UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2005/2006

April/Mei 2006

EEK 365 – SISTEM PENGAGIHAN ELEKTRIK KUASA

Masa : 3 jam

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH LAPAN** muka surat termasuk **EMPAT** mukasurat **FORMAT JAWAPAN AKHIR** dan **ENAM** mukasurat **Lampiran** bercetak sebelum anda memulakan peperiksaan ini.

Jawab **LIMA** soalan. Format Jawapan peperiksaan ini adalah

[i] Anda hendaklah menunjukkan jalan kerja jawapan dalam **Buku Jawapan**.

[ii] Jawapan-jawapan akhir kepada setiap soalan hendaklah diisi dalam kertas format jawapan akhir (FOJA) yang disediakan dan **mesti dikepilkan bersama dengan Buku Jawapan anda**.

Semua soalan hendaklah di jawab di dalam Bahasa Malaysia.

...2/-

1. (a) Mengikut takrifan yang luas, sistem agihan adalah bahagian daripada sistem utiliti elektrik antara sumber kuasa dan suis-suis perkhidmatan pengguna. Takrifan sistem agihan termasuk komponen-komponen berikut:
Isikan dalam borang FOJA

In a broad definition, the distribution system is part of the electric utility system between the bulk power source and the customer's service switches. This definition of distribution system includes the following components:
Fill in FOJA form. (10%)

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________

(b) Namakan jenis-jenis sistem subtransmisi yang berbeza.
Isikan dalam borang FOJA.
Name different types of subtransmission system.
Fill in FOJA form.

(10%)

1. ______________________
2. ______________________
3. ______________________
4. ______________________

...3/-

(c) Suatu sebstesyen distribusi tipikal boleh terdiri alat-alat berikut.
Tandakan mana satu yang sesuai.
Isikan dalam borang FOJA
A typical distribution substation may include the following equipments.
Tick any one which is appropriate.
Fill in FOJA form.

(10%)

1. Transformer kuasa [ ]
   Power transformer [ ]
2. Pemutus litar [ ]
   Circuit breakers [ ]
3. Suis Pemutus [ ]
   Disconnecting switches [ ]
4. Bas-bas dan penebat stesyen [ ]
   Station buses & insulators [ ]
5. Reaktor menghad arus [ ]
   Current limiting reactors [ ]
6. Reaktor pirau [ ]
   Shunt reactors [ ]
7. Transformer arus [ ]
   Current transformers [ ]
8. Transformer potensi [ ]
   Potential transformers [ ]
9. Voltan kapasitor [ ]
   Capacitors voltage [ ]
10. Kapasitor mengganding
    Coupling capacitors [ ]
11. Kapasitor siri [ ]
    Series capacitors [ ]
12. Kapasitor pirau [ ]
    Shunt capacitors [ ]
13. Sistem pembumian [ ]
    Grounding system [ ]
14. Penangkap dan/atau ruang kilat [ ]
    Lightning arresters and/or gaps [ ]
15. Perangkap talian [ ]
    Line traps [ ]
16. Geganti pelindung [ ]
    Protective relays [ ]
17. Bateri stesyen [ ]
    Station batteries [ ]
18. Alat-alat lain [ ]
    Other apparatus [ ]

...4/-

(d) Skim bus substesyen yang lazim digunakan termasuk.
Isikan dalam borang FOJA.

The most commonly used substation bus schemes includes.
Fill in FOJA form.

(10%)

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________

(e) Kes umum di mana kawasan perkhidmatan substesyen distribusi dilayan oleh n penyuap primer yang berasal daripada titik seperti dalam Rajah 1. Anggapkan bahawa beban dalam kawasan perkhidmatan diagihkan secara seragam dan setiap penyuap melayani kawasan berbentuk segitiga. Perbezaan beban yang dilayan oleh penyuap dalam perbezaan kawasan dA ialah

The general case in which the distribution substation service area is served by n primary feeders emanating from the point, as shown in Figure 1. Assume that the load in the service area is uniformly distributed and each feeder serves an area of triangular shape. The differential load served by the feeder in a differential area of dA is

$$dS = D\,dA \quad \text{kVA}$$

...5/-

dengan dS = perbezaan beban yang dilayan oleh penyuap dalam perbezaan masa dA, kVA.

where dS = differential load served by the feeder in the differential area of dA, kVA

D = ketumpatan beban kVA/mi^2

D = load density, kVA/mi^2

dA = perbezaan kawasan perkhidmatan penyuap mi^2

dA = differential service area of the feeder, mi^2

Dalam Rajah 1, hubungan berikut wujud:

In Figure 1, the following relationship exists:

$$\tan\theta = \frac{y}{x + dx}$$

Sahkan bahawa kejatuhan voltan dalam litar utama penyuap ialah

Verify that the voltage drop primary-feeder circuits is

$$\text{in } \%\ VDn = 2/3 \times K \times D \times \ln^3 \times \tan\theta$$

dengan K = % VD/(kVA.mi) ciri penyuap

where K = % VD/(kVA.mi) characteristic of the feeder

D = ketumpatan beban, kVA/mi^2

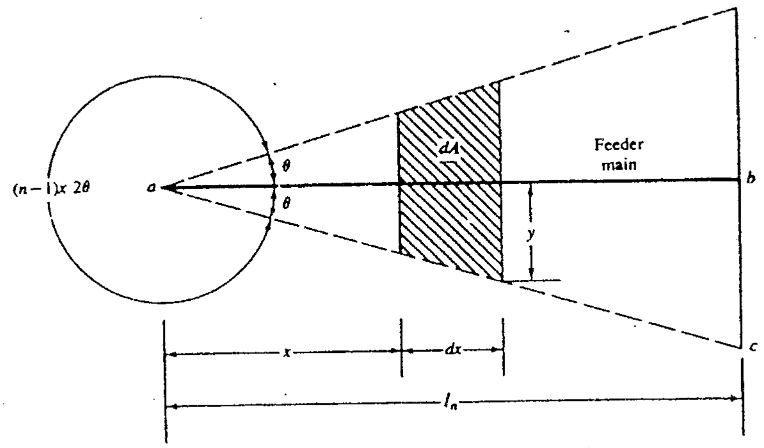
D = load density, kVA/mi^2

ℓ_n = panjang penyuap primer

ℓ_n = *length of primary feeder*

(60%)

...6/-



Rajah 1 : Kawasan perkhidmatan substesyen distribusi yang dilayan oleh n penyuap primer

Figure 1 : Distribution substation service area served by n primary feeders

2. (a) Suatu beban 250kW disambung kepada suatu substesyen distribusi. Penggunaan tenaga mingguan ialah 9240kW dan permintaan mingguan maksimum 20-min didapati 165kW. Cari faktor permintaan dan faktor beban mingguan 20-min untuk sistem distribusi.

A load of 250kW is connected at a distribution substation. The weekly energy consumption is 9240kW and 20-min weekly maximum demand is found to be 165kW. Find the demand factor and the 20-min weekly load factor for the distribution substation.

(30%)

...7/-

(b) Jika kadaran total semua transformer distribusi yang disambung ke penyuap ialah 4000 kVA. Hitung yang berikut:

If the total ratings of all distribution transformers connected to a feeder is 4000 kVA. Calculate the followings:

(i) Jumlah tenaga hilang teras tahunan pada penyuap jika purata hilang teras transformer ialah 0.49%

Total annual core loss energy on the feeder if the average core loss of transformer is 0.49%.

(ii) Nilai jumlah tenaga hilang teras yang didapati dalam bahagian (i) pada 30 sen (RM)kWh.

The value of the total core loss energy found in part (i) at 30 sen (RM)/kWh.

(30%)

(c) Rajah 2 menunjukkan sambungan T-T dan lakukan yang berikut.
Figure 2 shows the T-T connections and do the followings.

(i) Lakarkan diagram LV, betulkan orientasi atas rujukan kosong darjah seperti yang tertera.
Draw the LV diagram, correctly oriented on the zero degree reference shown.

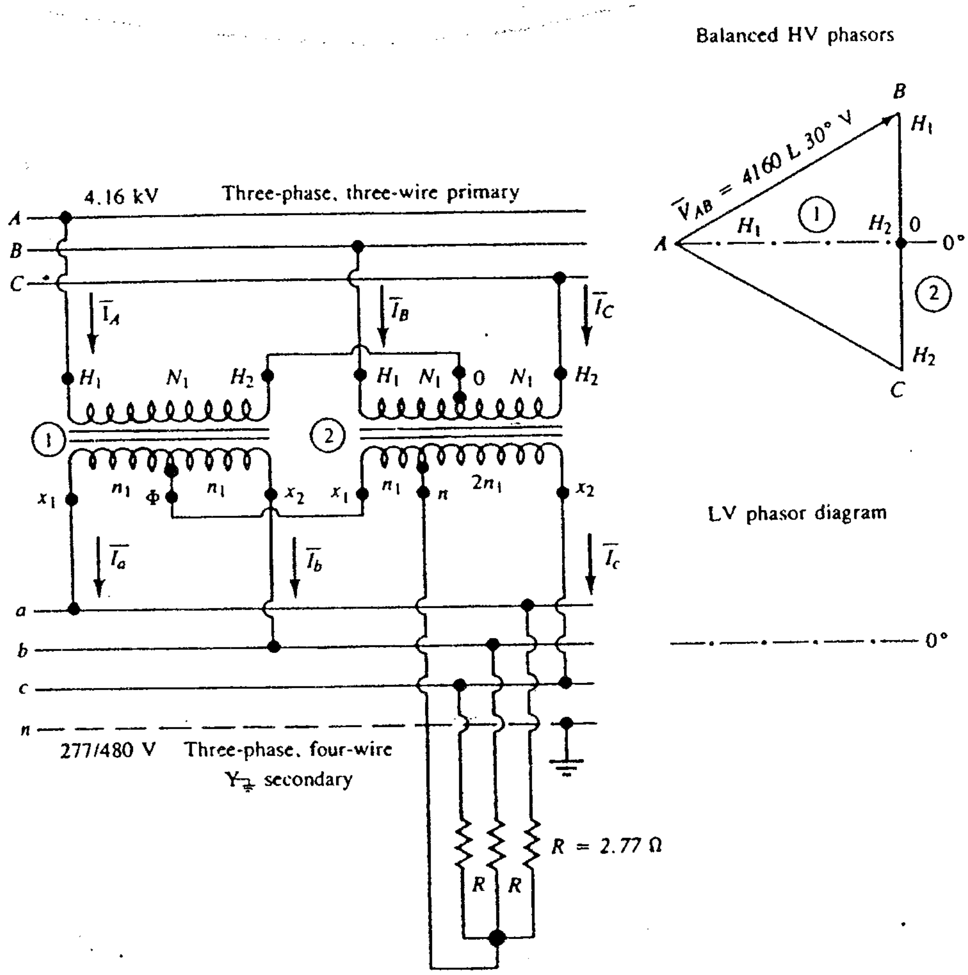
(ii) Cari nilai pemfasa V_{ab} dan V_{an}.
Find the value of the V_{ab} and V_{an} phasors.

...8/-

(iii) Cari magnitud voltan V_{nx_2} pada transformer 2.

Find the magnitude of the voltage V_{nx_2} on transformer 2.

(40%)



Rajah 2
Figure 2

...9/-

3. Tiga transformer satu fasa disambung delta-delta untuk membekalkan kuasa kepada beban 200 kVA, tiga fasa sambungan-wye yang mempunyai faktor kuasa 0.8 menyusul dan suatu beban ringan 80 kVA satu fasa dengan faktor kuasa 0.90 menyusul seperti dalam Rajah 3.

Three single-phase transformers are connected delta-delta to provide power for a three-phase wye-connected 200 kVA load with a 0.80 lagging power factor and a 80 kVA single-phase light load with a 0.90 lagging power factor, as shown in Figure 3.

Dengan menganggap bahawa tiga transformer satu fasa mempunyak peratus impedans yang sama dan nisbah peratus reaktans ke peratus rintangan yang sama. Voltan sebelah-primer bank ialah 7620/13,200 V dan voltan sebelah sekunder ialah 240 V. Anggap bahawa transformer satu fasa yang disambung antara fasa b dan c dikadarkan pada 100 kVA dan dua yang lain dikadarkan pada 75 kVA. Tentukan yang berikut:

Assume that the three single-phase transformers have equal percent impedance and equal ratios of percent reactance to percent resistance. The primary-side voltage of the bank is 7620/13,200 V and the secondary-side voltage is 240 V. Assume that the single-phase transformer connected between phases b and c is rated at 100 kVA and the other two are rated at 75 kVA. Determine the following:

(a) Arus talian $\bar{I}_a$
The line current $\bar{I}_a$ (20%)

(b) Arus belitan sekunder I_{ba}
The current in the secondary winding I_{ba} (20%)

...10/-

(c) Beban pada setiap transformer dalam kilovoltamperes

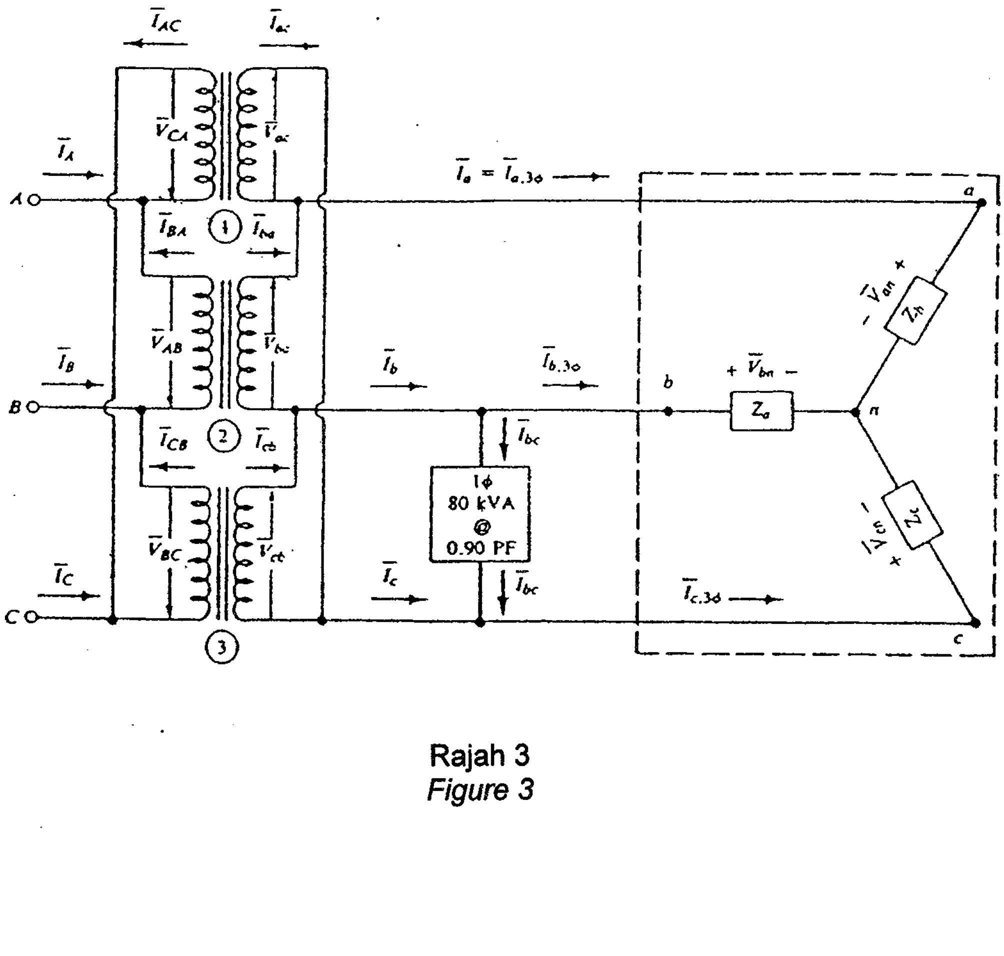
The load on each transformer in kilovoltamperes (20%)

(d) Arus belitan primer I_{CB}

The current primary winding I_{CB} (20%)

(e) Arus talian mengalir dalam wayar primer I_C

The line current flowing in primary-phase wire I_C (20%)



Rajah 3
Figure 3

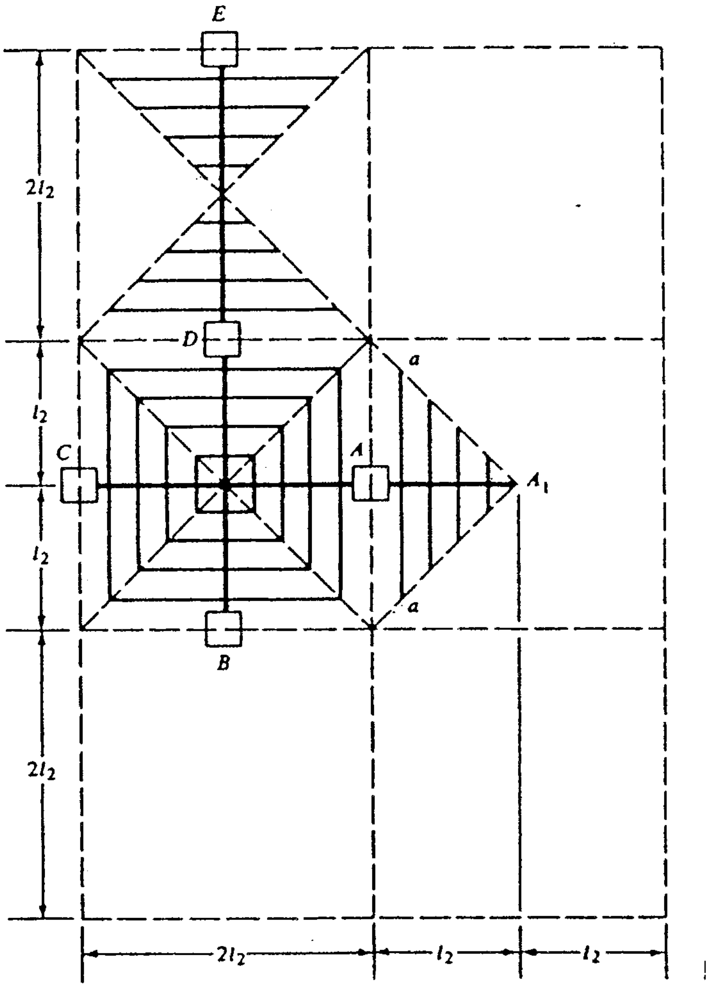
4. Rajah 4 menunjukkan suatu pola liputan kawasan perkhidmatan (tidak semesti pola yang sempurna) yang terdiri daripada penyuap primer utama dan lateral. Terdapat 5 substesyen yang tertera di dalam rajah, setiap satunya mempunyai dua penyuap utama. Sebagai contoh, substesyen A mempunyai dua utama dan setiap utama mempunyai banyak lateral yang diruangkan secara dekat seperti a-a.

Figure 4 shows a pattern of service area coverage (not necessarily a good pattern) with primary-feeder mains and laterals. There are five substations shown in the figure, each with two feeder mains. For example, substation A has two mains and each main has many closely spaced laterals such as a-a.

Jika lateral-lateral bukan tiga fasa, beban dalam utama dianggap betul-betul seimbang di antara ketiga-tiga fasa. Beban yang disambung daripada utama berkurang secara linear dengan jarak s, seperti yang ditunjukkan dalam Rajah 5.

If the laterals are not three-phase, the load in the main is assumed to be well-balanced among the three phases. The load tapped off the main decreases linearly with the distance s, as shown in Figure 5.

...12/-



Rajah 4
Figure 4

Dengan menggunakan notasi berikut dan notasi yang diberikan dalam rajah-rajah, analisis suatu penyuap utama.

Using the following notation and the notation given in the figures, analyze a feeder main..

D = kepadatan beban yang diagihkan secara seragam, kVA/mi^2
D = uniformly distributed load density, kVA/mi^2

V_{L-L} = Voltan bes dan voltan operasi nominal, talian-ke-talian kV
V_{L-L} = base voltage and nominal operating voltage, line-to-line kV

...13/-

A_2 = kawasan yang dibekalkan dengan satu penyuap utama
A_2 = area supplied by one feeder main

TA_2 = kawasan yang dibekalkan oleh satu stesyen
TA_2 = area supplied by one substation

S_2 = Input kVA pada substesyen ke satu penyuap utama
S_2 = kVA input at the substation to one feeder main

TS_2 = jumlah beban kVA yang dibekalkan oleh satu substesyen
TS_2 = total kVA load supplied by one substation

K_2 = % VD/(kVA . mi) untuk konduktor dan faktor kuasa beban yang dipertimbangkan
K_2 = % VD/(kVA . mi) for conductors and load power factor being considered

z_2 = impedans talian utama tiga fasa, Ω/(mi . fasa)
z_2 = impedance of three-phase main line, Ω/(mi . phase)

VD_2 = kejatuhan voltan pada hujung utama, sebagai contoh, A_1
VD_2 = voltage drop at end of main, for example, A_1

(a) Cari kawasan pembezaan dA dan kilovoltampere-beban-pembezaan yang dibekalkan d(S) yang ditunjukkan dalam Rajah 5.

Find the differential area dA and the differential kilovoltampere-load-supplied d(S) shown in Figure 5.

(20%)

...14/-

(b) Cari aliran beban kVA dalam utama dan sebarang titik s, yakni S_s. Ungkapkan S_s dalam sebutan S_2, s, dan l_2.

Find the kVA load flow in the main at any point s, that is, S_s. Express the S_s in terms of S_2, s, and l_2.

(20%)

(c) Cari kejatuhan voltan pembezaan pada titik s dan kemudian tunjukkan bahawa jumlah beban boleh dipusatkan pada $s = l_2/3$ untuk maksud menghitung VD_2.

Find the differential voltage drop at point s and then show that the total load may be concentrated at $s = l_2/3$ for the purpose of computing the VD_2.

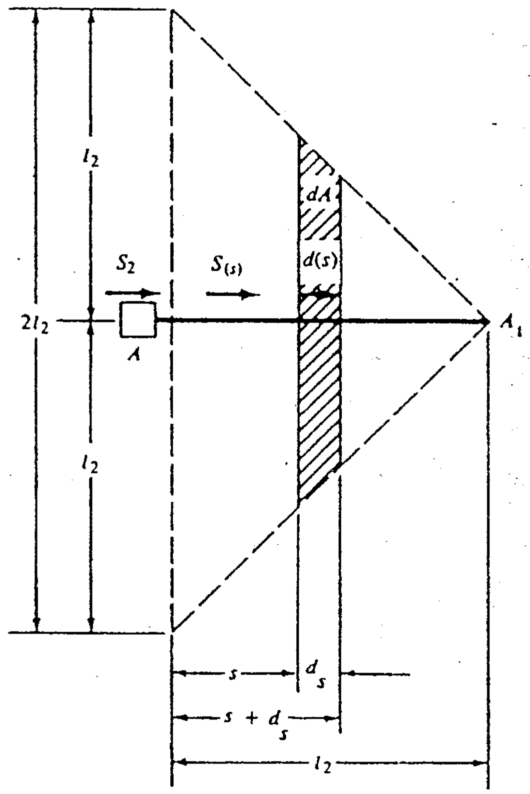
(30%)

(d) Andaikan pola dua-penyuap-per-substesyen ini akan dilaksanakan dengan penyuap had haba, yakni, pembebanan ampacity. Anggap bahawa ketumpatan beban ialah 500 kVA/mi^2, voltan talian-ke-talian ialah 12.42 kV, dan penyuap utama adalah talian wayar-terbuka #4/0 AWG ACSR. Cari peruang substesyen, kedua-dua hala, yakni, $2l_2$, dan beban ke atas transformer substesyen , yakni, TS_2. Ampacity konduktor untuk konduktor #4/0 AWG ACSR ialah 340A.

Suppose that this two-feeders-per-substation pattern is to be implemented with thermally limited, i.e., ampacity-loaded, feeders.
Assume that the load density is 500 kVA/mi^2, the line-to-line voltage is 12.47 kV, and the feeder mains are #4/0 AWG ACSR open-wire lines. Find the substation spacing, both ways, that is, $2l_2$, and the load on the substation transformers, that is, TS_2. The conductor ampacity for #4/0 AWG ACSR conductor is 340A.

(30%)

...15/-



Rajah 5
Figure 5

5. (a) Terbitan untuk voltan hujung penerima diberi dalam Lampiran sebagai fungsi sebutan-sebutan constant parameter K, P_r, Q_r dan A, B, C and D. Dengan mempelajari Lampiran tersebut, cari ungkapan berikut untuk V_r, K dan tan δ dalam sebutan R dan X.

The derivation for the receiving-end voltage is given in the Appendix in terms of constant K, P_r, Q_r and A, B, C and D parameter. By studying the Appendix, find the corresponding expressions for V_r, K and tan δ in terms of R and X.

(50%)

...16/-

(b) Anggap bahawa penyuap ekspres jejari, tertera dalam Rajah 8, digunakan pada distribusi luar Bandar dan disambung kepada beban tergumpal (atau terpusat) pada hujung penerima. Anggap impedans penyuap ialah 0.10 + j0.10 pu, voltan hujung penerima ialah 1.0 pu, P_r ialah 1.0 pu beban kuasa tetap dan fakctor kuasa pada hujung penerima ialah 0.8 menyusul. Maka data yang diberi dan persamaan-persamaan tepat untuk K, V_r dan tan δ yang diterbitkan di atas dan tentukan yang berikut:

Assume that the radial express feeder, shown in Figure 8, is used on rural distribution and is connected to a lumped-sum (or concentrated) load at the receiving end. Assume that the feeder impedance is 0.10 + j0.10 pu, the sending-end voltage is 1.0 pu, P_r is 1.0 pu constant power load and the power factor at the receiving end is 0.80 lagging. Use the given data and the exact equations for K, V_r and tan δ given previously and determine the following:

(i) Hitung V_r dan δ dengan menggunakan persamaan-persamaan tepat dan cari juga nilai-nilai arus yang bersepadanan I_r dan I_s.

Compute V_r and δ by using the exact equations and find also the corresponding values of the I_r and I_s currents.

(ii) Sahkan hasil-hasil nilai yang didapatkan dalam bahagian (a) dengan menggunakan hasil-hasil dalam $\overline{V}_s = \overline{V}_r + (R + jX)\,\overline{I}_r$.

Verify the numerical results found in part (a) by using those results in $\overline{V}_s = \overline{V}_r + (R + jX)\,\overline{I}_r$.

(50%)

6. Anggap bahawa suatu penyuap utama tiga fasa 4.16 kV terbumi-wye padu mempunyai pengalir tembaga dengan jarak setara 37 inci di antara fasa pengalir dan faktor kuasa beban menyusul 0.9.

Assume that a three-phase 4.16 kV wye-grounded feeder main has copper conductors until an equivalent spacing of 37 inch between phase conductors and a logging load power factor of 0.9.

(a) Tentukan constant K bagi penyuap utama dengan menggunakan persamaan berikut.
Determine the K constant of the main by employing the following formula.

$$K \cong \frac{(r\cos\theta + x\sin\theta)(\frac{1}{3} \text{ x } 1000)}{V_r V_b}$$

iaitu K = f (saiz pengalir, penjarakan, cos θ, V_b)
other K = f (conductor size, spacing, cos θ, V_b)

dengan z = r + jx = impedans talian utama (Ω/mi-pemfasa), r = Ω/mi, x = x_a

where z = r + jx = main line impedance (Ω/mi-phasor), r = Ω/mi, x = x_a

+ x_d = Ω/mi, x = x_a Reaktans induktif dari suatu fasa pengalir, Ω/mi,

+ x_d = Ω/mi, x = x_a Inductive reactance of a conductor phase, Ω/mi,

...18/-

x_d = Faktor penjarakan reaktans induktif, Ω/mi

x = Reaktans talian per unit panjang, r = 1.503 Ω/mi,

x_a = 0.609 Ω/mi dan x_d = 0.1366 Ω/mi.

x_d = Inductive reactance of spoling factor, Ω/mi

x = Line reactance per unit length, r = 1.503 Ω/mi,

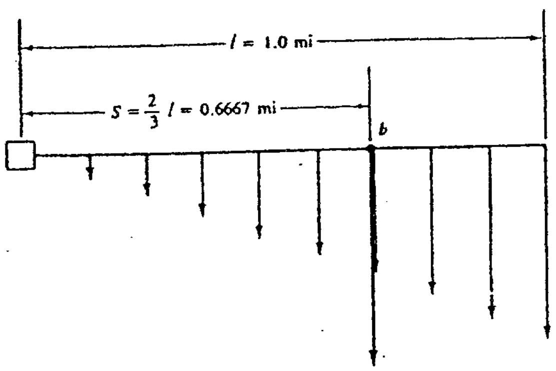
x_a = 0.609 Ω/mi dan x_d = 0.1366 Ω/mi.

(50%)

(b) Anggap bahawa penyuap yang tertera dalam Rajah 9 mempunyai ciri yang sama seperti dalam bahagian (a) di atas tetapi beban 500 kVA diagihkan secara pertambahan ketumpatan beban disepanjang penyuap utama. Hitung kejatuhan voltan dalam penyuap utama.

By assuming that the feeder shown in Figure 9 has the same characteristic as in part (a) above but 500 kVA load density and has an increasing lood along the feeder main. Calculate the percentage voltage drop in the main.

(50%)



Rajah 9
Figure 9

oooOooo

FORMAT JAWAPAN AKHIR (FOJA) [EEK 365]
(Mesti dikepilkan bersama Buku Jawapan)

ANGKA GILIRAN: ....

Jawapan Peperiksaan EEK 365 – Penghantaran Dan Pengagihan Sistem Kuasa
Semester II, Sidang Akademik 2005/2006 [April/Mei 2006]
Sesuaikan jawapan anda dengan unit-unit yang disediakan.

1. (a) 1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________ (10%)

(b) 1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________ (10%)

(c) 1. ____________ 2. ____________

3. ____________ 4. ____________

5. ____________ 6. ____________

7. ____________ 8. ____________

9. ____________ 10. ____________

11. ____________ 12. ____________

13. ____________ 14. ____________

15. ____________ 16. ____________

17. ____________ 18. ____________ (10%)

(d) 1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________ (10%)

(e) Sahkan ______________________________ (60%)

2. (a) (i) Faktor permintaan ____________

(ii) Faktor beban mingguan 20 min ____________ (30%)

(b) (i) Jumlah tenaga hilang teras ______________

(ii) Nilai jumlah tenaga hilang teras ____________ (30%)

(c) (i)

(Mesti dikepilkan bersama Buku Jawapan)

(ii) Vab ______________________

Van ______________________

(iii) V_{nx_2} ______________________ (40%)

3. (a) Arus talian $\bar{I}_a$ ______________________ (20%)

(b) Arus belitan sekunder I_{ba} ______________ (20%)

(c) Beban pada setiap transformer __________ (20%)

(d) Arus belitan primer I_{CB} ______________ (20%)

(e) Arus talian mengalir dalam wayar primer I_C

______________________________ (20%)

4. (a) Kawasan pembezaan dA ______________ (20%)

(b) Aliran beban kVA ______________ (20%)

(c) VD_2 ______________________ (30%)

(d) (i) Peruang substesyen ______________

(ii) Beban ke atas transformer substesyen

______________________________ (30%)

5. (a) Ungkapan Vr = ______________________

K = ______________________

tan δ = ______________________ (50%)

Assume a single-phase or balanced three-phase transmission or distribution circuit characterized by the $\overline{A}, \overline{B}, \overline{C}, \overline{D}$ general circuit constants, as shown in Figure 6. The mixed data assumed to be known, as commonly encountered in system design, are $|V_S|, P_r,$ and cos θ_r. Assume that all data represent either per phase dimensional values or per unit values.

As shown in Figure 7, taking phasor $\overline{V}_r$ as the reference,

$$\overline{V}_r = V_r\angle 0^\circ \quad (1)$$

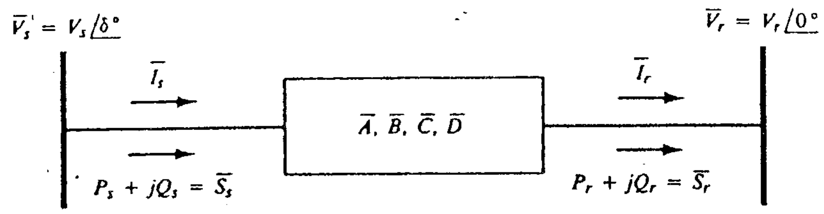
$$\overline{V}_s = V_s\angle \delta^\circ \quad (2)$$

$$\overline{I}_r = I_r\angle -\theta_r^\circ \quad (3)$$

where $\overline{V}_r$ = receiving-end voltage phasor

$\overline{V}_s$ = sending-end voltage phasor

$\overline{I}_r$ = receiving-end current phasor



Rajah 6
Figure 6

1

The sending-end voltage in terms of the general circuit constants can be expressed as

$$\overline{V}_s = \overline{A} \text{ x } \overline{V}_r + \overline{B} \text{ x } \overline{I}_r \qquad (4)$$

where

$$\overline{A} = A_1 + jA_2 \qquad (5)$$

$$\overline{B} = B_1 + jB_2 \qquad (6)$$

$$\overline{I}_r = I_r(\cos\theta_r - j\sin\theta_r) \qquad (7)$$

$$\overline{V}_r = V_r\angle 0^0 = V_r \qquad (8)$$

$$\overline{V}_s = V_s(\cos\delta + j\sin\delta) \qquad (9)$$

Therefore Eq. (4) can be written as

$$V_s \text{ x } \cos\delta + jV_s \text{ x } \sin\delta = (A_1 + jA_2)\, V_r + (B_1 + jB_2)(I_r \text{ x } \cos.\theta_r - jI_r \text{ x } \sin\theta_r)$$

from which

$$V_s \text{ x } \cos\delta = A_1 \text{ x } V_r + B_1 \text{ x } I_r \text{ x } \cos\theta_r + B_2 \text{ x } I_r \text{ x } \sin\theta_r \qquad (10)$$

and

$$V_s \text{ x } \sin\delta = A_2 \text{ x } V_r + B_2 \text{ x } I_r \text{ x } \cos\theta_r + B_1 \text{ x } I_r \text{ x } \sin\theta_r \qquad (11)$$

By taking squares of Eq. (10) and (11), and adding them side by side,

$$V_s^2 = (A_1 \text{ x } V_r + B_1 \text{ x } I_r \text{ x } \cos\theta_r + B_2 \text{ x } I_r \text{ x } \sin\theta_r)^2$$
$$+ (A_2 \text{ x } V_r + B_2 \text{ x } I_r \text{ x } \cos\theta_r - B_1 \text{ x } I_r \text{ x } \sin\theta_r)^2 \qquad (12)$$

Let

$$\hat{K} = V_s^2 - 2P_r\,[(A_1 \text{ x } B_1 + A_2 \text{ x } B_2) + (A_1 \text{ x } B_2 - B_1 \text{ x } A_2)\tan\theta_r] \tag{18}$$

Then Eq. (17) becomes

$$(A_1^2 + A_2^2)V_r^2 + (B_1^2 + B_2^2)(1 + \tan^2\theta_2)\frac{P_r^2}{V_r^2} - \hat{K} = 0 \tag{19}$$

or

$$(A_1^2 + A_2^2)V_r^2 + (B_1^2 + B_2^2)(\sec^2\theta_2)\frac{P_r^2}{V_r^2} - \hat{K} = 0 \tag{20}$$

Therefore, from Eq. (20), the receiving-end voltage can be found as

$$V_r = \left\{\frac{\hat{K} \pm [\hat{K}^2 - 4(A_1^2 + A_2^2)(B_1^2 + B_2^2) \text{ x } P_r^2 \text{ x } \sec^2\theta_r]^{1/2}}{2(A_1^2 + A_2^2)}\right\}^{1/2} \tag{21}$$

Also from Eq. (10) and (11).

$$V_s \text{ x } \sin\delta = A_2 \text{ x } V_r + B_2 \text{ x } I_r \text{ x } \cos\theta_r - B_1 \text{ x } I_r \text{ x } \sin\theta_r$$

and

$$V_s \text{ x } \cos\delta = A_1 \text{ x } V_r + B_1 \text{ x } I_r \text{ x } \cos\theta_r - B_1 \text{ x } I_r \text{ x } \sin\theta_r$$

where

$$I = \frac{P_r}{V_r \text{ x } \cos\theta_r} \tag{22}$$

Therefore

$$V_s \text{ x } \sin\delta = A_2 \text{ x } V_r + \frac{B_2 \text{ x } P_r}{V_r} - \frac{B_1 \text{ x } P_r}{V_r}\tan\theta_r \tag{23}$$

and

$$V_s \text{ x } \cos\delta = A_1 \text{ x } V_r + \frac{B_1 \text{ x } P_r}{V_r} - \frac{B_2 \text{ x } P_r}{V_r} \tan\theta_r \tag{24}$$

By dividing Eq. (23) by Eq. (24)

$$\tan\delta = \frac{A_2 \text{ x } V_r^2 + B_2 \text{ x } P_r - B_1 \text{ x } P_r \text{ x } \tan\theta_r}{A_1 \text{ x } V_r^2 + P_r \text{ x } B_1 - B_2 \text{ x } P_r \text{ x } \tan\theta_r} \tag{25}$$

or

$$\tan\delta = \frac{A_2 \text{ x } V_r^2 + P_r\,(B_2 - B_1 \text{ x } \tan\theta_r)}{A_1 \text{ x } V_r^2 + P_r\,(B_1 - B_2 \text{ x } \tan\theta_r)} \tag{26}$$

Equations (21) and (26) are found for a general transmission system. They could be adapted to the simpler transmission consisting of a short primary-voltage feeder where the feeder capacitance is usually negligible, as shown in Figure 8. To achieve the adaptation Eq. (18), (21) and (26) can be written in terms of R and X. Therefore, for the feeder shown in Figure 8, we have

$$[\bar{I}] = [\bar{Y}][\bar{V}]$$

or

$$\begin{bmatrix} \bar{I}_s \\ \bar{I}_r \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{bmatrix} \begin{bmatrix} \bar{V}_s \\ \bar{V}_r \end{bmatrix}$$