UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2016/2017

December 2016 / January 2017

EME 411 – Numerical Methods For Engineers
[Kaedah Berangka Untuk Jurutera]

Duration : 2 hours
Masa : 2 jam

Please check that this paper contains **FOUR(4)** printed pages, **ONE(1)** page appendix and **THREE(3)** questions before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **EMPAT(4)** mukasurat, **SATU(1)** mukasurat lampiran dan **TIGA(3)** soalan yang bercetak sebelum anda memulakan peperiksaan.]*

Appendix/*Lampiran :*

1. Useful formulas [**1 page**/*mukasurat*]

INSTRUCTIONS : Answer **ALL** questions.

*[**ARAHAN** : Jawab **SEMUA** soalan.]*

Answer questions in English OR Bahasa Malaysia.
[Jawab soalan dalam Bahasa Inggeris ATAU Bahasa Malaysia.]

Answer to each question must begin from a new page.
[Jawapan bagi setiap soalan mestilah dimulakan pada mukasurat yang baru.]

In the event of any discrepancies, the English version shall be used.
[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Q1. **[a]** **<u>Without writing any equation</u>, explain why the Robin boundary condition is necessary to model convective heat transfer**

<u>Tanpa menulis sebarang persamaan</u>, terangkan sebab syarat sempadan Robin perlu untuk memodelkan pemindahan haba olakan

$$u + \frac{k}{h}\nabla u \cdot \mathbf{n} = u_\infty$$

on a boundary.
pada sempadan.

(4 marks/*markah*)

[b] **Consider the steady-state heat conduction in 1D where the domain is comprised of three different materials. Explain how the standard finite difference equation for the problem with single material domain can be modified for multiple materials. You may use sketches and equations in your explanation.**

Pertimbangkan aliran haba mantap 1D di mana domain itu terbahagi kepada tiga bahan berbeza. Terangkan bagaimana persamaan pembezaan terhingga piawai bagi masalah domain bahan tunggal diubah kepada pelbagai bahan. Anda dibenarkan mengunakan lakaran dan persamaan dalam penerangan anda.

(6 marks/*markah*)

[c] **Consider the transient heat conduction problem in a rod of 1m length with thermal diffusivity of 1 m^2/s. The left end of the rod is prescribed with heat flux of 100 W/m while the right end is maintained at 20 °C. Assume the initial temperature is uniform at 20 °C.**

Use the FDM with IMPLICIT method to <u>solve</u> the value of the temperature at the left end after 0.4 seconds. You must use 3 grid points {x_0, x_1, x_2,} and 3 time levels {t_0, t_1, t_2}. You must also use finite difference of order $O(h^2)$ for the boundary value.

Pertimbangkan masalah aliran haba <u>fana</u> di dalam rod sepanjang 1m dengan kemeresapan haba 1 m^2/s. Hujung kiri ditetapkan menerima fluks haba sebanyak 100 W/m dan hujung kanan ditetapkan pada 20 °C. Anggap suhu awal rod adalah seragam pada 20 °C.

Dengan menggunakan FDM berserta kaedah TERSIRAT untuk <u>selesaikan</u> nilai suhu pada hujung kiri rod selepas 0.4 saat. Gunakan 3 titik grid {x_0, x_1, x_2,} dan 3 tahap masa {t_0, t_1, t_2}. Anda mesti gunakan juga pembezaan terhingga tahap $O(h^2)$ bagi nilai sempadan itu.

(20 marks/*markah*)

Q2. [a] Explain how the Galerkin method is related to the method of weighted residuals.

Terangkan bagaimana kaedah Galerkin adalah berhubungkait dengan kaedah reja berpemberat.

(5 marks/*markah*)

[b] By writing the linear basis functions in the local coordinate, show that the element stiffness matrix of element e, $\mathbf{K}^e$ is

Dengan menulis fungsi-fungsi basis linear dalam koordinat tempatan, tunjukkan bahawa matriks pegas unsur bagi unsur e, $\mathbf{K}^e$ *ialah*

$$\mathbf{K}^e = \frac{k^e}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

where k^e is the material coefficient of element e.
di mana k^e ialah pekali bahan bagi unsur e.

(10 marks/*markah*)

[c] Consider the displacement response of a 1 m bar due to the axial force P = 1000 N at its right end. The bar is fixed at the left end. Assume *$E \cdot A = 70\times10^3$ N.*

Pertimbangkan tindak balas sesaran bagi sebatang bar 1 m oleh daya paksi P=1000 N pada hujung kanan. Bar itu ditetapkan pada hujung kiri. Andaikan $E \cdot A = 70\times10^3$ N.

(i) Express completely the WEAK statement for the above problem that ensures a unique solution.

Ungkapkan masalah di atas secara lengkap dengan pernyataan LEMAH yang memastikan satu penyelesaian unik.

(5 marks/*markah*)

(ii) The domain is meshed with linear elements such that the nodal coordinates are $x = \{0, 0.1, 0.3, 0.6, 1.0\}$. Sketch the mesh with numbering of the nodes and elements.

Domain itu dibinakan jejaring dengan unsur-unsur linear di mana koordinat nod-nod ialah $x = \{0, 0.1, 0.3, 0.6, 1.0\}$. Lakarkan jejaring dengan menomborkan semua nod dan unsur.

(5 marks/*markah*)

(iii) **Set up the global linear system for the problem that includes the boundary conditions. Detailed derivation steps are not necessary.**

Binakan sistem linear global bagi masalah itu dengan memasukkan syarat-syarat sempadan. Langkah-langkah terbitan yang terperinci adalah tidak perlu.

(15 marks/*markah*)

(iv) **Without solving the linear system, sketch the plot of the expected FEM solution of the displacement.**

Tanpa menyelesaikan sistem linear itu, lakarkan penyelesaian FEM yang dijangka bagi sesaran itu.

(10 marks/*markah*)

Q3. [a] Consider the steady state heat conduction in a unit square domain. The problem is solved with FDM using 4×4 grid points with natural numbering. Assuming the temperature solution has been obtained, write MATLAB code to compute for the heat flux at the location index $[\boldsymbol{i}, \boldsymbol{j}] = [\mathbf{1}, \mathbf{2}]$ using second order accurate approximation. The origin is at $[\boldsymbol{i}, \boldsymbol{j}] = [\mathbf{0}, \mathbf{0}]$.

Pertimbangkan aliran haba mantap di dalam domain segi empat sama unit. Masalah ini diselesaikan dengan FDM menggunakan 4×4 titik grid. Dengan andaian penyelesaian suhu telah didapati, tuliskan kod MATLAB untuk menghitung fluks haba pada indeks lokasi $[i, j] = [1,2]$ *dengan menggunakan penghampiran jitu tahap kedua. Asalan adalah pada indeks* $[i, j] = [0,0]$.

(10 marks/*markah*)

[b] Consider the steady state heat conduction in a bar of unit length. Assume that:

- **the vector of nodal coordinates is `x=[0;0.4;0.5;0.6;1.0]`**
- **the FEM solution vector is `a=[0;60;90;75;0]`**

Write MATLAB code to compute for the value of the temperature at $\boldsymbol{x} = \mathbf{0.57}$.

Pertimbangkan aliran haba mantap di dalam bar sepanjang satu unit. Andaikan bahawa:

- *vektor koordinat-koordinat nod ialah* `x=[0;0.4;0.5;0.6;1.0]`
- *vektor penyelesaian FEM ialah* `a=[0;60;90;75;0]`

Tuliskan kod MATLAB untuk menghitung nilai suhu pada $x = 0.57$.

(10 marks/*markah*)

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APPENDIX 1
LAMPIRAN 1

Useful formulas

Forward differences:

$$u_i' = \frac{u_R - u_i}{h} + O(h)$$

Centered Differences:

$$u_i'' = \frac{u_R - 2u_i + u_L}{h^2} + O(h^2) \qquad u_i' = \frac{u_R - u_L}{2h} + O(h^2)$$

2D Stencil for Poisson's equation:

$$-u_{E,j} - u_{W,j} + 4u_{i,j} - u_{i,N} - u_{i,S} = \frac{h^2 f_i}{k}$$

Robin boundary condition

$$p \cdot u + q \cdot \nabla u \cdot \mathbf{n} = g$$

1D heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

FDM equation with variable material

$$-k_{i-\frac{1}{2}} u_{i-1} + \left(k_{i-\frac{1}{2}} + k_{i+\frac{1}{2}}\right) u_i - k_{i+\frac{1}{2}} u_{i+1} = h^2 f_i$$

Implicit method for heat equation

$$-\lambda u_{i+1}^{l+1} + (1+2\lambda) u_i^{l+1} - \lambda u_{i-1}^{l+1} = u_i^l \qquad \text{where } \lambda = \frac{\alpha s}{h^2}$$

Explicit method for heat equation:

$$\alpha \frac{u_R^l - 2u_i^l + u_L^l}{h^2} = \frac{u_i^{l+1} - u_i^l}{s}$$

Load vector with point load P at ρ_0:

$$\mathbf{b}^{(e)} = P \begin{bmatrix} \xi_1(\rho_0) \\ \xi_2(\rho_0) \end{bmatrix}$$

Basis functions in local coordinate

$$\xi_1(\rho) = \frac{1}{2}(1-\rho) \qquad \xi_2(\rho) = \frac{1}{2}(1+\rho)$$

Local-global coordinates mapping

$$x = \frac{l}{2}\rho + \frac{1}{2}(x_i + x_j)$$

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