UNIVERSITI SAINS MALAYSIA

First Semester Examination
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EEE 443/3 – DIGITAL SIGNAL PROCESSING
[PEMPROSESAN ISYARAT DIGIT]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **THIRDTEEN (13)** pages of printed material and **FIVE (5)** pages of Appendix before you begin the examination. English version from page **TWO (2)** to page **SEVEN (7)** and Malay version from page **EIGHT (8)** to page **THIRDTEEN (13)**.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TIGA BELAS (13)** muka surat bercetak beserta Lampiran **LIMA (5)** mukasurat bercetak sebelum anda memulakan peperiksaan ini. Versi Bahasa Inggeris daripada muka surat **DUA (2)** sehingga muka surat **TUJUH (7)** dan versi Bahasa Melayu daripada muka surat **LAPAN (8)** sehingga muka surat **TIGA BELAS (13)**.*

Instructions: This question paper consists of **SIX (6)** questions. Answer **FIVE** questions. All questions carry the same marks.
*[**Arahan:** Kertas soalan ini mengandungi **ENAM (6)** soalan. Jawab **LIMA** soalan. Semua soalan membawa jumlah markah yang sama]*

Answer to any question must start on a new page
[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baharu].

"In the event of any discrepancies, the English version shall be used".
[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

ENGLISH VERSION

1. (a) An electroencephalogram (EEG) signal is useful in monitoring brain activities. EEG signal contains useful frequencies up to 100 Hz. If we want to process EEG signal using digital signal processor, analog-to-digital (A/D) conversion needs to be done.

   (i) Draw the basic block diagram of an A/D and explain the function of each block.

   (20 marks)

   (ii) What is the Nyquist rate for this EEG signal?

   (5 marks)

   (iii) If we use a sampling rate of 400 samples/s, what is the highest EEG frequency that can be represented uniquely at this sampling rate?

   (10 marks)

   (b) A linear time-invariant (LTI) discrete-time system is built from two blocks. These two blocks are connected in parallel. The first block has an impulse response $h_1(n)$ while the second block has an impulse response $h_2(n)$. The input for this system is $x(n)$, whereas the output of this system is $y(n)$.

   (i) Draw the block diagram for this system.

   (10 marks)

   (ii) Determine the output of the system, if:

$$h_1(n) = \{1,2,-2,-1\}$$
$$h_2(n) = \{1,-2,2,2\}$$
$$x(n) = \{4,5,2,1,3,6\}$$

   (25 marks)

   (c) Give one of the usages of the autocorrelation. Then, calculate the autocorrelation of signal $x(n)$ and $y(n)$, $r_{xy}(l)$, given:

$$x(n) = \{10,20,10,30\}$$
$$y(n) = \{1,1,8,16,6,25\}$$

   (30 marks)

2. Question 2 is based on Figure 2. This figure shows the ROC for signal $x(n)$, together with its pole-zero plot. In this plot, there are two zeros (i.e., z_1 and z_2), and two poles (i.e., p_1 and p_2). The ROC is given as .

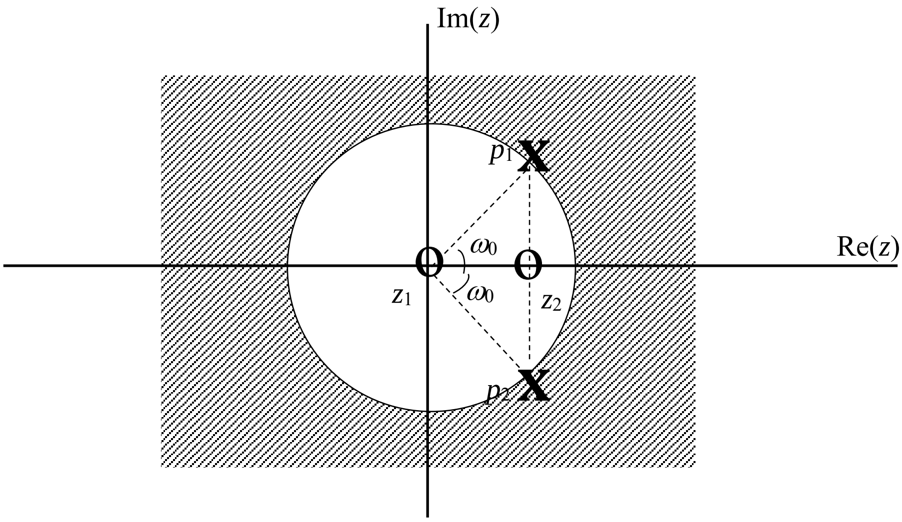

Figure 2

(a) Determine all zeros and poles, in terms of .

(10 marks)

(b) Give one possible expression for $X(z)$.

(15 marks)

(c) Give one possible expression for $x(n)$.

(15 marks)

(d) Analyze the signal:

(i) Is the signal stable? Give your reason for your answer.

(10 marks)

(ii) Is the signal causal? Give your reason for your answer.

(10 marks)

(iii) If is equal to 0.25, is the signal periodic? If it is periodic, determine its period.

(10 marks)

(iv) If is equal to 0.25π, is the signal periodic? If it is periodic, determine its period.

(10 marks)

(v) Can $x(n)$ be transformed into frequency domain by using Fourier transform? Give the reason for your answer. If $x(n)$ has a Fourier transform pair, determine $X(\omega)$.

(20 marks)

3. (a) Find $y(n)=x_1(n)④x_2(n)$, if $x_1(n)=\{1,-1,2,1\}$ and $x_2(n)=\{3,4\}$.

(20 marks)

(b) Direct implementation of discrete Fourier transform (DFT) requires a lot of calculations. To reduce the computational time, the fast Fourier transform (FFT) has been introduced.

(i) Draw eight-point FFT.

(20 marks)

(ii) Draw eight-point inverse FFT (IFFT).

(20 marks)

(c) You want to develop a speech recognition system. The discrete-time input to your system is an aperiodic digital voice signal $x(n)$ with the sample length L equal to 1800. You found out that $x(n)$ suffers from additive noise. To remove this noise, you plan to do noise filtering process in frequency domain. The noise reduction filter that you want to use has a finite impulse response, with length M equal to 7. The impulse response of this filter in frequency domain is $H(k)$. To save computational time, you are provided with a programming library with functions of FFT for 4-point, 16-point, 32-point, 64-point, and 128-points. By using appropriate figure, explain how you will process this voice signal.

(40 marks)

4. (a) Assuming that you must design a digital filter for an application in which the phase-distortion is not tolerable, which filter type amongst FIR or IIR will you select for that application? Explain your answer?

(20 marks)

(b) Design a bandstop filter that satisfies the following specifications:

(i) Estimate the order of the equiripple filter required to meet these specifications.

(20 marks)

(ii) What weighting function, , should be used to design this filter?

(10 marks)

(iii) What is the minimum number of extremal frequencies that the optimal filter must have?

(20 marks)

(c) Design a linear phase digital FIR low-pass filter with the following specifications;

using windows design method.

(30 marks)

5. (a) Determine the coefficients of an IIR filter which are obtained using bilinear transformation from a second-order Butterworth analog prototype filter with a 3-dB cut-off frequency of 3 kHz. The sampling rate for the digital filter is 30,000 samples per second.

(50 marks)

(b) A second-order lowpass IIR digital filter with a 3-dB cutoff frequency at has following transfer function. Design a second-order lowpass filter with a 3-dB cutoff frequency by transforming the above lowpass filter function using a lowpass-to-lowpass spectral transformation.

(20 marks)

(c) Design a digital low-pass filter using Butterworth and Chebyshev filters that has a passband cutoff frequency with and a stopband cutoff frequency with . The filter is to be designed using the bilinear transformation.

(30 marks)

6. (a) Given a system with difference equation given below, find a transposed direct form II for this system.

(10 marks)

(b) The unit sample response of an FIR filter is

(i) Draw the direct form implementation of this system.

(20 marks)

(ii) Show that the corresponding system function is

and use this to draw a flowgraph that is a cascade of an FIR system with an IIR system.

(30 marks)

(c) Given a network shown in Figure 6, find the system function and the unit sample response. Then, draw an equivalent direct form II structure.

(40 marks)

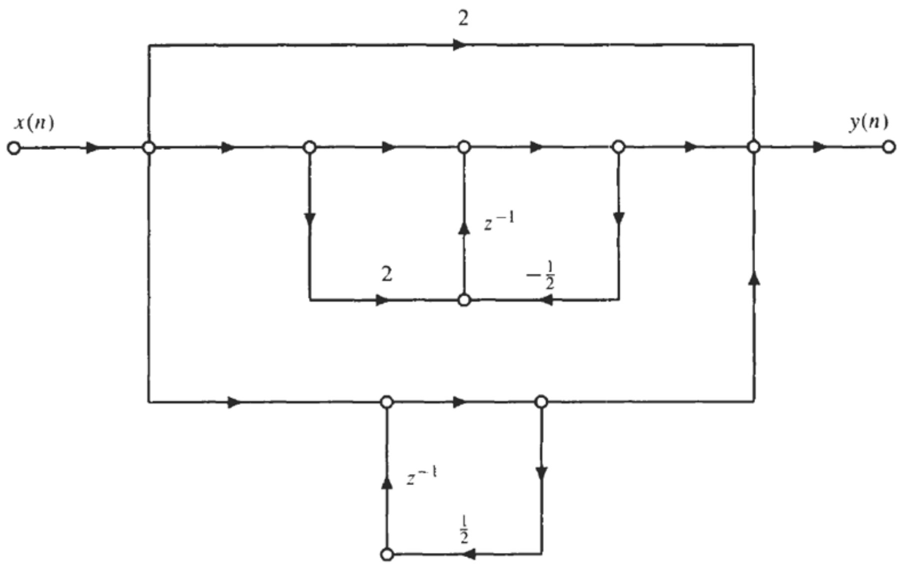

Figure 6

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VERSI BAHASA MELAYU

1. (a) Isyarat elektroencephalogram (EEG) berguna dalam memantau aktiviti otak. Isyarat EEG mengandungi frekuensi berguna sehingga 100 Hz. Jika kita mahu *memproses isyarat EEG menggunakan pemproses isyarat digital, penukaran analog-ke-digital (A/D) perlu dilakukan.*

(i) Lukiskan rajah blok asas A/D dan terangkan fungsi setiap blok tersebut.

(20 markah)

(ii) Apakah kadar Nyquist bagi signal EEG ini?

(5 markah)

(iii) Jika kita menggunakan kadar pensampelan 400 sampel/s, apakah frekuensi tertinggi EEG yang boleh diwakili secara unik pada kadar pensampelan ini?

(10 markah)

(b) Satu sistem diskret linear masa tak berubah (LTI) dibina daripada dua blok. Kedua-dua blok disambungkan secara selari. Blok pertama mempunyai sambutan dedenyut $h_1(n)$ manakala blok kedua mempunyai sambutan dedenyut $h_2(n)$. Input bagi sistem ini adalah x(n), manakala output sistem ini ialah y(n).

(i) Lukiskan rajah blok bagi system ini.

(10 markah)

(ii) Tentukan keluaran sistem, jika:

$$h_1(n) = \{1,2,-2,-1\}$$
$$h_2(n) = \{1,-2,2,2\}$$
$$x(n) = \{4,5,2,1,3,6\}$$

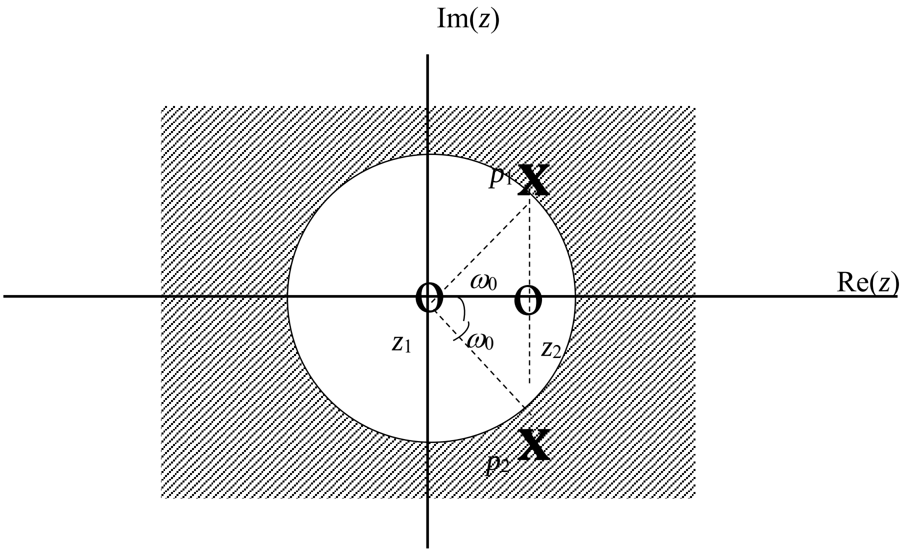
(25 markah)

(c) Berikan satu daripada kegunaan-kegunaan autokorelasi. Kemudian, kira autokorelasi signal x(n) dan y(n), $r_{xy}(l)$, diberikan:

$$x(n) = \{10,20,10,30\}$$
$$y(n) = \{1,1,8,16,6,25\}$$

(30 markah)

2. *Soalan 2 adalah berdasarkan Rajah 2. Rajah ini menunjukkan ROC bagi isyarat x(n), bersama dengan plot kutub-sifar. Dalam plot ini, terdapat dua sifar (iaitu z_1 dan z_2), dan dua kutub (iaitu p_1 dan p_2). ROC diberikan sebagai .*



Rajah 2

(a) *Tentukan kesemua sifar dan kutub, dalam terma ω_0.*

(10 markah)

(b) *Berikan satu ungkapan yang mungkin bagi X(z).*

(15 markah)

(c) *Berikan satu ungkapan yang mungkin bagi x(n).*

(15 markah)

(d) *Analisa signal tersebut:*

(i) *Adakah signal tersebut stabil? Berikan alasan kepada jawapan anda.*

(10 markah)

(ii) *Adakah signal tersebut kausal? Berikan alasan kepada jawapan anda.*

(10 markah)

(iii) *Jika ω_0 bersamaan 0.25, adakah signal tersebut berkala? Jika ianya berkala, tentukan tempoh kalaan tersebut.*

(10 markah)

(iv) *Jika ω_0 bersamaan 0.25 π, adakah signal tersebut berkala? Jika ianya berkala, tentukan tempoh kalaan tersebut.*

(10 markah)

(v) *Bolehkah x(n) diubah ke domain frekuensi menggunakan jelmaan Fourier? Berikan alasan kepada jawapan anda. Jika x(n) mempunyai pasangan jelmaan Fourier, tentukan $X(\omega)$.*

(20 markah)

3. (a) *Dapatkan $y(n)=x_1(n) \textcircled{4} x_2(n)$, jika $x_1(n)=\{1,-1,2,1\}$ dan $x_2(n)=\{3,4\}$.*

(20 markah)

(b) *Implementasi terus jelmaan Fourier diskret (DFT) memerlukan banyak pengiraan. Untuk mengurangkan masa pengiraan, jelmaan Fourier pantas (FFT) telah diperkenalkan.*

(i) *Lukiskan lapan-titik FFT.*

(20 markah)

(ii) *Lukiskan lapan-titik FFT songsang (IFFT).*

(20 markah)

(c) *Anda mahu membangunkan satu sistem pengecaman pertuturan. Masukan masa diskret kepada sistem anda adalah isyarat suara digital tak berkala x(n) dengan panjang sampel L bersamaan 1800. Anda mendapati bahawa x(n) mempunyai hingar tambah. Untuk membuang hingar ini, anda merancang untuk melakukan proses penapisan hingar dalam domain frekuensi. Penapis pengurangan hingar yang anda mahu gunakan mempunyai sambutan dedenyut, dengan panjang M sama dengan 7. Sambutan dedenyut penapis ini dalam domain frekuensi ialah H(k). Untuk menjimatkan masa pengiraan, anda disediakan dengan sebuah perpustakaan pengaturcaraan dengan fungsi FFT untuk 4-titik, 16-titik, 32-titik, 64-titik, dan 128-titik. Dengan menggunakan rajah yang sesuai, terangkan bagaimana anda akan memproses isyarat suara ini.*

(40 markah)

4. (a) *Dengan mengandaikan bahawa anda perlu mereka bentuk penapis digital untuk sebuah aplikasi di mana fasa-penyelewengan tidak boleh dipertimbangkan, apakah jenis penapis di kalangan FIR atau IIR yang anda akan pilih untuk aplikasi tersebut? Terangkan jawapan anda?*

(20 markah)

(b) *Reka bentuk penapis jalur henti yang memenuhi spesifikasi berikut:*

(i) *Anggarkan tertib penapis sama riak yang diperlukan untuk memenuhi spesifikasi ini.*

(20 markah)

(ii) *Apakah fungsi pemberat, , yang boleh digunakan untuk mereka bentuk penapis ini?*

(10 markah)

(iii) *Apakah bilangan minimum frekuensi extremal yang penapis optimum mesti perolehi?*

(20 markah)

(c) *Reka bentuk satu fasa lelurus penapis digital FIR laluan rendah dengan spesifikasi berikut:*

menggunakan kaedah tetingkap.

(30 markah)

5. (a) *Tentukan pekali penapis IIR yang diperolehi dengan menggunakan transformasi dwilelurus daripada tertib-kedua prototaip penapis analog Butterworth dengan 3-dB frekuensi potong 3 kHz. Kadar pensampelan untuk penapis digital ialah 30,000 sampel sesaat. Tentukan pekali penapis IIR ini.*

(50 markah)

(b) *Satu tertib-kedua penapis digital IIR laluan rendah dengan frekuensi potong 3-dB pada mempunyai fungsi pindah berikut. Reka bentuk satu tertib-kedua penapis laluan rendah dengan 3-dB frekuensi potong dengan mengtransformasikan fungsi penapis laluan rendah di atas dengan menggunakan transformasi spektrum laluan rendah ke laluan rendah.*

(20 markah)

(c) *Reka bentuk penapis digital laluan rendah menggunakan penapis Butterworth dan Chebyshev yang mempunyai frekuensi potong jalur lulus dengan dan frekuensi potong jalur henti dengan . Penapis ini mestilah direka bentuk menggunakan transformasi dwilelurus.*

(30 markah)

6. (a) *Diberi sistem dengan persamaan perbezaan diberikan di bawah, cari satu bentuk langsung II untuk sistem ini.*

(10 markah)

(b) *Sambutan unit sampel penapis FIR adalah*

(i) *Lukis bentuk langsung perlaksanaan sistem ini.*

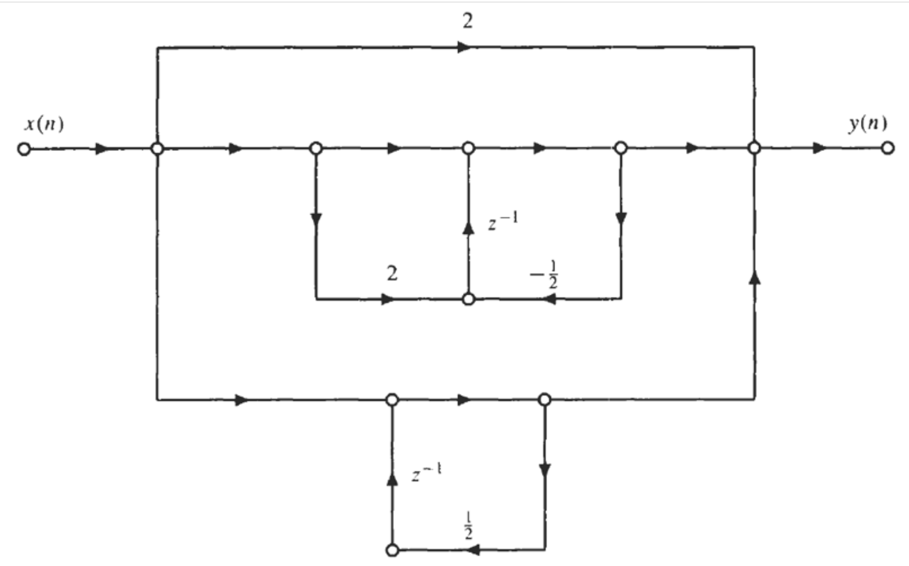
(20 markah)

(ii) *Tunjukkan bahawa fungsi sistem bagi sistem yang sama adalah* *dan gunakan fungsi ini untuk melukis gambarajah aliran yang lata sistemnya daripada sistem FIR dengan IIR.*

(30 markah)

(c) *Diberi rangkaian yang ditunjukkan dalam Rajah 6, cari fungsi sistem dan sambutan sampel unit. Kemudian, lukiskan satu struktur bentuk langsung II yang setara.*

(40 markah)



Rajah 6

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Appendix A/Lampiran A

Table 1: Summary of analysis and synthesis formulas

		Continuous-time signal: Time-domain	Continuous-time signal: Frequency-domain	Discrete-time signals: Time-domain	Discrete-time signals: Frequency-domain
Periodic signals	Fourier series	$c_k = \frac{1}{T_p}\int_{T_p} x_a(t)e^{-j2\pi kF_0 t}dt$	$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kF_0 t}$	$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$	$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$
		Continuous and periodic	Discrete and aperiodic	Discrete and periodic	Discrete and periodic
Aperiodic signal	Fourier transform	$X_a(F) = \int_{-\infty}^{\infty} x_a(t)e^{-j2\pi Ft}dt$	$x_a(t) = \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Ft}dF$	$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$	$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}d\omega$
		Continuous and aperiodic	Continuous and aperiodic	Discrete and aperiodic	Continuous and periodic

Discrete Fourier Transform (DFT):

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \qquad k = 0,1,2,\ldots,N-1$$

Inverse Discrete Fourier Transform (IDFT):

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \qquad n = 0,1,2,\ldots,N-1$$

Table 2: Some common *z*-transform pairs.

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
3	$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
4	$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|>\|a\|$
5	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
6	$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|<\|a\|$
7	$(\cos\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\|>1$
8	$(\sin\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\|>1$
9	$(a^n\cos\omega_0 n)u(n)$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\|>\|a\|$
10	$(a^n\sin\omega_0 n)u(n)$	$\frac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\|>\|a\|$

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Table 3: Properties of the *z*-transform.

Property	Time domain	z-domain	ROC

Notation	$x(n)$ $x_1(n)$ $x_2(n)$	$X(z)$ $X_1(z)$ $X_2(z)$	ROC: $r_2 < \|z\| < r_1$ ROC_1 ROC_2
Linearity	$ax_1(n)+bx_2(n)$	$aX_1(z)+bX_2(z)$	At least the intersection of ROC_1 and ROC_2
Time-shifting	$x(n-k)$	$z^{-k}X(z)$	That of $X(z)$ except z=0 if k>0 and $z=\infty$ if k<0.
Scaling in the z-domain	$a^n x(n)$	$X(a^{-1}z)$	$\|a\|r_2 < \|z\| < \|a\|r_1$
Time reversal	$x(-n)$	$X(z^{-1})$	$(1/r_1) < \|z\| < (1/r_2)$
Conjugation	$x^*(n)$	$X^*(z^*)$	ROC
Real part	$\mathrm{Re}\{x(n)\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Includes ROC
Imaginary part	$\mathrm{Im}\{x(n)\}$	$\frac{1}{2}j[X(z)-X^*(z^*)]$	Includes ROC
Differentiation in the z-domain	$nx(n)$	$-z\frac{dX(z)}{dz}$	$r_2 < \|z\| < r_1$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)*X_2(z)$	At least the intersection of ROC_1 and ROC_2
Correlation	$r_{x_1x_2}(l)=x_1(l)*x_2(-l)$	$R_{x_1x_2}(z)=X_1(z)*X_2(z^{-1})$	At least the intersection of ROC of $X_1(z)$ and ROC of $X_2(z^{-1})$
Initial value theorem	If $x(n)$ causal	$x(0)=\lim_{z\to\infty} X(z)$	
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{2\pi j}\oint_C X_1(v)X_2\left(\frac{z}{v}\right)v^{-1}dv$	At least, $r_{1l}\, r_{2l} < \|z\| < r_{1v}\, r_{2v}$
Parseval's relation	$\sum_{n=-\infty}^{\infty} x_1(n)x_2^*(n)=\frac{1}{2\pi j}\oint_C X_1(v)X_2^*\left(\frac{1}{v^*}\right)v^{-1}dv$		

Appendix B/Lampiran B

i. Information on FIR filter design using common windows

Table A.1 Some common windows

Rectangular	$w(n) = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hanning[1]	$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hamming	$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Blackman	$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{N}\right) + 0.08\cos\left(\frac{4\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$

Table A.2 The Peak side-lobe amplitude of some common windows and the approximate transition width and stopband attenuation of an Nth-Order low-pass filter designed using the given window

Window	Side-Lobe Amplitude (dB)	Transition Width (Δf)	Stopband Attenuation (dB)
Rectangular	−13	$0.9/N$	−21
Hanning	−31	$3.1/N$	−44
Hamming	−41	$3.3/N$	−53
Blackman	−57	$5.5/N$	−74

ii. Information on FIR and IIR filter design

Table A.3 Equations used to design FIR and IIR filters

Unit sample response for an ideal lowpass filter	$h_d(n) = \frac{\sin(n-\alpha)\omega_c}{\pi(n-\alpha)}$
Equiripple filter	$N = \frac{-10\log(\delta_s\delta_p) - 13}{14.6\Delta f}$
Discrimination Factor	$d = \left[\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}\right]^{1/2} = \frac{\epsilon}{\sqrt{A^2 - 1}}$
Selectivity Factor	$k = \frac{\Omega_p}{\Omega_s}$
Butterworth	$\lvert H_a(j\Omega)\rvert^2 = \frac{1}{1 + (\frac{j\Omega}{j\Omega_c})^{2N}}$ $\lvert H_a(j\Omega)\rvert^2 = \frac{1}{1 + \epsilon^2(\frac{j\Omega}{j\Omega_p})^{2N}}$ where,

	$$\epsilon = (\frac{\Omega_p}{\Omega_c})^N$$ $$\lvert H_a(j\Omega)\rvert^2 = H_a(s)H_a(-s)\rvert_{s=j\Omega}$$ The poles: $$s_k = (-1)^{1/2N}(j\Omega_c) = \Omega_c \exp\left[j\frac{(N+1+2k)\pi}{2N}\right], \quad k = 1, 2, \ldots, 2N-1$$ The order: $$N \geq \frac{\log d}{\log k}$$
Chebyshev	$$\lvert H_a(j\Omega)\rvert^2 = \frac{1}{1+\epsilon^2 T_N^2(\frac{\Omega}{\Omega_p})}$$ System function of a type I Chebyshev filter $$H_a(s) = H_a(0)\prod_{k=0}^{N-1}\frac{-s_k}{s-s_k}$$ where, $$H_a(0) = (1-\epsilon^2)^{-1/2}, \quad N : \text{even}$$ $$H_a(0) = 1, \quad N : \text{odd}$$ The order: $$N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$$
Elliptic	$$\lvert H_a(\Omega)\rvert^2 = \frac{1}{1+\epsilon^2 U_N^2(\frac{\Omega}{\Omega_p})}$$ where $$U_N(\frac{1}{\Omega}) = \frac{1}{U_N(\Omega)}$$ The order: $$N \geq \frac{\log(\frac{16}{d^2})}{\log(\frac{1}{q})}$$ where $$q = q_0 + 2q_0^5 + 15q_0^9 + 150q_0^{13}$$ where $$q_0 = \frac{1}{2}\frac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}$$

iii. Information on bilinear and frequency transformation

Table A.4 Information used for bilinear and frequency transformation

Bilinear Z-transform	$H(z) = H(s)\|_{s=\frac{2}{T}(\frac{z-1}{z+1})}$
Pre-warped analog frequency	$\Omega = \frac{2}{T}\tan(\frac{\omega}{2})$

Table A.5 The transformation of an analog low-pass filter with a 3-dB cutoff frequency to other frequency selective filters

Transformation	Mapping	New Cutoff Frequencies
Low-pass	$s \to \frac{\Omega_p}{\Omega'_p} s$	Ω'_p
High-pass	$s \to \frac{\Omega_p \Omega'_p}{s}$	Ω'_p
Bandpass	$s \to \Omega_p \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}$	Ω_l, Ω_u
Bandstop	$s \to \Omega_p \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l \Omega_u}$	Ω_l, Ω_u

Table A.6 The transformation of a digital low-pass filter with a cutoff frequency to other frequency selective filters

Filter Type	Mapping	Design Parameters
Low-pass	$z^{-1} \to \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$	$\alpha = \frac{\sin[(\omega_c - \omega'_c)/2]}{\sin[(\omega_c + \omega'_c)/2]}$ ω'_c = desired cutoff frequency
High-pass	$z^{-1} \to -\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}$	$\alpha = -\frac{\cos[(\omega_c + \omega'_c)/2]}{\cos[(\omega_c - \omega'_c)/2]}$ ω'_c = desired cutoff frequency
Bandpass	$z^{-1} \to -\frac{z^{-2} - [2\alpha\beta/(\beta+1)]z^{-1} + [(\beta-1)/(\beta+1)]}{[(\beta-1)/(\beta+1)]z^{-2} - [2\alpha\beta/(\beta+1)]z^{-1} + 1}$	$\alpha = \frac{\cos[(\omega_{c2} + \omega_{c1})/2]}{\cos[(\omega_{c2} - \omega_{c1})/2]}$ $\beta = \cot[(\omega_{c2} - \omega_{c1})/2]\tan(\omega_c/2)$ ω_{c1} = desired lower cutoff frequency ω_{c2} = desired upper cutoff frequency
Bandstop	$z^{-1} \to \frac{z^{-2} - [2\alpha/(\beta+1)]z^{-1} + [(1-\beta)/(1+\beta)]}{[(1-\beta)/(1+\beta)]z^{-2} - [2\alpha/(\beta+1)]z^{-1} + 1}$	$\alpha = \frac{\cos[(\omega_{c1} + \omega_{c2})/2]}{\cos[(\omega_{c1} - \omega_{c2})/2]}$ $\beta = \tan[(\omega_{c2} - \omega_{c1})/2]\tan(\omega_c/2)$ ω_{c1} = desired lower cutoff frequency ω_{c2} = desired upper cutoff frequency