UNIVERSITI SAINS MALAYSIA

UNIVERSITY RESEARCH GRANT FINAL REPORT
Geran Penyelidikan Universiti Laporan Akhir

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A.	PARTICULARS OF RESEARCH / *MAKLUMAT PENYELIDIKAN:*
(i)	Title of Research: *Tajuk Penyelidikan:* **ON PLANAR HARMONIC MAPPINGS AND MINIMAL SURFACES**
(ii)	Account Number: **1001/PMATHS/811190** *Nombor Akaun:*
B.	PERSONAL PARTICULARS OF RESEARCHER / *MAKLUMAT PENYELIDIK:*
(i)	Name of Research Leader: *Nama Ketua Penyelidik:* **PROF DATO' ROSIHAN M. ALI**
	Name of Co-Researcher: *Nama Penyelidik Bersama:* Dr. Lee See Keong; Prof. V. Ravichandran; Prof. K. G. Subramanian
(ii)	School/Institute/Centre/Unit: *Pusat Pengajian /Institut/Pusat/Unit:* Sains Matematik
D.	Duration of this research: *Tempoh masa penyelidikan ini:* *Duration : 36 months *Tempoh :* **From** : December 1, 2011 **To** : November 30, 2014 *Dari:* *Ke :*

E.	**ABSTRACT OF RESEARCH** (An abstract of between 100 and 200 words must be prepared in **Bahasa Malaysia and in English**. This abstract will be included in the Annual Report of the Research and Innovation Section at a later date as a means of presenting the project findings of the researcher/s to the University and the community at large)	

The project seeks to study the geometric properties of planar harmonic mappings (and related mappings), and aims to apply the findings in the investigation of minimal surfaces.

Harmonic mappings in the plane are univalent (one-to-one) complex-valued solutions of the Laplace's equation. Of particular interest will be functions f in the unit disk D which are solutions of the elliptic partial differential equation

$$\overline{f_{\bar{z}}}(z) = \omega(z) f_z(z), \quad \omega \in D,$$

where ω is the analytic dilatation of f from D into itself. A harmonic function $f = u + iv$ in D is related to analytic functions, and have the canonical decomposition $f(z) = h(z) + \overline{g(z)},$ where h, g are both analytic in D. A conformal analytic mapping is a special case where its real and imaginary parts are conjugate harmonic functions satisfying the Cauchy–Riemann equations.

One interesting use of ideas from complex analysis is in the study of minimal surfaces. These are geometric objects that minimize surface area locally. It is known that a surface with an isothermal parametrization is minimal if and only if each coordinate function is harmonic, and the projection of the surface onto its base plane defines a harmonic univalent map. This project studies minimal Laguerre surfaces, and its connections with the lifts of biharmonic mappings, that is, mappings f satisfying $\Delta(\Delta f) = 0,$ where Δ is the Laplacian operator.

The project also studies normalized univalent logharmonic mappings $f(z) = zh(z)\overline{g(z)}$ in the unit disk D. This class has only been studied extensively in recent years, and as further evidence of its importance, note that the maps $F(\varsigma) = \log f(e^{\varsigma})$ are univalent harmonic mappings in the left half-plane.

The project also seek to establish an extension of the important duality principle from the class of analytic functions to the class H of normalized complex-valued harmonic mappings in the disk D. The dual class of a compact and complete subset is investigated vis-à-vis to a complex-valued continuous linear functional on H.

F.	**SUMMARY OF RESEARCH FINDINGS** *Ringkasan dapatan Projek Penyelidikan* A Laguerre surface is known to be minimal if and only if its corresponding isotropic map is biharmonic. To every Laguerre surface Φ is its associated surface Ψ = (1+ |u|^2)Φ, where |u| lies in the unit disk. The projection of the surface Ψ associated to a Laguerre minimal surface is shown to be biharmonic. A complete characterization of Ψ is obtained under the assumption that the corresponding isotropic map of the Laguerre minimal surface is harmonic. A sufficient and necessary condition is also derived for Ψ to be a graph. Estimates of the Gaussian curvature to the Laguerre minimal surface are obtained, and several illustrative examples are given. In another work, the existence of the Landau constant is obtained for functions with logharmonic Laplacian of the form F(z) = |z|^2L(z) + K(z), where z lies in the unit disk, L is logharmonic, and K is harmonic. The problem of minimizing the area was also solved. A comprehensive survey article was also written describing recent advances on univalent logharmonic mappings defined on a simply or multiply connected domain. Topics discussed include mapping theorems, logharmonic automorphisms, univalent logharmonic extensions onto the unit disk or the annulus, univalent logharmonic exterior mappings, and univalent logharmonic ring mappings. Logharmonic polynomials were also discussed, along with several important subclasses of logharmonic mappings. We also extended the important duality principle for analytic functions to the class *H* of normalized complex-valued harmonic mappings *f* in the unit disk. If *H_{1}* is the set of functions *f* in *H* satisfying *f*(0) = 0, $f_z(0)=1$ and $f_{\bar{z}}(0)=0$, and *V* a subset of *H_{1}*, then its dual *V** is the set consisting of functions *g* in *H_{1}* satisfying $(f * g) \neq 0$ for all nonzero *z* in the unit disk, and for all *f* in *V*. Here * denotes the convolution between harmonic functions. If *V* is compact and *V ^{T}* is complete, it is shown that $\lambda(V)=\lambda(V^{**})$ for each continuous complex-valued linear functional λ on *H*. Moreover, *f* in *V*** if and only if $\lambda(f) \in \{\lambda(f) : f \in V\}$.

G. **COMPREHENSIVE TECHNICAL REPORT**

Laporan Teknikal Lengkap

Applicants are required to prepare a comprehensive technical report explaining the project.
(This report must be attached separately)
Sila sediakan laporan teknikal lengkap yang menerangkan keseluruhan projek ini.
[Laporan ini mesti dikepilkan]

List the key words that reflect our research:
Senaraikan kata kunci yang mencerminkan penyelidikan anda:

English	Bahasa Malaysia
Harmonic mappings	Pemetaan harmonik
Logharmonic mappings	Pemetaan logharmonik
Biharmonic mappings	Pemetaan biharmonik
Minimal surfaces	Permukaan minimum
Hyperbolic metric	Metrik hiperbolik
Subordination	Subordinasi
Convolution	Konvolusi
Duality principle	Prinsip kedualan