UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2002/2003

April 2003

JEE 543 – PEMPROSESAN ISYARAT DIGIT

Masa : 3 jam

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka berserta Lampiran (4 mukasurat) bercetak dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan ini.

Jawab **LIMA (5)** soalan.

Agihan markah bagi soalan diberikan disut sebelah kanan soalan berkenaan.

Jawab semua soalan di dalam Bahasa Malaysia.

1. (a) Dapatkan jelmaan-z songsang yang dinyatakan oleh jelmaan-z berikut dengan memecahkan kepada siri kuasa menggunakan keadah pembahagian panjang.

Inverse z-transform represented by the following z-transform by expanding it into a power series using long division:

$$X(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - z^{-1} + 0.3561z^{-2}}$$

(50%)

(b) Dapatkan jelmaan-z songsang berikut:
Find the inverse z-transform of the following:

$$X(z) = \frac{z - 1}{1 - 0.25z^{-1} - 0.375z^{-2}}$$

(50%)

2. (a) Pertimbangkan jujukan berikut:
Consider the following sequence:

$$f(n) = \{1,\ 0,\ 0,\ 1,\ 1\}$$

Dapatkan jelmaan Fourier diskrit untuk jujukan tersebut.
Find the discrete Fourier transform of the sequence.

(50%)

(b) Diberi satu komponen DFT:
Given a DFT component:

$$X(k) = [2, 1 + j, 0, 1 - j]$$

Dapatkan Fourier Diskrit songsang.
Find the inverse discrete Fourier.

(50%)

3. Nilai voltan tersampel bagi satu isyarat lebarjalur 10Hz disampelkan pada 125Hz adalah (0, 5, 1, 1, 0.5).

The sampled voltage values of a 10Hz bandwidth signal sampled at 125Hz were (0, 5, 1, 1, 0.5).

(a) Tunjukkan bagaimana jelmaan Fourier Diskrit bagi jujukan ini boleh diperolehi menggunakan jelmaan fourier pantas.

Demonstrate how the discrete Fourier Transform of this sequence may be obtained using the fast Fourier transform.

(70%)

(b) Dapatkan jelmaan Fourier untuk data di atas.
Obtain the Fourier transform of the data.

(30%)

4. Pertimbangkan penuras anjakan-tak-berbeza kausal lelurus dengan sistem fungsi.
Consider the causal linear shift-invariant filter with system function.

$$H(z) = \frac{1 + 0.237z^{-1}}{(1 + 0.4z^{-1} - 0.8z^{-2})(1 + 0.32z^{-1})}$$

Lakarkan graf aliran isyarat untuk sistem ini menggunakan
Draw a signal flowgraph for this system using

(a) Bentuk terus I
Direct form I (30%)

(b) Bentuk terus II
Direct form II (30%)

(c) Satu kaskad bagi sistem peringkat pertama dan kedua dalam bentuk terus II.
A cascade of first and second-order systems realized in direct form II.
(40%)

5. (a) Dengan menganggap satu pendaraban kompleks memerlukan 10μs dan jumlah masa untuk mengira DFT ditentukan oleh jumlah masa yang diambil untuk menjalankan kesemua pendaraban.

Assume that a complex multiply takes 10μs and that the amount of time to compute a DFT is determined by the amount of time it takes to perform all of the multiplication.

(i) Berapakah masa yang diambil untuk mengira 512-titik DFT secara terus.
How much times does it take to compute a 512-point DFT directly?

(ii) Berapakah masa yang diperlukan jika FFT digunakan.
How much time is required if an FFT is used.

(iii) Ulangi bahagian (i) dan (ii) untuk 1024-titik DFT.
Repeat part (i) and (ii) for 1024-point DFT.

(50%)

(b) Pertimbangkan jujukan panjang-terhad.
Consider the finite-length sequence.

$$X(u) = \delta(n) + 2\delta\,(n-5)$$

(i) Dapatkan jelmaan Fourier diskrit 10-titik untuk x(n).
Find the 10-point discrete Fourier transform of x(n).

(ii) Dapatkan jujukan yang mempunyai satu jelmaan Fourier Diskrit.
Find the sequence that has a discrete Fourier transform.

$$Y(k) = e^{j2k\frac{2\pi}{10}} X(k)$$

di mana X(k) adalah DFT 10-titik bagi x(n).
where X(k) is the 10-point DFT of x(n).

(50%)

6. Fungsi pindah berikut menunjukkan dua penuras yang berbeza yang memenuhi spesifikasi sambutan amplitud-frekuensi.

The following transfer functions represent two different filters meeting identical amplitude-frequency response specifications:

(i) $$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} + \frac{b_3 + b_4 z^{-1} + b_5 z^{-2}}{1 + a_3 z^{-1} + a_4 z^{-2}}$$

di mana
where

$b_0 = 3.136\,362 \times 10^{-1}$
$b_1 = 5.456\,657 \times 10^{-2}$
$b_2 = 4.635\,728 \times 10^{-1}$
$b_3 = -5.456\,657 \times 10^{-2}$
$b_4 = 3.136\,362 \times 10^{-1}$
$b_5 = 4.635\,728 \times 10^{-1}$
$a_1 = -8.118\,702 \times 10^{-1}$
$a_2 = 3.339\,288 \times 10^{-1}$
$a_3 = 2.794\,577 \times 10^{-1}$
$a_4 = 3.030\,631 \times 10^{-1}$

(ii) $H(z) = \sum_{k=0}^{22} h_k z^{-k}$

dimana
where

h_0 = 0.398 264 80 x 10^{-1} = h_{22}

h_1 = -0.168 743 80 x 10^{-1} = h_{21}

h_2 = 0.347 811 30 x 10^{-1} = h_{20}

h_3 = 0.120 528 90 x 10^{-1} = h_{19}

h_4 = -0.447 318 60 x 10^{-1} = h_{18}

h_5 = 0.278 946 10 x 10^{-1} = h_{17}

h_6 = -0.875 733 60 x 10^{-1} = h_{16}

h_7 = -0.909 720 60 x 10^{-1} = h_{15}

h_8 = -0.156 675 50 x 10^{-1} = h_{14}

h_9 = -0.284 995 60 x 100 = h_{13}

h_{10} = 0.740 350 30 x 10^{-1} = h_{12}

h_{11} = 0.623 495 60 x 10^{0}

Untuk setiap penuras:
For each filter:

(a) Nyatakan sama ada ianya penuras FIR atau IIR.
State whether it is an FIR or IIR filter. (20%)

(b) Tunjukkan operasi penurasan dalam bentuk gambarajah blok dan tuliskan persamaan perbezaan.

Represent the filtering operation in a block diagram form and write down the difference equation, and

(50%)

(c) Tentukan dan berikan komen anda ke atas keperluan pengiraan dan penyimpanan.

Determine and comment on the computational and storage requirements.

(30%)

ooo0ooo

Property	Fourier Transform $x(t) \xleftrightarrow{FT} X(j\omega)$ $y(t) \xleftrightarrow{FT} Y(j\omega)$	Fourier Series $x(t) \xleftrightarrow{FS;\,\omega_o} X[k]$ $y(t) \xleftrightarrow{FS;\,\omega_o} Y[k]$ Period = T
Linearity	$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$	$ax(t) + by(t) \xleftrightarrow{FS;\,\omega_o} aX[k] + bY[k]$
Time shift	$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o}X(j\omega)$	$x(t - t_o) \xleftrightarrow{FS;\,\omega_o} e^{-jk\omega_o t_o}X[k]$
Frequency shift	$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$	$e^{jk_o\omega_o t}x(t) \xleftrightarrow{FS;\,\omega_o} X[k - k_o]$
Scaling	$x(at) \xleftrightarrow{FT} \frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$	$x(at) \xleftrightarrow{FS;\,a\omega_o} X[k]$
Differentiation-time	$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$	$\frac{d}{dt}x(t) \xleftrightarrow{FS;\,\omega_o} jk\omega_o X[k]$
Differentiation-frequency	$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$	—
Integration/ Summation	$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$	—
Convolution	$\int_{-\infty}^{\infty} x(\tau)y(t-\tau)\,d\tau \xleftrightarrow{FT} X(j\omega)Y(j\omega)$	$\int_{\langle T\rangle} x(\tau)y(t-\tau)\,d\tau \xleftrightarrow{FS;\,\omega_o} TX[k]Y[k]$
Modulation	$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\nu)Y(j(\omega-\nu))\,d\nu$	$x(t)y(t) \xleftrightarrow{FS;\,\omega_o} \sum_{l=-\infty}^{\infty} X[l]Y[k-l]$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	$\frac{1}{T}\int_{\langle T\rangle} \lvert x(t)\rvert^2\,dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$
Duality	$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$
Symmetry	$x(t)$ real $\xleftrightarrow{FT} X^*(j\omega) = X(-j\omega)$ $x(t)$ imaginary $\xleftrightarrow{FT} X^*(j\omega) = -X(-j\omega)$ $x(t)$ real and even $\xleftrightarrow{FT} \text{Im}\{X(j\omega)\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FT} \text{Re}\{X(j\omega)\} = 0$	$x(t)$ real $\xleftrightarrow{FS;\,\omega_o} X^*[k] = X[-k]$ $x(t)$ imaginary $\xleftrightarrow{FS;\,\omega_o} X^*[k] = -X[-k]$ $x(t)$ real and even $\xleftrightarrow{FS;\,\omega_o} \text{Im}\{X[k]\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FS;\,\omega_o} \text{Re}\{X[k]\} = 0$

Discrete-Time FT	Discrete-Time FS
$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$	$x[n] \xleftrightarrow{DTFS;\ \Omega_o} X[k]$ $y[n] \xleftrightarrow{DTFS;\ \Omega_o} Y[k]$ Period $= N$
$ax[n] + by[n] \xleftrightarrow{DTFT} aX(e^{j\Omega}) + bY(e^{j\Omega})$	$ax[n] + by[n] \xleftrightarrow{DTFS;\ \Omega_o} aX[k] + bY[k]$
$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o}X(e^{j\Omega})$	$x[n - n_o] \xleftrightarrow{DTFS;\ \Omega_o} e^{-jk\Omega_o n_o}X[k]$
$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega-\Gamma)})$	$e^{jk_o\Omega_o n}x[n] \xleftrightarrow{DTFS;\ \Omega_o} X[k - k_o]$
$x_z[n] = 0, \quad n \neq lp$ $x_z[pn] \xleftrightarrow{DTFT} X_z(e^{j\Omega/p})$	$x_z[n] = 0, \quad n \neq lp$ $x_z[pn] \xleftrightarrow{DTFS;\ p\Omega_o} pX_z[k]$
—	—
$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega}X(e^{j\Omega})$	—
$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DTFT} \frac{X(e^{j\Omega})}{1 - e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$	—
$\sum_{l=-\infty}^{\infty} x[l]y[n-l] \xleftrightarrow{DTFT} X(e^{j\Omega})Y(e^{j\Omega})$	$\sum_{l=\langle N\rangle} x[l]y[n-l] \xleftrightarrow{DTFS;\ \Omega_o} NX[k]Y[k]$
$x[n]y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\Gamma})Y(e^{j(\Omega-\Gamma)})\, d\Gamma$	$x[n]y[n] \xleftrightarrow{DTFS;\ \Omega_o} \sum_{l=\langle N\rangle} X[l]Y[k-l]$
$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} \lvert X(e^{j\Omega})\rvert^2\, d\Omega$	$\frac{1}{N}\sum_{n=\langle N\rangle} \lvert x[n]\rvert^2 = \sum_{k=\langle N\rangle} \lvert X[k]\rvert^2$
$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$	$X[n] \xleftrightarrow{DTFS;\ \Omega_o} \frac{1}{N}x[-k]$
$x[n]$ real $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = X(e^{-j\Omega})$ $x[n]$ imaginary $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = -X(e^{-j\Omega})$ $x[n]$ real and even $\xleftrightarrow{DTFT} \text{Im}\{X(e^{j\Omega})\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFT} \text{Re}\{X(e^{j\Omega})\} = 0$	$x[n]$ real $\xleftrightarrow{DTFS;\ \Omega_o} X^*[k] = X[-k]$ $x[n]$ imaginary $\xleftrightarrow{DTFS;\ \Omega_o} X^*[k] = -X[-k]$ $x[n]$ real and even $\xleftrightarrow{DTFS;\ \Omega_o} \text{Im}\{X[k]\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFS;\ \Omega_o} \text{Re}\{X[k]\} = 0$

E.1 *Basic z-Transforms*

Signal	*Transform*	*ROC*
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$\lvert z \rvert > 1$
$\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$\lvert z \rvert > \lvert \alpha \rvert$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\lvert z \rvert > \lvert \alpha \rvert$
$[\cos(\Omega_1 n)]u[n]$	$\frac{1 - z^{-1}\cos\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z \rvert > 1$
$[\sin(\Omega_1 n)]u[n]$	$\frac{z^{-1}\sin\Omega_1}{1 - z^{-1}2\cos\Omega_1 + z^{-2}}$	$\lvert z \rvert > 1$
$[r^n \cos(\Omega_1 n)]u[n]$	$\frac{1 - z^{-1}r\cos\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2 z^{-2}}$	$\lvert z \rvert > r$
$[r^n \sin(\Omega_1 n)]u[n]$	$\frac{z^{-1}r\sin\Omega_1}{1 - z^{-1}2r\cos\Omega_1 + r^2 z^{-2}}$	$\lvert z \rvert > r$

■ BILATERAL TRANSFORMS FOR SIGNALS THAT ARE NONZERO FOR $n < 0$

Signal	*Bilateral Transform*	ROC
$u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$

E.2 *z-Transform Properties*

Signal	*Unilateral Transform*	*Bilateral Transform*	ROC
$x[n]$	$X(z)$	$X(z)$	R_x
$y[n]$	$Y(z)$	$Y(z)$	R_y
$ax[n] + by[n]$	$aX(z) + bY(z)$	$aX(z) + bY(z)$	At least $R_x \cap R_y$
$x[n-k]$	See below	$z^{-k}X(z)$	R_x except possibly $\lvert z\rvert = 0, \infty$
$\alpha^n x[n]$	$X\left(\frac{z}{\alpha}\right)$	$X\left(\frac{z}{\alpha}\right)$	$\lvert\alpha\rvert R_x$
$x[-n]$	—	$X\left(\frac{1}{z}\right)$	$\frac{1}{R_x}$
$x[n] * y[n]$	$X(z)Y(z)$	$X(z)Y(z)$	At least $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	$-z\frac{d}{dz}X(z)$	R_x except possibly addition or deletion of $z = 0$