = 4.0 x 10^{-4}m^2, dan L = 2m, dan sudut di antara x dan untuk bar tersebut $\hat{x}$ ialah 50°. Dengan mengandaikan anjakan global telah ditentukan sebelum ini iaitu d_{1x} = 0.3 mm, d_{1y} = 0.0 mm dan d_{2x} = 0.6 mm, d_{2y} = 0.8 mm, tentukan tegasan paksi untuk bar tersebut.

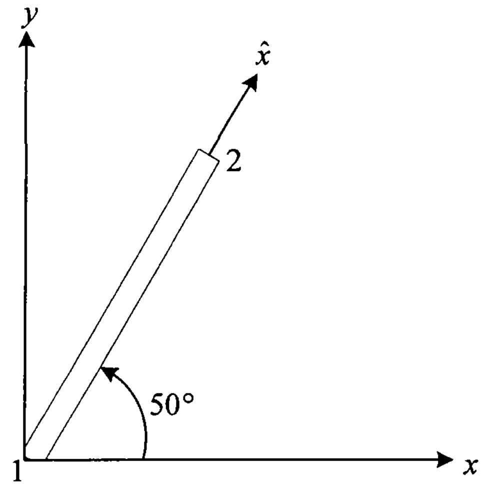

Figure Q2[c]
Rajah S2[c]

(30 marks/*30 markah*)

Q3. [a] For the plane trusses shown in Figure Q3[a], determine the horizontal and vertical displacements of node 1 and the stresses in each element. All elements have E = 210 GPa and cross sectional area, A = 4.0 x 10^{-4}m^2.

Untuk kekuda satah yang ditunjukkan dalam Rajah S3[a], tentukan anjakan menegak dan melintang pada nod 1 dan tegasan di setiap elemen. Semua elemen mempunyai E = 210 GPa dan luas keratan rentas A = 4.0 x 10^{-4}m^2.

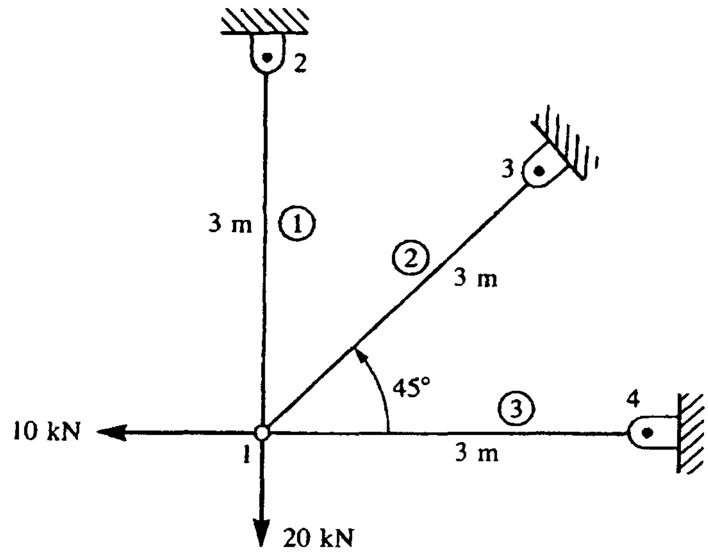

Figure Q3[a]
Rajah S3[a]

(70 marks/*70 markah*)

[b] Derive the stiffness matrix, k for a linear elastic bar element by using Galerkin's method where the governing equation is given by $\frac{d}{dx}\left(AE\frac{du}{dx}\right)=0$. A is cross-sectional area, E is Young's modulus and u is the displacement function.

Terbitkan matriks kekakuan, k untuk elemen bar linear elastik dengan menggunakan kaedah Galerkin di mana persamaan diberi oleh $\frac{d}{dx}\left(AE\frac{du}{dx}\right)=0$. A ialah luas keratan rentas, E ialah modulus Young dan u ialah fungsi anjakan.

(30 marks/*30 markah*)

Q4. [a] For the beam shown in Figure Q4[a], with the node numbers 1-2-3, determine

(i) The displacements
(ii) The slopes at the node
(iii) The forces in each element
(iv) The reactions

Untuk rasuk yang ditunjukkan dalam Rajah S4[a] dengan nombor-nombor nod 1-2-3, tentukan

(i) Anjakan
(ii) Kecerunan pada nod
(iii) Daya pada setiap element
(iv) Daya tindakbalas

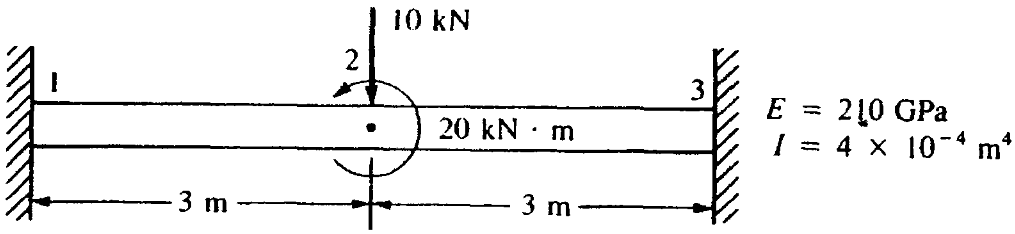

Figure Q4[a]
Rajah S4[a]

(60 marks/*60 markah*)

[b] **Consider the following displacement function for the two-noded bar element:**

$$u = a_1 + a_2 x^2$$

Discuss whether this displacement function is a valid or not valid displacement function. State the reasons.

Fungsi anjakan untuk 2 nod bar elemen diberikan seperti berikut:

$$u = a_1 + a_2 x^2$$

Bincangkan samada fungsi anjakan ini sah atau tidak. Nyatakan alasannya.

(40 marks/*40 markah*)

Q5. [a] **For the composite wall idealized by the one-dimensional model shown in Figure Q5[a], determine the interface temperature between elements 1 & 2, and 2 &3. For element 1, let thermal conductivity K_{xx} = 15 W/(m·°C); for element 2, K_{xx} = 10 W/(m·°C); and for element 3, K_{xx} = 25 W/(m·°C). The left end has a constant temperature of 200°C and the right end has a constant temperature of 600°C.**

Untuk dinding komposit yang diambil sebagai model satu dimensi seperti yang ditunjukkan dalam Rajah 5(a), tentukan suhu diantara permukaan elemen 1 & 2, dan 2 & 3. Untuk elemen 1, ambil aliran terma, K_{xx} = 15 W/(m·°C); untuk elemen 2, K_{xx} = 10 W/(m·°C); dan untuk elemen 3, K_{xx} = 25 W/(m·°C). Suhu tetap dihujung kiri ialah 200°C manakala suhu ditetap dihujung kanan ialah 600°C.

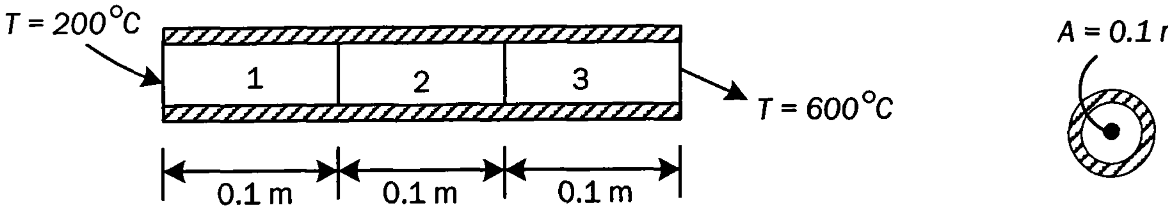

Figure Q5[a]
Rajah S5[a]

(40 marks/*40 markah*)

[b] **Calculate the stiffness matrix k and the force matrix f for the element shown in Figure Q5[b]. The conductivities are $K_{xx} = K_{yy}$ = 15 W/(m·°C) and the convection coefficient is h = 20 W/(m^2·°C). Convection occurs across the *i-m* surface. The free-stream temperature is T_∞ = 20°C. The coordinate are shown expressed in units of meters. Let the line source be Q^* = 75 W/m as located in the figure. Take the thickness of the element to be 1 m.**

Untuk elemen segitiga seperti yang ditunjukkan dalam Rajah S5[b], tentukan matriks kekakuan k dan matriks daya f. Kekonduksian ialah $k_{xx} = k_{yy}$ = 15 W/(m. °C) dan pemalar perolakan ialah h = 20 W/(m^2.°C). Perolakan berlaku pada permukaan nod i-m. Suhu sekeliling ialah T_∞ = 20°C. Sumber haba Q ialah sumber garisan Q* = 75 W/m dilokasi yang ditunjukkan dalam rajah. Koordinat ditunjukkan dalam unit m. Ambil ketebalan elemen 1 m.*

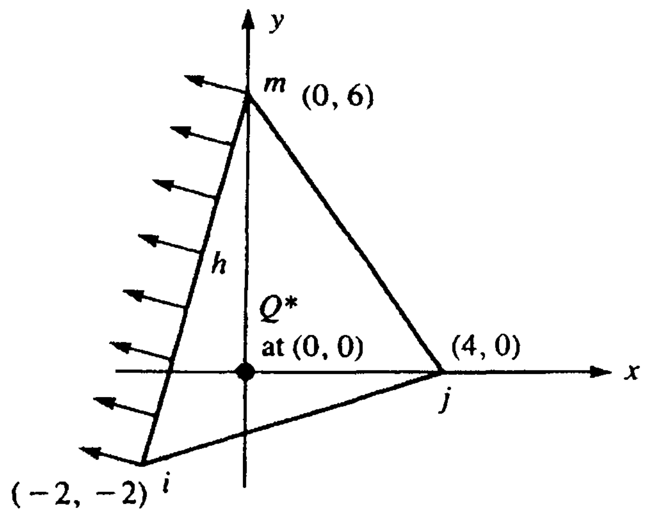

Figure Q5[b]
Rajah S5[b]

(60 marks/*60 markah*)

Q6. [a] **For the smooth pipe shown discretized in Figure Q6[a] with uniform cross section of 1 cm^2, determine the flow velocities at the center and right end, knowing the velocity at the left end is v_x = 2 cm/s.**

Untuk satu paip licin yang dibahagikan seperti yang ditunjukkan dalam Rajah S6[a] dengan keratan rentas seragam 1 cm^2, tentukan had laju aliran pada pusat dan hujung sebelah kanan, dengan mengetahui had laju pada sebelah kiri ialah v_x = 2 cm/s.

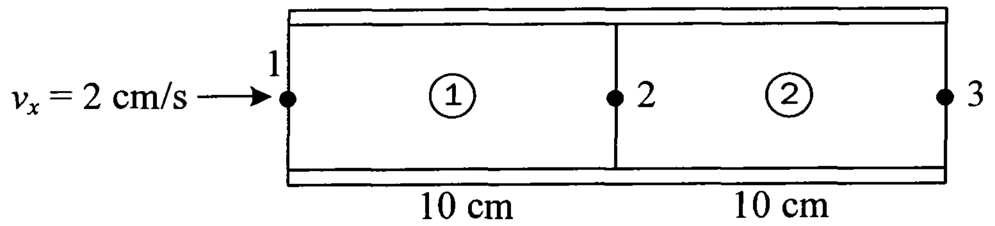

Figure Q6[a]
Rajah S6[a]

(30 marks/*30 markah*)

[b] **For the plane truss shown in Figure Q6[b], determine the displacements at node 1 and the axial stresses in each bar. Bar 1 is subjected to a temperature rise of 30°C. Let *E =210 GPa*, coefficient of thermal expansion $\alpha = 12 \times 10^{-6}$ *(mm/mm)/°C*, and cross sectional area $A = 1 \times 10^{-2}$ for both bar elements.**

Untuk kekuda satah yang ditunjukkan dalam Rajah S6[b], tentukan anjakan pada nod 1 dan tegasan paksi dalam setiap bar. Bar elemen 1 dikenakan suhu meningkat 30°C. Ambil modulus Young E = 210 GPa, pekali pengembangan terma $\alpha = 12 \times 10^{-6}$ *(mm/mm)/°C dan luas keratan rentas* $A = 1 \times 10^{-2}\ m^2$ *untuk kedua dua bar elemen.*

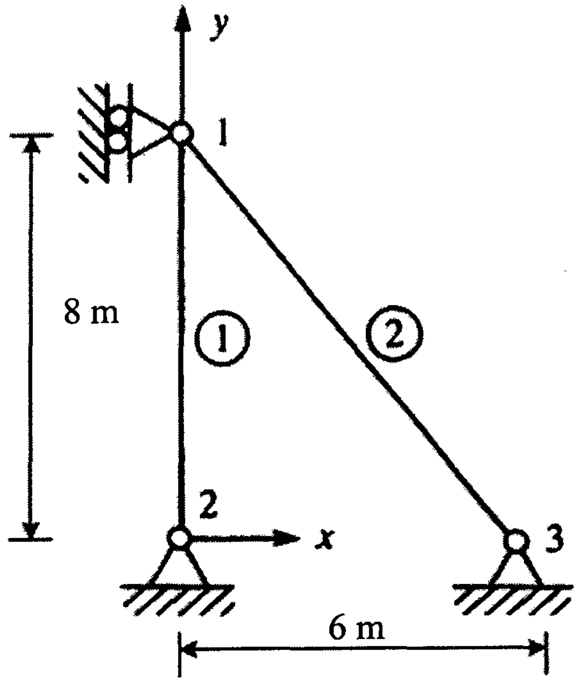

Figure Q6[b]
Rajah S6[b]

(70 marks/*70 markah*)

-oooOOooo-

The given selected formula

Bar and Truss:

$$\begin{Bmatrix} \hat{d}_x \\ \hat{d}_y \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} \qquad \underline{\hat{d}} = \underline{T}\,\underline{d} \qquad \underline{T} = \begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix}$$

$$\underline{k} = \frac{AE}{L} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ & S^2 & -CS & -S^2 \\ & & C^2 & CS \\ Symmetry & & & S^2 \end{bmatrix} \qquad \underline{\sigma} = \underline{C}'\,\underline{d} \qquad \underline{C}' = \frac{E}{L}\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix}$$

Beam and Frame:

$$\hat{\underline{k}} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Plane Stress

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$[k] = tA[B]^T[D][B]$$

Plane Strain

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Triangular element strain

$$\{\varepsilon\} = \frac{1}{2A} \begin{bmatrix} \beta_i & 0 & \beta_j & 0 & \beta_m & 0 \\ 0 & \gamma_i & 0 & \gamma_j & 0 & \gamma_m \\ \gamma_i & \beta_i & \gamma_j & \beta_j & \gamma_m & \beta_m \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{Bmatrix}$$

$$\{\varepsilon\} = \begin{bmatrix} \underline{B}_i & \underline{B}_j & \underline{B}_m \end{bmatrix} \begin{Bmatrix} \underline{d}_i \\ \underline{d}_j \\ \underline{d}_m \end{Bmatrix}$$

LAMPIRAN

$$[B_i]=\frac{1}{2A}\begin{bmatrix}\beta_i & 0\\ 0 & \gamma_i\\ \gamma_i & \beta_i\end{bmatrix} \qquad [B_j]=\frac{1}{2A}\begin{bmatrix}\beta_j & 0\\ 0 & \gamma_j\\ \gamma_j & \beta_j\end{bmatrix} \qquad [B_m]=\frac{1}{2A}\begin{bmatrix}\beta_m & 0\\ 0 & \gamma_m\\ \gamma_m & \beta_m\end{bmatrix}$$

Characteristics of triangular element

$$2A=\begin{bmatrix}1 & x_i & y_i\\ 1 & x_j & y_j\\ 1 & x_m & \gamma_m\end{bmatrix} \qquad 2A = x_i(y_j - y_m) + x_j(y_m - y_i) + x_m(y_i - y_j)$$

$$\alpha_i = x_j y_m - y_j x_m \qquad \alpha_j = y_i x_m - x_i y_m \qquad \alpha_m = x_i y_j - y_i x_j$$

$$\beta_i = y_j - y_m \qquad \beta_j = y_m - y_i \qquad \beta_m = y_i - y_j$$

$$\gamma_i = x_m - x_j \qquad \gamma_j = x_i - x_m \qquad \gamma_m = x_j - x_i$$

x displacement:

$$u(x,y)=\frac{1}{2A}\{(\alpha_i+\beta_i x+\gamma_i y)u_i+(\alpha_j+\beta_j x+\gamma_j y)u_j+(\alpha_m+\beta_m x+\gamma_m y)u_m\}$$

y displacement:

$$v(x,y)=\frac{1}{2A}\{(\alpha_i+\beta_i x+\gamma_i y)v_i+(\alpha_j+\beta_j x+\gamma_j y)v_j+(\alpha_m+\beta_m x+\gamma_m y)v_m\}$$

$$N_i = \frac{1}{2A}(\alpha_i + \beta_i x + \gamma_i y)$$

$$N_j = \frac{1}{2A}(\alpha_j + \beta_j x + \gamma_j y)$$

$$N_m = \frac{1}{2A}(\alpha_m + \beta_m x + \gamma_m y)$$