UNIVERSITI SAINS MALAYSIA

First Semester Examination
2016/2017 Academic Session

December 2016 / January 2017

EKC 314 – Transport Phenomena
[Fenomena Pengangkutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>ELEVEN</u> pages of printed material and <u>SEVEN</u> pages of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>SEBELAS</u> muka surat yang bercetak dan <u>TUJUH</u> muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

<u>Instruction:</u> Answer **<u>ALL</u>** (4) questions.

*[**<u>Arahan:</u>** Jawab **<u>SEMUA</u>** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Answer ALL questions.

1. [a] τ_{yx} is defined as the momentum flux of a flow of a liquid in x-direction. Compute the steady-state, τ_{yx} in lb_f/ft^2 when the lower plate velocity V in Figure Q.1.[a] is 1 ft/s in the positive x-direction, the plate separation Y is 0.001 ft and the fluid viscosity, μ is 0.7 cp.

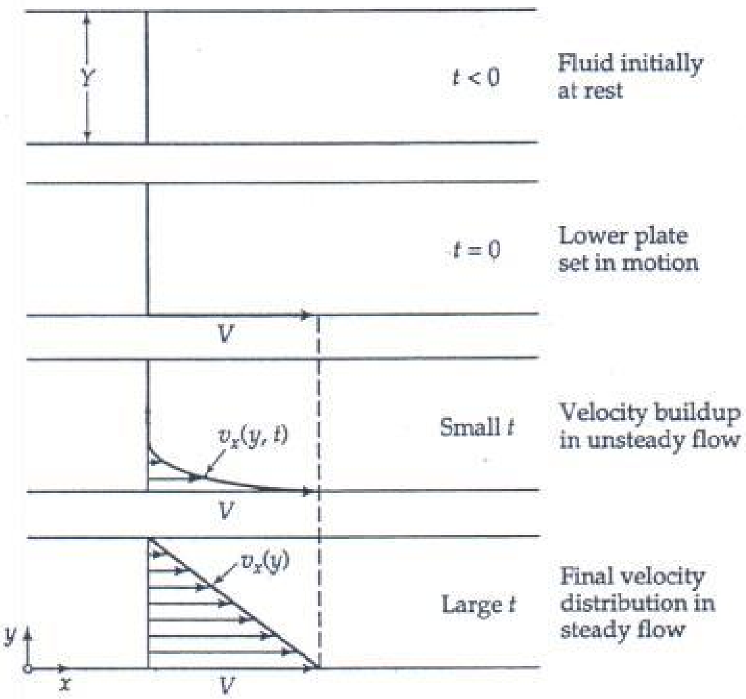


Figure Q.1.[a]. The formation of laminar velocity profile for a fluid contained between two plates

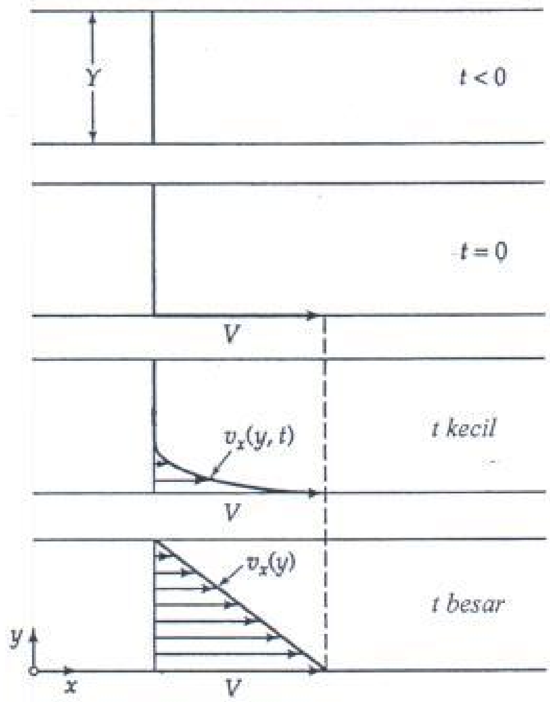
[5 marks]

[b] Describe in detail the molecular theory of the viscosity of gases at low density in terms of the average velocity, $\bar{u}$, the frequency of the bombarded molecules, Z, and the distance, a, experienced during the last collision between molecules. Define all the terms used.

[5 marks]

Jawab SEMUA soalan.

1. [a] *Terma τ_{yx} dimaksudkan sebagai fluks momentum aliran bendalir pada arah-x. Kirakan pada keadaan mantap, τ_{yx} dalam $lb_f/kaki^2$ apabila plat pada halaju V dalam Rajah S.1.[a] adalah 1 kaki/s pada arah-x positif, plat pemisah Y adalah 0.001 kaki dan kelikatan bendalir, μ adalah 0.7 cp.*



Rajah S.1.[a]. : Pembentukan profil halaju laminar bagi bendalir di antara dua plat.

[5 markah]

[b] *Perihalkan dengan lengkap teori molekular bagi kelikatan gas-gas pada ketumpatan rendah bagi terma-terma, halaju purata, $\bar{u}$, frekuensi molekul terbedil, Z, dan jarak, a yang dilalui semasa perlanggaran terakhir di antara molekul. Takrifkan semua terma yang digunakan.*

[5 markah]

[c] In a gas absorption experiment, a viscous fluid flows upward through a small circular tube and then downward in laminar flow on the outside. Set up a momentum balance over a shell of thickness Δr in the film as shown in Figure Q.1.[c]. Note that the momentum in and momentum out arrows are always taken in the positive coordinate direction, even though in this problem the momentum is flowing through the cylindrical surfaces in the negative r direction. Show that the velocity distribution in the falling film is given by;

$$v_z = \frac{\rho g R^2}{4\mu}\left[1-\left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right]$$

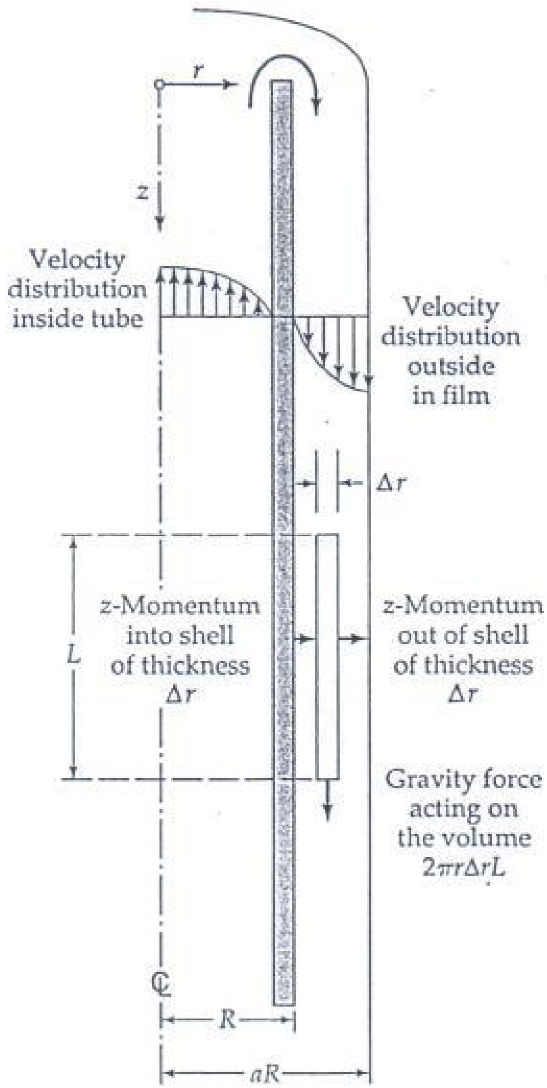
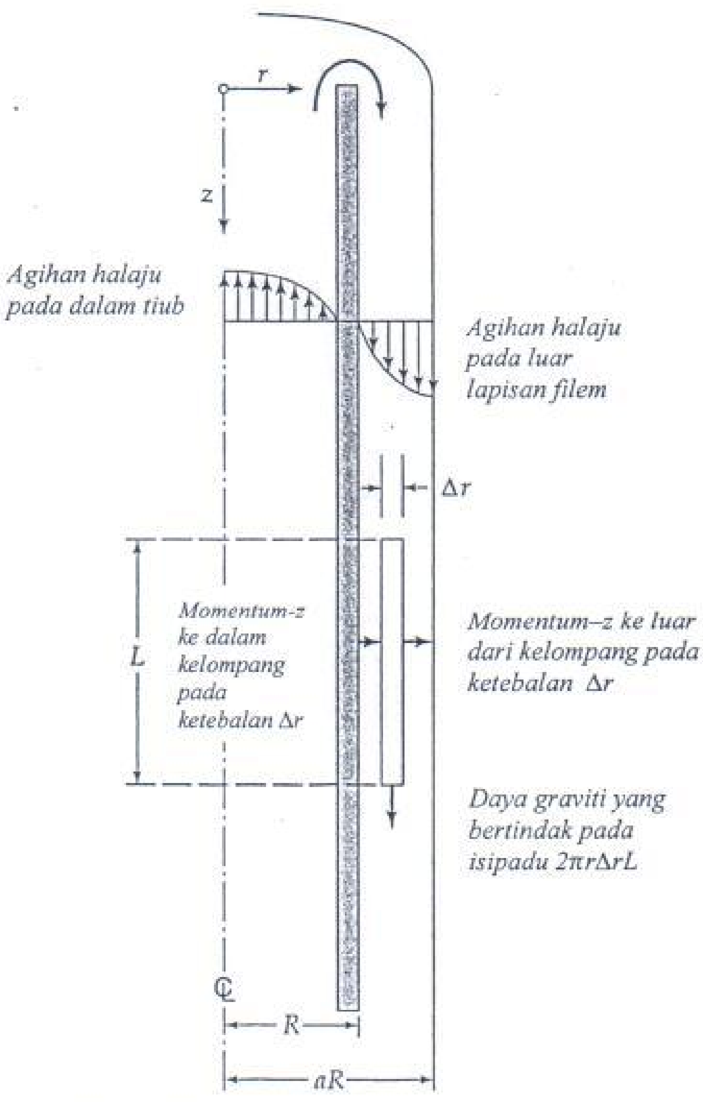


Figure Q.1.[c].: Flow through annulus

[15 marks]

[c] *Dalam satu ujikaji penyerapan gas, satu bendalir likat mengalir kearah atas melalui satu tiub bulatan kecil dan kemudian ke arah bawah pada aliran laminar di bahagian luar tiub. Berikan satu imbangan momentum merangkumi kelompang pada ketebalan Δr pada lapisan filem seperti pada Rajah S.1.[c]. Diingatkan bahawa anak panah momentum ke dalam dan momentum ke luar selalunya diambil pada arah koordinat positif walaupun dalam masalah ini momentum mengalir melalui permukaan silinder pada arah-r negatif. Tunjukkan bahawa agihan halaju pada lapisan filem terjatuh diberikan oleh;*

$$v_z = \frac{\rho g R^2}{4\mu}\left[1-\left(\frac{r}{R}\right)^2 + 2a^2 \ln\left(\frac{r}{R}\right)\right]$$



Rajah S.1.[c].: Aliran melalui anulus

[15 markah]

2. [a] The Navier-Stokes equation may be derived by considering a momentum balance on an element of volume $\Delta x \Delta y \Delta z$. The rate of momentum accumulation in the volume element is given by the balance between the rates at which momentum is transferred into and out of the box and the sum of the forces (pressure and gravitational) on the box.

[i] Sketch an appropriate diagram showing a particular momentum balance and use the diagram to obtain the continuity equation.

[5 marks]

[ii] Momentum is also transferred to the element by both convection and molecular processes (viscosity) arising from the gradient of velocity. Using both types of balances, derive in detail, the Navier-Stoke equation in the x-direction, where μ is the viscosity of the fluid.

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + vz\frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial \rho}{\partial x} + \rho g_x$$

ρ is the fluid density and g_x is the acceleration due to gravity in the direction of x. Note that you should use the continuity expression derived in part [ii].

[8 marks]

[b] Determine the overall heat transfer coefficient for the composite cylindrical pipe as shown in Figure Q.2.[b].

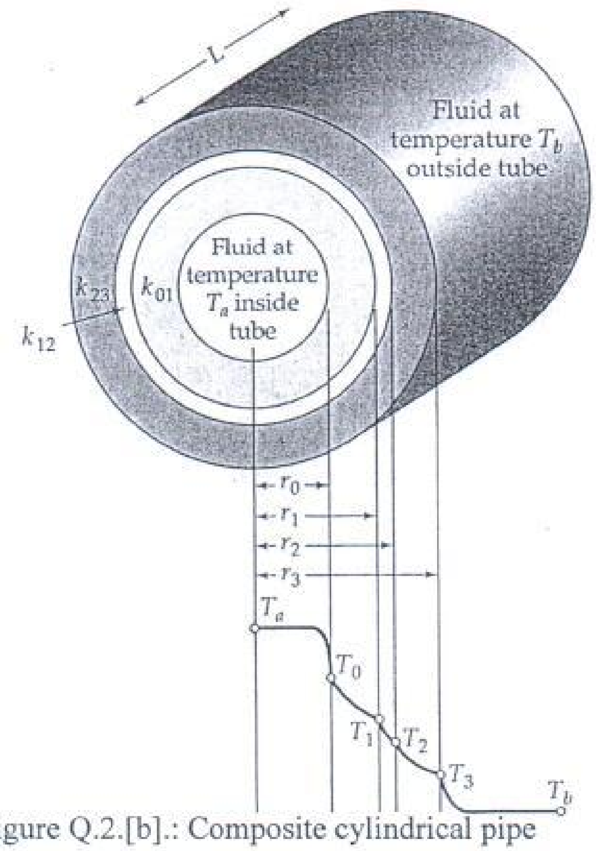


Figure Q.2.[b].: Composite cylindrical pipe

[12 marks]

2. [a] *Persamaan Navier-Stokes mungkin diterbitkan dengan mengambil imbangan momentum pada elemen dengan isipadu $\Delta x \Delta y \Delta z$. Kadar penumpukan momentum pada isipadu elemen diberikan dengan mengimbangkan antara kadar di mana momentum dipindahkan ke dalam dan ke luar dari kotak dan jumlah daya (tekanan dan graviti) ke atas kotak.*

[i] *Lakarkan gambarajah yang bersesuaian yang menunjukkan setiap imbangan momentum dan gunakan gambarajah tersebut untuk mendapatkan persamaan keselanjaran.*

[5 markah]

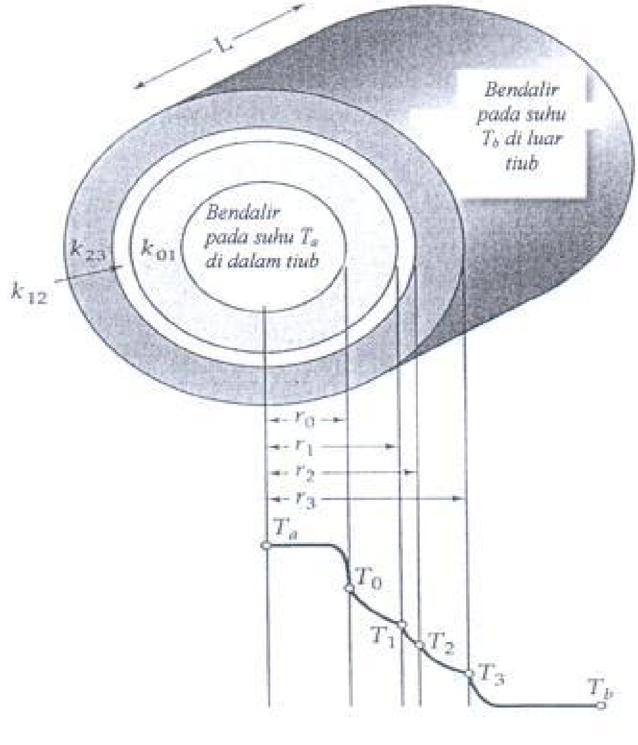
[ii] *Momentum juga dipindahkan ke elemen melalui perolakan dan proses-proses molekul (kelikatan) yang disebabkan oleh kecerunan halaju. Dengan menggunakan kedua-dua imbangan, terbitkan dengan lengkap, persamaan Navier-Stokes pada arah-x, dimana μ adalah kelikatan bendalir.*

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + vz\frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial \rho}{\partial x} + \rho g_x$$

ρ adalah ketumpatan bendalir dan g_x adalah pecutan graviti pada arah-x. Diingatkan bahawa anda perlu menggunakan persamaan keselajaran yang diterbitkan pada bahagian [ii].

[8 markah]

[b] *Tentukan pekali pemindahan haba keseluruhan untuk rencam paip silinder yang diberikan pada Rajah S.2.[b].*



Rajah S.2.[b].: Rencam paip silinder

[12 markah]

3. A heat conduction equation is given as follow:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{2}\frac{\partial T}{\partial t}$$

and is valid for the case where $0 < x < 3$, $t > 0$. The boundary conditions are $T(0,t) = T(3,t) = 0$ and the initial condition is $T(x,0) = 5 \sin(4\pi x)$.

[a] By using separation of variable by taking $T(x,t) = M(x)N(t)$, find the expression for both $\frac{d^2M}{dx^2}$ and $\frac{dN}{dt}$ in the general form of M and N.

[8 marks]

[b] By defining separation constant first, solve the *ODE* of $\frac{d^2M}{dx^2}$ and $\frac{dN}{dt}$ by getting the general expression of these equations.

[4 marks]

[c] From answer of part [b], simplified your arbitrary constant to get an equation to define $T(x,t)$. Remember $T(x,t) = M(x)N(t)$

[3 marks]

[d] By using the *B.C.* and *I.C.*, find the arbitrary constants you have defined in part [c] and prove that $T(x,0) = 5sin(4\pi x)e^{-32\pi^2 t}$

[10 marks]

4. [a] [i] For Stokes-Einstein equation, what is the physical meaning of k_BT?

[1 mark]

[ii] Fick's Second Law gives information of the change of concentration of a diffusive mass with respect to two independent variables. What are the two independent variables?

[1 mark]

[iii] From the thermodynamic perspective, what is the external factor(s) that driven diffusion?

[2 marks]

[iv] Sherwood number (*Sh*) is defined as $Sh = k_cL/D$. What is the physical meaning of this dimensionless number?

[2 marks]

[v] A small spherical particulate matter is having a diameter of 10 μm and diffusivity of 5 mm^2/s. Calculate the 1-dimensional mean square displacement at *10 s* and comment on the rate of this process happening within this time scale.

[4 marks]

3. *Persamaan konduksi haba seperti berikut:*

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{2}\frac{\partial T}{\partial t}$$

adalah sah untuk kes $0 < x < 3$, $t > 0$. *Keadaan sempadan adalah* T (0,t) = T (3,t) = 0 *dan keadaan awal ialah* $T(x,0) = 5 \sin(4\pi x)$.

[a] *Dengan menggunakan cara pemisahan pembolehubah dengan* $T(x,t) = M(x)N(t)$, *cari persamaan untuk* $\frac{d^2M}{dx^2}$ *and* $\frac{dN}{dt}$ *dalam bentuk am M dan N.*

[8 markah]

[b] *Bermula dengan menentukan pemalar pemisahan, selesaikan ODE untuk* $\frac{d^2M}{dx^2}$ *dan* $\frac{dN}{dt}$ *dengan mendapatkan persamaan am.*

[4 markah]

[c] *Dari jawapan bahagian [b], memudahkan pemalar sembarangan anda untuk mendapatkan satu persamaan demi menentukkan* $T(x,t)$. *Diingatkan bahawa* $T(x,t) = M(x)N(t)$.

[3 markah]

[d] *Dengan menggunakan B.C. dan I.C., cari pemalar sembarangan yang anda telah tentukan dalam bahagian [c] dan buktikan bahawa* $T(x,0) = 5\sin(4\pi x)e^{-32\pi^2 t}$

[10 markah]

4. [a] [i] *Untuk persamaan Stokes-Einstein, apakah maksud fisikal untuk* $k_B T$?

[1 markah]

[ii] *Hukum Fick Kedua membekalkan maklumat mengenai perubahan kepekatan untuk jisim resapan melalui kaitan dengan dua pembolehubah tak bersandar. Apakah dua pembolehubah tak bersandar tersebut?*

[1 markah]

[iii] *Dari perspektif termodinamik, apakah faktor luaran yang menyebabkan resapan untuk berlaku?*

[2 markah]

[iv] *Nombor Sherwood (Sh) ditakrifkan sebagai* $Sh = k_c L/D$. *Apakah maksud fisikal untuk nombor tak berdimensi ini?*

[2 markah]

[v] *Satu zarah kecil sfera memiliki garis pusat 10 μm dan kemeresapan 5* mm^2/s. *Kirakan min anjakan kuasa dua satu dimensi dalam 10 saat dan beri keterangan mengenai kadar proses ini berlaku.*

[4 markah]

[b] A gas phase polymer (species P) is irreversibly and very rapidly cracked on a heated metal plate (reaction on a solid surface) in an experimental reactor. The cracking reduces the molecular weight by an average factor of three ([species P]→3*[species Q]) and the governing equation is:

$$n_P = -D_{AP}\frac{dc_{AP}}{dz} + c_P v_0$$

[i] List down the boundary conditions for this problem at z direction (away from the surface).

[4 marks]

[ii] Derive the rate equation for this process, assuming that both the reactant and the product diffuse through a thin unstirred film of thickness l near the plate. Note that the reactant must be constantly diffusing against product moving away from the plate and assuming $-dn_p/dz = 0$.

[8 marks]

[iii] From the answer in part [ii], what will be happening if the bulk phase polymer is very dilute (*so,* $y_{p,l} \approx 0$)?

[3 marks]

[b] *Satu polimer dalam fasa gas (spesis P) melalui tindak balas tidak berbalik dengan kadar cepat atas logam panas (tindak balas atas permukaan pepejal) dalam reaktor eksperimen. Proses pemecahan ini mengurangkan berat molekul spesis P sebanyak 3 faktor ([spesis P]→3*[spesis Q]) dan persamaan yang mengawal proses ini ialah:*

$$n_p = -D_{AP}\frac{dc_{AP}}{dz} + c_P v_0$$

[i] *Senaraikan keadaan sempadan bagi process ini pada arah z (dari permukaan).*

[4 markah]

[ii] *Terbitkan persamaan kadar untuk proses ini, dengan anggapan bahawa kedua-dua reaktor dan produk meresap melalui satu filem nipis tanpa aduk dengan ketebalan l berhampiran plat. Perhatikan bahawa reaktor mesti sentiasa meresap terhadap produk beralih daripada plat dan menganggap bahawa* $-dn_p/dz = 0$.

[8 markah]

[iii] *Dari jawapan dalam bahagian [ii], apa yang akan berlaku jika polimer fasa pukal adalah sangat cair (jadi,* $y_{p,l} \approx 0$)?

[3 markah]

-oooOooo-

APPENDICES

Appendix A: Conversion Factors

Given a quantity in these units:	Multiply by:	To get quantity in these units:
Pounds	453.59	Grams
Kilograms	2.2046	Pounds
Inches	2.5400	Centimeters
Meters	39.370	Inches
Gallons (U.S.)	3.7853	Liters
Gallons (U.S.)	231.00	Cubic inches
Gallons (U.S.)	0.13368	Cubic feet
Cubic feet	28.316	Liters
Kelvins	1.800000	Degrees Rankine
Degrees Rankine	0.555556	Kelvins

Table A.1

Given a quantity in this units	Multiply by table value to convert to these units	$N = kg{\cdot}m/s^2$ (Newtons)	$g{\cdot}cm/s^2$	$lb_m{\cdot}ft/s^2$	lb_f
$N = kg{\cdot}m/s^2$	(Newtons)	1	10^5	7.2330	2.24881×10^{-1}
$g{\cdot}cm/s^2$	(dynes)	10^{-5}	1	7.2330×10^{-5}	2.24881×10^{-6}
$lb_m{\cdot}ft/s^2$	(poundals)	1.3826×10^{-1}	1.3826×10^4	1	3.1081×10^{-2}
lb_f		4.4482	4.4482×10^5	32.1740	1

Table A.2

<u>Appendix B</u>: Equation of Motion in Terms of τ

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p - [\nabla\cdot\tau] + \rho g$$

Cartesian coordinates (x, y, z):[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.1

Cylindrical coordinates (r, θ, z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.2

Spherical coordinates (r, θ, ϕ):[c]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$- \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi$$

Table B.3

Appendix C: Equation of Motion for a Newtoniam Fluid with Constant ρ and μ

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho \mathbf{g}$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.1

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.2

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$

Table C.3

Appendix D: Lennard-Jones Potential Parameters and Critical Properties

Substance	Molecular Weight M	Lennard-Jones parameters σ (Å)	ε/κ (K)	Ref.	Critical properties[g,h] T_c (K)	p_c (atm)	$\tilde{V}_c$ (cm³/g-mole)	$\mu_c \times 10^6$ (g/cm·s)	$k_c \times 10^6$ (cal/cm·s·K)
Light elements:									
H_2	2.016	2.915	38.0	*a*	33.3	12.80	65.0	34.7	—
He	4.003	2.576	10.2	*a*	5.26	2.26	57.8	25.4	—
Noble gases:									
Ne	20.180	2.789	35.7	*a*	44.5	26.9	41.7	156.	79.2
Ar	39.948	3.432	122.4	*b*	150.7	48.0	75.2	264.	71.0
Kr	83.80	3.675	170.0	*b*	209.4	54.3	92.2	396.	49.4
Xe	131.29	4.009	234.7	*b*	289.8	58.0	118.8	490.	40.2
Simple polyatomic gases:									
Air	28.964[i]	3.617	97.0	*a*	132.4[i]	37.0[i]	86.7[i]	193.	90.8
N_2	28.013	3.667	99.8	*b*	126.2	33.5	90.1	180.	86.8
O_2	31.999	3.433	113.	*a*	154.4	49.7	74.4	250.	105.3
CO	28.010	3.590	110.	*a*	132.9	34.5	93.1	190.	86.5
CO_2	44.010	3.996	190.	*a*	304.2	72.8	94.1	343.	122.
NO	30.006	3.470	119.	*a*	180.	64.	57.	258.	118.2
N_2O	44.012	3.879	220.	*a*	309.7	71.7	96.3	332.	131.
SO_2	64.065	4.026	363.	*c*	430.7	77.8	122.	411.	98.6
F_2	37.997	3.653	112.	*a*	—	—	—	—	—
Cl_2	70.905	4.115	357.	*a*	417.	76.1	124.	420.	97.0
Br_2	159.808	4.268	520.	*a*	584.	102.	144.	—	—
I_2	253.809	4.982	550.	*a*	800.	—	—	—	—
Hydrocarbons:									
CH_4	16.04	3.780	154.	*b*	191.1	45.8	98.7	159.	158.
CH≡CH	26.04	4.114	212.	*d*	308.7	61.6	112.9	237.	—
CH_2=CH_2	28.05	4.228	216.	*b*	282.4	50.0	124.	215.	—
C_2H_6	30.07	4.388	232.	*b*	305.4	48.2	148.	210.	203.
CH_3C≡CH	40.06	4.742	261.	*d*	394.8	—	—	—	—
CH_3CH=CH_2	42.08	4.766	275.	*b*	365.0	45.5	181.	233.	—
C_3H_8	44.10	4.934	273.	*b*	369.8	41.9	200.	228.	—
n—C_4H_{10}	58.12	5.604	304.	*b*	425.2	37.5	255.	239.	—
i—C_4H_{10}	58.12	5.393	295.	*b*	408.1	36.0	263.	239.	—
n—C_5H_{12}	72.15	5.850	326.	*b*	469.5	33.2	311.	238.	—
i—C_5H_{12}	72.15	5.812	327.	*b*	460.4	33.7	306.	—	—
$C(CH_3)_4$	72.15	5.759	312.	*b*	433.8	31.6	303.	—	—
n—C_6H_{14}	86.18	6.264	342.	*b*	507.3	29.7	370.	248.	—
n—C_7H_{16}	100.20	6.663	352.	*b*	540.1	27.0	432.	254.	—
n—C_8H_{18}	114.23	7.035	361.	*b*	568.7	24.5	492	259.	—
n—C_9H_{20}	128.26	7.463	351.	*b*	594.6	22.6	548.	265.	—
Cyclohexane	84.16	6.143	313.	*d*	553.	40.0	308.	284.	—
Benzene	78.11	5.443	387.	*b*	562.6	48.6	260.	312.	—
Other organic compounds:									
CH_4	16.04	3.780	154.	*b*	191.1	45.8	98.7	159.	158.
CH_3Cl	50.49	4.151	355.	*c*	416.3	65.9	143.	338.	—
CH_2Cl_2	84.93	4.748	398.	*c*	510.	60.	—	—	—
$CHCl_3$	119.38	5.389	340.	*e*	536.6	54.	240.	410.	—
CCl_4	153.82	5.947	323.	*e*	556.4	45.0	276.	413.	—
C_2N_2	52.034	4.361	349.	*e*	400.	59.	—	—	—
COS	60.076	4.130	336.	*e*	378.	61.	—	—	—
CS_2	76.143	4.483	467.	*e*	552.	78.	170.	404.	—
CCl_2F_2	120.91	5.116	280.	*b*	384.7	39.6	218.	—	—

Table D.1

Collision Integrals for use with the Lennard-Jones Potential for the Prediction of Transport Properties of Gases at Low Densities

$\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathfrak{D},AB}$ (for diffusivity)	$\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathfrak{D},AB}$ (for diffusivity)
0.30	2.840	2.649	2.7	1.0691	0.9782
0.35	2.676	2.468	2.8	1.0583	0.9682
0.40	2.531	2.314	2.9	1.0482	0.9588
0.45	2.401	2.182	3.0	1.0388	0.9500
0.50	2.284	2.066	3.1	1.0300	0.9418
0.55	2.178	1.965	3.2	1.0217	0.9340
0.60	2.084	1.877	3.3	1.0139	0.9267
0.65	1.999	1.799	3.4	1.0066	0.9197
0.70	1.922	1.729	3.5	0.9996	0.9131
0.75	1.853	1.667	3.6	0.9931	0.9068
0.80	1.790	1.612	3.7	0.9868	0.9008
0.85	1.734	1.562	3.8	0.9809	0.8952
0.90	1.682	1.517	3.9	0.9753	0.8897
0.95	1.636	1.477	4.0	0.9699	0.8845
1.00	1.593	1.440	4.1	0.9647	0.8796
1.05	1.554	1.406	4.2	0.9598	0.8748
1.10	1.518	1.375	4.3	0.9551	0.8703
1.15	1.485	1.347	4.4	0.9506	0.8659
1.20	1.455	1.320	4.5	0.9462	0.8617
1.25	1.427	1.296	4.6	0.9420	0.8576
1.30	1.401	1.274	4.7	0.9380	0.8537
1.35	1.377	1.253	4.8	0.9341	0.8499
1.40	1.355	1.234	4.9	0.9304	0.8463
1.45	1.334	1.216	5.0	0.9268	0.8428
1.50	1.315	1.199	6.0	0.8962	0.8129
1.55	1.297	1.183	7.0	0.8727	0.7898
1.60	1.280	1.168	8.0	0.8538	0.7711
1.65	1.264	1.154	9.0	0.8380	0.7555
1.70	1.249	1.141	10.0	0.8244	0.7422
1.75	1.235	1.128	12.0	0.8018	0.7202
1.80	1.222	1.117	14.0	0.7836	0.7025
1.85	1.209	1.105	16.0	0.7683	0.6878
1.90	1.198	1.095	18.0	0.7552	0.6751
1.95	1.186	1.085	20.0	0.7436	0.6640
2.00	1.176	1.075	25.0	0.7198	0.6414
2.10	1.156	1.058	30.0	0.7010	0.6235
2.20	1.138	1.042	35.0	0.6854	0.6088
2.30	1.122	1.027	40.0	0.6723	0.5964
2.40	1.107	1.013	50.0	0.6510	0.5763
2.50	1.0933	1.0006	75.0	0.6140	0.5415
2.60	1.0807	0.9890	100.0	0.5887	0.5180

Table D.2

<u>Appendix E</u>: Some Ordinary Differential Equations and Their Solutions

Equation	Solution
$\frac{dy}{dx} = \frac{f(x)}{g(y)}$	$\int g\, dy = \int f\, dx + C_1$
$\frac{dy}{dx} + f(x)y = g(x)$	$y = e^{-\int f dx}(\int e^{\int f dx} g\, dx + C_1)$
$\frac{d^2y}{dx^2} + a^2y = 0$	$y = C_1 \cos ax + C_2 \sin ax$
$\frac{d^2y}{dx^2} - a^2y = 0$	$y = C_1 \cosh ax + C_2 \sinh ax$ or $y = C_3e^{+ax} + C_4e^{-ax}$
$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) + a^2y = 0$	$y = \frac{C_1}{x} \cos ax + \frac{C_2}{x} \sin ax$
$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) - a^2y = 0$	$y = \frac{C_1}{x} \cosh ax + \frac{C_2}{x} \sinh ax$ or $y = \frac{C_3}{x}e^{+ax} + \frac{C_4}{x}e^{-ax}$
$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$	Solve the equation $n^2 + an + b = 0$, and get the roots $n = n_+$ and $n = n_-$. Then (a) if n_+ and n_- are real and unequal, $y = C_1\exp(n_+x) + C_2\exp(n_-x)$ (b) if n_+ and n_- are real and equal to n, $y = e^{nx}(C_1x + C_2)$ (c) if n_+ and n_- are complex: $n_\pm = p \pm iq$, $y = e^{px}(C_1 \cos qx + C_2 \sin qx)$
$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$	$y = C_1\int_0^x \exp(-\bar{x}^2)\, d\bar{x} + C_2$
$\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} = 0$	$y = C_1\int_0^x \exp(-\bar{x}^3)\, d\bar{x} + C_2$
$\frac{d^2y}{dx^2} = f(x)$	$y = \int_0^x\int_0^{\bar{x}} f(\bar{\bar{x}})d\bar{\bar{x}}\, d\bar{x} + C_1x + C_2$
$\frac{1}{x}\frac{d}{dx}\left(x\frac{dy}{dx}\right) = f(x)$	$y = \int_0^x \frac{1}{\bar{x}}\int_0^{\bar{x}} \bar{\bar{x}}f(\bar{\bar{x}})d\bar{\bar{x}}d\bar{x} + C_1 \ln x + C_2$
$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) = f(x)$	$y = \int_0^x \frac{1}{\bar{x}^2}\int_0^{\bar{x}} \bar{\bar{x}}^2f(\bar{\bar{x}})d\bar{\bar{x}}d\bar{x} - \frac{C_1}{x} + C_2$
$\frac{d^2y}{dx^2} = h(y)$	$x = \int_0^y \frac{d\bar{y}}{\sqrt{2\int_0^{\bar{y}} h(\bar{\bar{y}})d\bar{\bar{y}} + C_1}} + C_2$
$x^3\frac{d^3y}{dx^3} + ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$	$y = C_1x^{n_1} + C_2x^{n_2} + C_3x^{n_3}$, where the n_k are the roots of the equation $n(n-1)(n-2) + an(n-1) + bn + c = 0$, provided that all roots are distinct.

Table E

<u>Appendix E</u>: Some Ordinary Differential Equations and Their Solutions (cont'd)

Error Function:

The error function is defined as

$$\operatorname{erf} x = \frac{\int_0^x \exp(-\bar{x}^2)\, d\bar{x}}{\int_0^\infty \exp(-\bar{x}^2)\, d\bar{x}} = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\bar{x}^2)\, d\bar{x}$$

$$\frac{d}{dx} \operatorname{erf} u = \frac{2}{\sqrt{\pi}} \exp(-u^2) \frac{du}{dx}$$