AN OPTIMAL DESIGN OF S^2-EWMA CONTROL CHART BASED ON MEDIAN RUN LENGTH (MRL) USING MARKOV CHAIN APPROACH

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by

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TABLE OF CONTENTS

CHAPTER 2 - LITERATURE REVIEW

CHAPTER 3 - METHODOLOGY

APPENDIX B

LIST OF TABLES

LIST OF FIGURES

LIST OF NOTATIONS

The list of the main abbreviations and notations used in this thesis:-

ARL	average run length
ARL_0	in-control average run length
ARL_1	out-of-control average run length
MRL	median run length
MRL_0	in-control median run length
MRL_1	out-of-control median run length
n	sample size
τ	magnitude of shift (σ_1 / σ_0)
σ_0	the nominal standard deviation of the process
σ_1	new process standard deviation
S_k^2	sample variance of kth subgroup
T_k	the "transformed" in-control range of the kth subgroup
$T_k^{'}$	the "transformed" out-of-control range of the kth subgroup
$A(n)$, $B(n)$, $C(n)$	the basic parameters of the logarithmic transformation
a, b, c	the three parameters of the logarithmic transformation
$f_{T_k}\left(t \middle| n\right)$	the pdf of T_k
$f_{T'_k}\left(t \middle| n\right)$	the pdf of $T_k^{'}$
Z_k	the EWMA statistic

Z_0	the initial value for Z_k
λ, K	the EWMA smoothing parameter and "scale factor" of the control chart
λ^*, K^*	the optimal values for λ and K that minimize the ARL/ MRL
UCL	upper control limit
LCL	lower control limit
UCL_S	upper control limit for S-chart
LCL_S	lower control limit for S-chart
$E(T_k)$	theoretical mean of T_k
$\sigma(T_k)$	theoretical standard deviation of T_k

LIST OF PUBLICATIONS

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Teh, S. Y., Khoo, M. B. C., **Ong, K. H.** & Soh, K. L. (2014). *A comparative study of the median run length (MRL) performance of the Max-DEWMA and SS-DEWMA control charts*. In T. I. Mohd, A. Syakila, & A. R. Rosmanjawati (Eds.). Proceedings of the 21th National Symposium of Mathematical Sciences. AIP Conference Proceedings (Vol. 1605, pp. 900-905). Melville, NY: American Institute of Physics Publishing. [ISBN: 978-0-7354-1239-2, ISSN:0094-243X, indexed & abstracted in Scopus and ISI]

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Teh, S. Y., Khoo, M. B. C., **Ong, K. H.**, Soh, K. L. & Teoh, W. L. (2015). A study on the S^2-EWMA chart for monitoring the process variance based on the MRL performance. *Sains Malaysiana*, *44*(7), 1067-1075.

SUATU REKA BENTUK OPTIMUM CARTA KAWALAN S^2-EWMA BERDASARKAN PANJANG LARIAN MEDIAN (MRL) DENGAN MENGGUNAKAN PENDEKATAN RANTAI MARKOV

ABSTRAK

Kualiti amat mustahak bagi sesuatu organisasi untuk terus bersaing dalam pasaran yang kompetitif pada hari ini. "*Six Sigma*" memainkan peranan yang penting dalam mengurangkan variasi proses dan memupuk budaya kualiti sesebuah organisasi. Hal ini merupakan usaha dari segi kualiti untuk memberi tumpuan kepada pengurangan kebarangkalian menghasilkan item yang cacat dalam proses pengeluaran dan bukannya mengurangkan jumlah kecacatan item yang terhasil dalam proses tersebut. Ia menyingkirkan variasi proses melalui pengenalpastian dan pengurangan sumber variasi. Carta kawalan merupakan sejenis alat yang boleh diaplikasikan dalam projek "*Six Sigma*". Suatu carta kawalan yang berkesan amat diperlukan dalam industri supaya titik yang berada di luar had-had kawalan boleh dikesan dengan kadar segera dan sumber variasi boleh disingkirkan dengan secepat mungkin daripada proses tersebut. Tesis ini mencadangkan reka bentuk optimum bagi carta kawalan S^2-EWMA berdasarkan kepada kriteria panjang larian median (MRL) sebagai carta kawalan kemajuan yang boleh diaplikasikan dalam projek "*Six Sigma*" kerana carta kawalan ini mempunyai daya kepekaan yang tinggi. Carta kawalan ini dapat mencegahkan proses daripada menghasilkan item-item yang cacat dan juga mengurangkan kos operasi seperti kos kerja semula dan kos bahan buangan. Kriteria MRL merupakan ukuran prestasi yang lebih berkesan kalau dibandingkan dengan sekadar bergantung kepada kriteria panjang larian purata (ARL). Suatu pendekatan rantai Markov ditubuhkan untuk menganalisis dan

mereka bentuk carta kawalan S^2-EWMA secara optima. Prestasi carta kawalan Shewhart S dan carta kawalan S^2-EWMA telah dinilai dan dibanding berdasarkan kepada kriteria ARL dan MRL. ARL dan MRL merupakan pengukur prestasi carta kawalan. Keputusan MRL (yang mana MRL_0 terkawal bersamaan dengan 200 dan MRL_0 terkawal bersamaan dengan 370) menunjukkan bahawa carta kawalan S^2-EWMA mempunyai daya kepekaan yang lebih tinggi untuk mengesan anjakan varians (τ) yang kecil dan sederhana kalau dibandingkan dengan carta kawalan Shewhart S klasik yang mana $\tau \in [0.50, 0.95]$ dan $\tau \in [1.05, 1.50]$ bagi saiz sampel (n) bersamaan dengan 3, 5, 7 dan 9. Manakala carta kawalan S^2-EWMA adalah setanding dengan carta kawalan Shewhart S bagi mengesan anjakan varians yang besar.

AN OPTIMAL DESIGN OF S^2-EWMA CONTROL CHART BASED ON MEDIAN RUN LENGTH (MRL) USING MARKOV CHAIN APPROACH

ABSTRACT

Quality is necessary for business organizations to survive in today's competitive market place. Six Sigma plays a vital role when it comes to reducing process variations and enhancing quality culture for an organization. It is a quality initiative that focuses on reducing the probability of delivering defective items within a production process instead of reducing the number of defective items produced in a process. It eliminates process variation by identifying and reducing the source of variations. Control chart is a tool that can be applied in Six Sigma projects. An effective control chart is necessary for the industry so that the out-of-control points can be detected promptly and the source of variations can be removed as soon as possible from a process. This thesis proposes an optimally designed S^2-EWMA control chart based on median run length (MRL) criterion as an improvement chart that can be applied in Six Sigma project due to its good sensitivity. The chart helps to prevent a process from delivering defective items as well as reducing operations cost such as rework and scrap costs. Instead of relying solely on average run length (ARL) criterion, the MRL criterion is indeed a better performance measure for control charts. A Markov chain approach is established to optimally analyze and design the S^2-EWMA control chart. The performances of the Shewhart S control chart and S^2-EWMA control chart are evaluated and compared based on both ARL and MRL criteria where ARL and MRL are the performance measures for control charts. The MRL results (i.e. in-control MRL_0 equals to 200 and in-control MRL_0 equals to

370) show that S^2-EWMA control chart has higher sensitivity in signalling out-of-control signals compared to the classical Shewhart S control chart in detecting small and moderate process variance shifts (τ) where $\tau \in [0.50, 0.95]$ and $\tau \in [1.05, 1.50]$ when sample sizes (n) equal to 3, 5, 7 and 9. Whereas, the S^2-EWMA control chart is comparable to the Shewhart S control chart in detecting large process variance shifts.

CHAPTER 1

INTRODUCTION

1.1 Overview

Operations management plays a big part in terms of planning, managing and organizing business activities in the business world today. According to Waters (2006), all organizations supply products. Regardless of tangible goods or intangible benefits that end users experienced, every product includes some combination of goods and services that brings a series of benefits. Operations are the key activities which focus on making or producing an organization's products. Generally, an organization will collect a variety of inputs, using operations to transform them and produce a range of outputs.

This is further supported by Russell and Taylor (2009) as according to them, operations management does not only design but also operates and improves productive systems. Operations can be described as a transformation process as it is a function or system which changes inputs into outputs of greater value. In order to survive in the competitive market today, products sold at the lowest price is no longer the customer's main concern. The consumer market today is experiencing an ever increasing demand for products with high quality. Customers would rather go for products with the right quality and the right price. Quality now plays an important role as it contributes to customer satisfaction and eventually to customer loyalty (Waters, 2006). Thus, organizations need to improve continuously in terms of quality to remain competitive in the market.

Continuous improvement (CI) is an important strategy in improving the performances for an organization (Witell et al., 2005). CI can be defined as a culture of sustained improvement that targets at eliminating waste in every systems and processes for a business (Bhuiyan and Baghel, 2005). CI can be classified as either an innovative step change (process re-engineering) or small incremental change (kaizen). The Deming wheel or in other words, the plan-do-check-act (PDCA) cycle forms the essential of virtuous cycle of improvement. It is the fundamental requirement for today's quality management systems (Russell and Taylor, 2009). The PDCA cycle can be explained as below:

- Plan - study existing situation and identify changes for improvement.
- Do - implement the plan on trial basis, evaluate the improvement.
- Check - test effect of changes to check on whether the desired result is accomplished.
- Action - standardise the change on a permanent basis.

According to Martins and Toledo (2000), total quality management (TQM) can be viewed as a practice that stresses on CI and customer focus in an organization which helps to be the source of competitive advantage. In order to produce a quality product, TQM is important (Russell and Taylor, 2009). TQM originated as a Japanese-style management approach to quality improvement in 1980s. In the context of TQM, the word "total" means that it has to include everything and everybody in an organization, while "quality" is defined as meeting the customer's expectations and "management" contains support from the top management and leadership (Toremen et al., 2009). TQM is an integrated management philosophy which aimed to accomplish and exceed customer’s expectation through CI on the performance of products, processes or services (Bayazit, 2003). It aims to increase customer’s satisfaction and at the same time minimizes the production cost (Khan, 2003). In short, TQM requires active involvement, participation and cooperation of everyone in an organization to monitor all the activities and operations (Russell and Taylor, 2009).

Besides, Six Sigma is another methodology that stresses on CI. Six Sigma is an approach which aims to achieve excellent organizational performance through continuous process improvement (Antony et al., 2012; Savolainen and Haikonen, 2007). It is a methodology that aims to produce less than 3.4 defects or errors per million opportunities. Referring to Banuelas and Antony (2004), Six Sigma is a process that develops and delivers virtually perfect products or services through its applications on project basis. Six Sigma eliminates variations by identifying and reducing the source of variations. Variation is the extent of how much a product differs from one another. At the heart of Six Sigma, there are five important steps to improvement projects. They are define, measure, analyze, improve and control (DMAIC) are explained as follows (Russell and Taylor, 2009):

- Define - Identifying and defining what is the problem, what needs to improve.
- Measure - Measuring the process, collect data and compare to the desired state.
- Analyze - Analyze the data to identify the root cause of a problem.
- Improve - Brainstorm, make changes and measure results to check on whether the problems have been removed.
- Control - Monitor to ensure that the improvement is sustained when the performance of the process is operating at a desired level.

Martorell et al. (2011) outlined that business decisions in organizations are no longer made based on opinions, it has to be made based on facts. Therefore, the process has to be measured, data must be collected and analyzed in order to understand and improve upon the current process. This is where Statistical Quality Control (SQC) takes place. According to Pan (2006), the principle of SQC is to monitor the process for the consistency of products so that products do not differ too much from the requirements defined by customers. SQC can be categorised into three broad categories. They are descriptive statistics, statistical process control (SPC) and acceptance sampling.

Process control is important to produce quality products through reducing variation (Pan, 2006). SPC is a major quality control tool and is one of the greatest technological developments as it is easy to use. It has major impact with sound underlying principles and is applicable to any process (Montgomery, 2009). SPC can be said to be the most powerful tool in terms of reducing variations in a process (Antony et al., 2000). Castagliola et al. (2007a) further agreed that SPC is a statistical technique used to monitor and control processes. Prajapati and Mahapatra (2008) however said that SPC is an analytical decision making tool that helps organizations check whether a process is working correctly and tells when it is not.

The main difference between SQC and SPC is that SQC is the scientific method that focuses on data analysis. For SQC, measures are taken accordingly in order to maintain the product's quality. SPC on the other hand is one of the techniques used to monitor and provide feedbacks on process. The feedback is then used to maintain and improve the process capability and ensure product produced conforms to its design (Srinivasu et al., 2009).

According to Vassilakis and Besseris (2010), a process can be controlled by taking samples periodically from the process. Plotting the sample points on a control chart helps practitioners to decide whether the process is within statistical control limit(s). Russell and Taylor (2009) highlighted that a sample can consist of either a single item or a group of items. A process is considered out-of-control if a sample point is plotted outside the limit(s). Thus, the root cause must be identified immediately to correct the problem. The process will continue without interference if the sample is within the control limits. Nevertheless, the process will still be continuously monitored to prevent quality issues. In this way, quality problems are identified and solved by correcting the process before defective products are being produced.

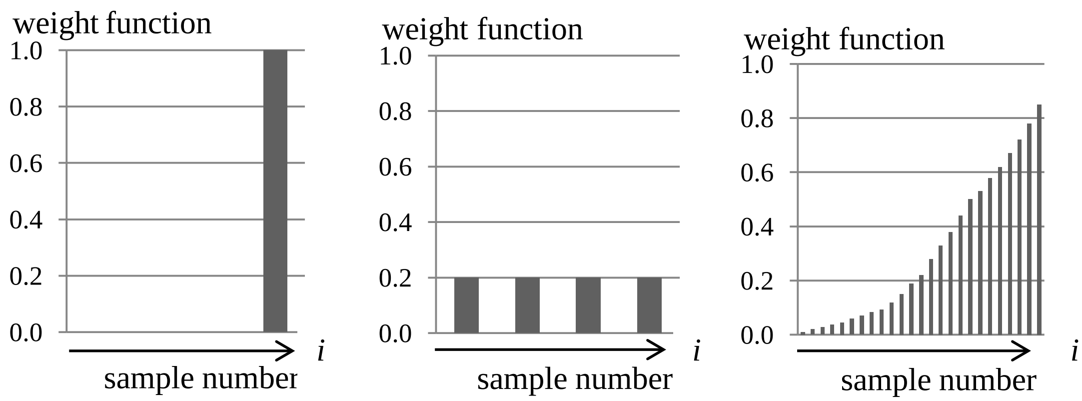
Montgomery (2009) proposed four well known control charts which are used within the manufacturing industry. They are:

- Control chart for variables. This chart is used when a quality characteristic can be measured based on a continuous scale. Examples include aspects such as width or length, volume and temperature. The $\overline{X}$ -, *R*- and *S*-charts are widely applied in this case. The $\overline{X}$ -chart uses the process mean, *R*-chart is based on the range of sample taken and the *S*-chart uses the standard deviation; they all identify whether a process is in-control.

- Control chart for attributes. This chart is used when the quality characteristic cannot be represented by a number. It has the advantage of considering several quality characteristics together and if the unit fails to meet the specification on any one characteristic, then it will be classified as nonconforming. The *np*-chart, *p*-chart, *u*-chart and the *c*-chart are normally used as attributes control charts. The *p*-chart uses the proportion defective in a sample to determine whether the process is in-control, while the *np*-chart uses the amount of defectives in each subgroup, *c*-chart uses the number of defective items and the *u*-chart uses the number of defects per unit sampled in each subgroup. Normally, the *c*-chart is used when the size of the subgroup is constant while the *u*-chart is used when the size of the subgroup varies (Gebus, 2000). Generally, the variables control chart and the attribute control chart are referred to as the Shewhart control chart.

- Cumulative sum (CUSUM) control chart. The CUSUM control chart has an advantage of increased sensitivity as the entire sample is used to conclude on a process. On the other hand, the Shewhart control chart tends to be insensitive to shifts on the order of 1.5 standard deviations or less because only data of the most recent measurements are used to conclude about the process.

- Exponentially weighted moving average (EWMA) control chart. The EWMA control chart was first introduced by Roberts (1959). EWMA chart is a chart much like the CUSUM chart. The difference between EWMA and CUSUM chart is that recent measurements will be weighted more heavily for EWMA chart, whereas all measurements are weighted equally for CUSUM chart. EWMA chart detect much smaller process shifts than a normal control chart would as it utilizes information from entire sample. Figures 1.1 (a) – 1.1 (c) display the difference of weighted function for Shewhart, CUSUM and EWMA charts, respectively. Note that λ is the weight in designing the S^2-EWMA for this study and it is selected optimally via the Markov chain approach. Beyond these four types of control charts, the control charts will become complicated or in other words, there will be mixtures of more than one type of control charts.



(a) Shewhart (b) CUSUM (c) EWMA

Figure 1.1. The weighted functions for the Shewhart, CUSUM and EWMA charts

Source: Cheng et al. (2007)

Traditionally, the samples in control charts will be based on a fixed size *n*, with a fixed sampling interval *d*, between successive sampling points. In the year 2005, the fixed sample size and interval (FSSI) S^2-EWMA (called S square exponentially weighted moving average) control chart was proposed by Castagliola. Later on, a variable sampling interval (VSI) which is the extension for the FSSI S^2-EWMA chart was developed by Castagliola et al. (2007b). After a few years, Castagliola et al. (2008) discussed the construction of a variable sample size (VSS) version of a static FSSI S^2-EWMA chart to monitor the process dispersion. VSS and VSI control charts changes the sampling rate in a process.

1.2 Importance of Statistical Process Control (SPC) Chart

Quality Management cannot be assured when there is no application of correct tools or no proper statistical method to follow in an organization (Ahmed and Hassan, 2003). The implementation of Six Sigma quality grows as a means to improve company's competitive position since the last two decades (Aboelmaged, 2011). Six Sigma projects lead to quality improvement. Advanced technologies, new philosophies and the latest statistical methods must be used to produce high-quality products at low cost. Booker (2003) cited that the Six Sigma statistically based problem solving methods are strong in dealing with variations. Besides, according to Antony (2011), Six Sigma thinking needs to be supported by the learning and application of a variety of tools and techniques, not by simple attitude changes or will power. By delivering reliable data that drives good solutions, the statistically based problem solving methodology of Six Sigma produces impressive bottom line results. A Six Sigma DMAIC process control phase is the key to keep up the improvements secured from projects. It requires a lot of hard work to effectively execute sustainable control methods.

Control chart is one of the commonly used tools in the DMAIC cycle (Rantamaki et al., 2013). Besides, control chart is one of the statistical tools that have been widely used for quality control purpose (Xu, 2001). Control chart aids process improvement and process control in DMAIC cycle for Six Sigma (MacCarthy and Wasusri, 2002). Chan et al. (2003) cited a control chart consists of a graph where measurements such as the sample mean, sample median, sample range or sample standard deviation which characterizes the product quality are plotted on the vertical axis and time on the horizontal axis. Control limits help to ensure the stability of a process. When points are spotted outside the chart’s control limit(s), the process is considered out-of-control. In other words, a control chart helps to determine whether variations happened are due to a specific correctable cause (assignable cause) or an inherent variability of the process (common cause).

Thus, with these characteristics, control charts help management team in making managerial decisions (such as decisions related to costs, profits and revenues) and they ease the engineering team to make engineering decisions (such as decisions related to process efficiency) when a process goes out-of-control. Besides, as control limit needs to be set in the control charts, this also helps the management team in defining the customer’s needs and requirements. Therefore, the products will meet customers’ expectations.

Since quality plays an important role in both servicing and manufacturing organizations to gain competitive advantage for their businesses, according to Tsai and Lin (2009), control chart indirectly helps to improve product quality. This is because quality is inversely proportional to variability (Montgomery, 2009). Variations in a certain process often impact the quality of products. The objective of control chart application is to remove assignable causes of variation in the process so that stability can be achieved for the process (Kawamura et al., 2012). Variation reduction and process stability helps to increase quality of product which will then contribute to increasing customer satisfaction and loyalty (Waters, 2006).

According to Guh (2002), in a process, the special disturbances (out-of-control) data can be separated from inherent variability (in-control) data by using a control chart. This separation eases the finding and correction of production problems thereby always results in improving the quality of products and at the same time reduces scrap and rework costs. This is further supported by Shamsuzzaman and Zhang (2012) as they mentioned that control charts are popular in the industries in terms of improving productivity. The number of defectives can be reduced as a good control chart helps signalling the out-of-control cases quickly. Defectives, scrap or rework are the key productivity-killers in operations of any businesses (Shamsuzzaman and Zhang, 2012). The productivity for an operation increased when rework and scrap are reduced.

1.3 Applications of Statistical Process Control (SPC) Chart

SPC chart has been historically used to examine both the quality of manufacturing processes and the quality of services. A few examples of real world applications of SPC chart will be discussed in this section chronologically. SPC charts can be used in the field of chemical industry, mobile device manufacturing, health care, pharmaceutical company, supermarket chain's suppliers, radiotherapy, software industry and media industry.

SPC chart is useful in enhancing product quality at Slave Lake Pulp Corporation, Alberta (Ho and Henriksson, 1993). Started in December 1990, the Slave Lake Pulp Corporation is a company that works on greenfield bleached chemi-thermo-mechanical pulp (BCTMP) mill. With the help of SPC chart, the operator's understanding of the process has improved. Besides, the response time required to return the process to target conditions has reduced. Overall result shows that by using the SPC chart, the freeness and brightness of the products have been improved by 26 % and 60 % respectively.

Gebus (2000) mentioned that SPC chart has been used as a tool to improve the quality of manufacturing in Nokia. Nokia as one of the popular mobile phone manufacturers utilizes the application of SPC chart in order to understand the process and its problems, achieve faster detection of problems and even motivate line workers. SPC provides line workers a clear picture on the process control and thus they develop a greater interest to involve themselves in quality assurance efforts.

SPC chart is also useful in health care. MacCarthy and Wasusri (2002) highlighted SPC charts can be used to check on the performance of a health care process. For example, the performance of a health care unit such as pneumonia clinic; the control of asthma in a patient and the satisfaction levels of a patient with a particular health care unit such as the outpatients clinic can be monitored via the SPC charts.

SPC chart is applicable in the quality assurance of radiotherapy field too (Gerard et al., 2009). In the article, one of the radiotherapy techniques called the Intensity-Modulated Radiation Therapy (IMRT) was introduced. With the help of SPC chart, the reliability and the quality of pre-treatment quality controls of IMRT was evaluated and improved. The objective of the study is to determine whether the controls can be reduced and at the same time quality can be maintained. The findings of the study indicate that SPC chart significantly contribute in improving treatment process security. Therefore, SPC also contributes in the IMRT quality controls.

According to Evans and Lindsay (2011), a control chart has high contributions in identifying assignable causes and improving manufacturing process. For example, the manufacturing process for a pharmaceutical company that produces a self-contained syringes with a single dose of an inject-able drug requires the containment cap to be tacked accurately at a desired length of the syringe. When a syringe is tacked by the containment cap, the measurement of the length of the syringe always shows excessive variations. This causes potential loss of cap or cartridge during handling and shipment. To prevent this, a 100% inspection has to be done on the tacked syringes, this process will increase the production cost. Thus, for this case, the $\overline{X}$ -chart and R-chart can be applied to detect unacceptable variations early so that corrective actions could be undertaken.

Mahanti and Evans (2012) presented a paper on SPC chart in the software industry. The objective of the paper is to understand and identify the critical success factors for successful deployment of SPC in the software industry. From the study, it is shown that commitment and involvement from the management are the most critical success factors. Then, the selection of control charts came into the picture.

Furthermore, the delivery chain can be monitored using the multivariate control chart too (Faraz et al., 2013). In the paper, the multivariate T^2 control charts are designed via the economic-statistical design method. When the control chart indicates delay on the delivery time, managers may make further plans to reduce delays. It is found that the application of this control chart does not only benefit the company economically, it also increases customer satisfaction.

Last but not least, control charts can be also applied to control and improve quality of bolt. Touqir et al. (2014) shows how to develop the $\bar{X}$ -, *R*- and *S*-charts to inspect the height, diameter and weight for bolts. Figure 1.2 and Figure 1.3 show the application of $\bar{X}$ chart in controlling the height and diameter of bolts. When a sample point falls outside the control limit(s), an out-of-control point is indicated. Root cause for this out-of-control point has to be investigated and removed from the process. Referring to Figures 1.2 and 1.3, it can be seen that there is no sample point falling outside the control limits. Hence, the process is in-control.

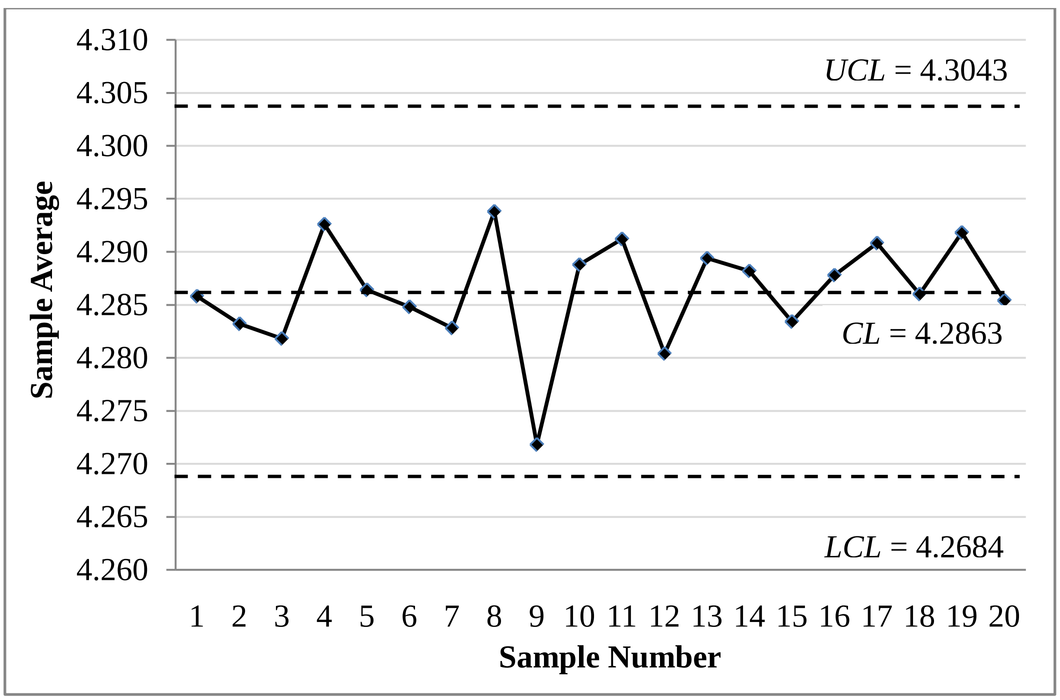


Figure 1.2. Application of $\bar{X}$ chart in controlling the height of bolts (in cm)

Source: Touqir et al. (2014)

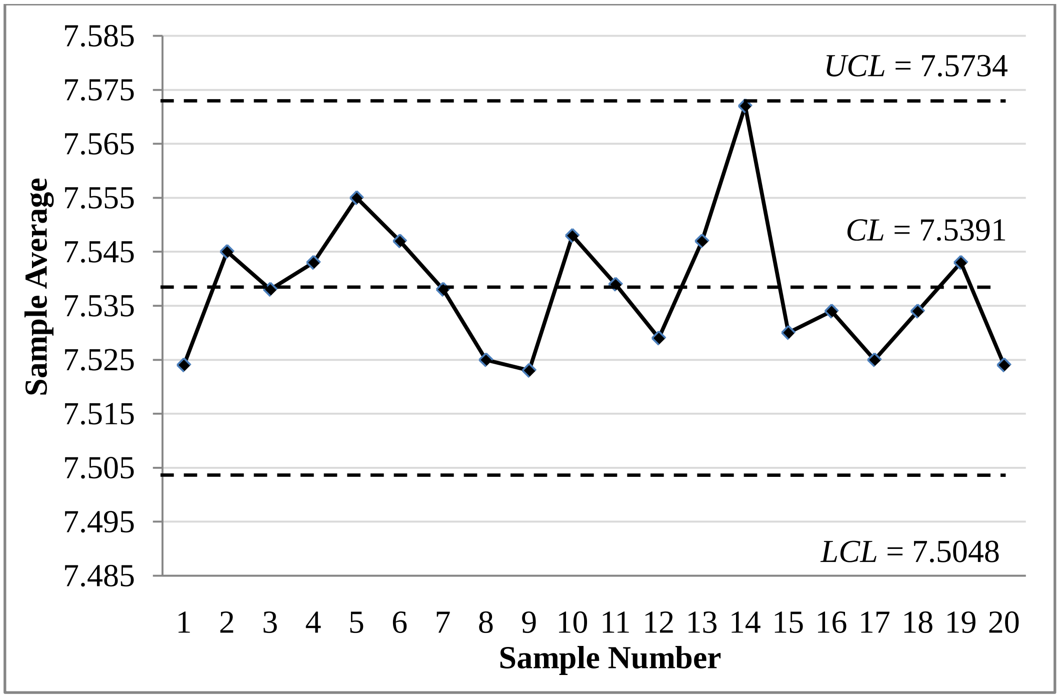


Figure 1.3. Application of $\bar{X}$ chart in controlling the diameter of bolts (in cm)

Source: Touqir et al. (2014)

1.4 Applications of Exponentially Weighted Moving Average (EWMA) Chart

As mentioned in Section 1.3, the EWMA chart is efficient in detecting small process shifts. Instead of solely discussing the applications of SPC chart, this section focuses on the applications of the EWMA charts. The EWMA $\bar{X}$ -R chart combines both $\bar{X}$ chart and R chart into a single chart. This type of chart provides an equal chance of signalling a decrease or an increase in the process mean and process variance. It also enables the VSS problem to be handled easily. In other words, the benefit of this EWMA $\bar{X}$ -R chart is that only a single chart is needed for monitoring both process mean and variance shifts while retaining the desirable properties of the EWMA chart. When a single chart is used, the operations and design of the monitoring scheme could be simplified. EWMA control chart could be widely applied in the manufacturing industry. As an example, the EWMA

$\overline{X}$ -*R* chart is used in controlling the filling process that fills 600 ml bottle with a certain type of liquid (Khoo et al., 2010).

Besides, the EWMA control chart performs very well in monitoring small process shifts. According to Shamsuzzaman and Zhang (2012), EWMA control chart could be used to minimize the mean number of defective units (denoted by MD) for manufacturing processes. For example, both the Shewhart MD- $\overline{X}$ and MD-EWMA charts are applied to monitor the production of a special type of bearing. Although both the charts have the ability to control a manufacturing process, however, it is found that compared to the Shewhart MD- $\overline{X}$ chart, the average number of defective units produced is further reduced by 22% using the MD-EWMA chart. Moreover, the reduction of number of defective units also contributes to lowering the quality cost of the organization immediately (Shamsuzzaman and Zhang, 2012). This further reduction is attributable to the optimization of the weight factor λ as well as the charting parameters such as the sample size, sampling interval, Lower Control Limit (*LCL)* and Upper Control Limit (*UCL*). Although the design of the EWMA chart is much more complicated compared to the traditional Shewhart chart, studies have found from an overall point of view, the MD-EWMA chart outperforms the Shewhart MD- $\overline{X}$ chart.

In addition, EWMA charts could also be used to evaluate and monitor the environmental performance (Liu and Xue, 2015). Environmental pollution leads to drastic changes such as the depletion of ozone layer, natural disasters and global warming. As a result, there is a need for organizations to continuously monitor their environmental performance indicators so that it will not go beyond the regulatory limits. Referring to Liu and Xue (2015), ML-EWMA chart is optimally designed to monitor the environmental pollution processes. From the study, it is found that the ML-EWMA chart is a good choice to monitor environmental data as it is very effective in reducing loss in environmental pollution process. Note that the loss function is used broadly to estimate the cost caused by poor quality. The smaller the loss, the better the product quality is. Therefore, reducing the loss corresponds with the effort of continuous quality improvement. Likewise, the loss function can also be applied in monitoring the environmental pollution process. The smaller the loss, the better it is in monitoring environmental data.

In short, the study on the construction of EWMA control charts is very important and worthy of various industry applications be they manufacturing or service. Practitioners must carefully and clearly define the process to be measured and select the most appropriate type of control chart to measure the process. A proper selection of control chart helps to analyze the pattern of process variation accurately. Based on the out-of-control signal, the root cause of the out-of-control signal should be investigated and removed from the process. This effort will contribute to process improvement and maintain good process control. All these are essential for organizations in delivering high quality products to customers.

1.5 Problem Statement

As mentioned in Section 1.2, the purpose of using a control chart is to remove assignable causes of variation in the process so that a process can achieve stability (Kawamura et al., 2012). A good control chart signals the out-of-control points promptly so that the number of defective units in a process can be reduced. ARL is the average number of the successive control chart points that has been plotted before a point is detected outside the control limit(s). Traditionally, ARL is widely used in measuring the performance of control charts.

However, Cox (2010) outlined that the ARL sometimes brings false alarms. This is due to the in-control run length distribution of an EWMA chart is highly skewed. When the magnitude of the shift in the variance change, the shape of the run length distribution could also varies. Therefore, the MRL gives a better explanation regarding the in-control and out-of-control performances of a control chart compared to ARL (Gan, 1993a). Teoh and Khoo (2012) further supported this as according to them, the skewness of the run length distribution changes with the size of the process mean shifts. When the standard deviation of run length is large, it is an indication that the data points are distributed over a large range of values (Montgomery, 2009). Thus, ARL is not necessarily a good representative of the run length distribution. Instead, when the standard deviation is large, the MRL is better in terms of providing a more reliable interpretation for the in-control and out-of-control performances of a control chart.

Chin and Khoo (2012) propose the optimal design of EWMA *t* chart based on the MRL instead the ARL to monitor process mean. This EWMA *t* chart can be applied to monitor torque measurements for a screwing process in car radio manufacturing. For example, when a process is in-control, the in-control MRL (MRL_0) of 200 signifies that at half (or 50%) of the time, an out-of-control signal of failure screwing process will be signalled by the 200th sample. In contrast, for in-control ARL (ARL_0) of 200, an out-of-control signal of failure screwing process will be given by the 200th sample point at 60%

to 70% of total time due to the highly skewed in-control run length distribution. Therefore, the MRL measurement is more reliable.

However, no work has been done on S^2-EWMA control chart in monitoring process variance based on MRL measurement. The FSSI S^2-EWMA chart proposed by Castagliola (2005) is designed optimally based on the ARL. The need for a more reliable measurement is the motivation of this study to develop a Markov chain procedure to optimally design the FSSI S^2-EWMA chart for the process variance of Castagliola (2005) using MRL requirements described in Gan (1993a, 1993b and 1994). This thesis aims to propose the S^2-EWMA chart based on MRL as an improvement chart that has better sensitivity and able to prompt out-of-control signals accurately.

1.6 Objectives of the Thesis

The following objectives are targeted:

i. to construct an optimally designed fixed sample size and interval (FSSI) S^2-EWMA chart based on MRL using Markov chain approach.

ii. to study the MRL and ARL performances of the S^2-EWMA chart in the detection of various sizes of process variance shifts.

iii. to compare the performances of the S^2-EWMA and *S*-charts based on MRL.

iv. to provide an optimally designed procedure and tables of optimal parameters to assist practitioners in the design of an optimal S^2-EWMA chart based on MRL accurately.

1.7 Research Questions

The findings of this study attempts to answer the following questions:

i. How to establish a Markov chain model which is able to optimally designed FSSI S^2-EWMA chart based on MRL?

ii. Does the MRL performance of the S^2-EWMA chart perform better than the ARL performance in the detection of various sizes of process variance shifts?

iii. Does S^2-EWMA chart perform better than the *S*-chart based on MRL measurement?

iv. What are the possible procedures to optimally design an optimal S^2-EWMA chart based on MRL and what are the corresponding optimal parameters?

1.8 Organization of the Thesis

The organization of this thesis is as follows:

Chapter 2 discusses the overview of Six Sigma, works on statistical related Six Sigma projects and existing literatures of EWMA charts. Besides, a review on the *S*-chart and S^2-EWMA chart will also be described. Then, the performance measures of the *S*-chart and S^2-EWMA chart i.e. the ARL and MRL will also be explained in this chapter.

Chapter 3 justifies the use of Monte Carlo simulation approach to compute the MRL and ARL of the *S*-chart and also the use of Markov Chain approach to compute the MRL and ARL of S^2-EWMA chart. Besides, it also describes the optimal design of the *S*-chart based on MRL and ARL. Furthermore, this chapter includes the optimal design of the S^2-EWMA control chart based on MRL and ARL as well.

In Chapter 4, the MRL and ARL performances of S^2-EWMA chart will be presented together with their optimal combination of parameters. The MRL performances of S^2-EWMA chart will be compared to the ARL performances of S^2-EWMA chart. Then, the performances of the MRL and ARL for both the *S*-chart and S^2-EWMA chart in detecting process variance shifts will also be presented, studied and compared. Furthermore, graphical illustration will also be shown in this chapter so that the readers can have a better understanding on this study.

Lastly, Chapter 5 consists of conclusions, contributions and suggestions for further research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Variations exist in all types of processes. Six Sigma plays an important role in reducing variations and creating quality culture (Nakhai and Neves, 2009). Control chart however is a tool that can be applied in the Six Sigma DMAIC cycle. This chapter will be discussing the Exponentially Weighted Moving Average (EWMA) type control charts and the performance measures for the *S*-chart (called standard deviation chart) and S^2-EWMA (called *S* square exponentially weighted moving average) chart. Section 2.2 will explain on the overview of Six Sigma and Section 2.3 will describe the previous works on statistical related Six Sigma projects. Then, Section 2.4 will be a brief review on previous work on EWMA charts. Sections 2.5 and 2.6 will contain detailed explanations for *S*-chart and S^2-EWMA chart. Lastly, the performance measures of *S*-chart and S^2-EWMA chart based on average run length (ARL) and median run length (MRL) will be explained in Section 2.7.

2.2 Overview of Six Sigma

Manufacturing process variation is the central issue to achieve good product quality. It is associated with higher prices, higher costs and customer dissatisfaction (Tannock et al., 2007). Understanding variation is essential in making good decisions (Grigg and Walls, 2007). Variations can only be reduced when one understands what variation is and how variation happens. Traditionally, products are inspected at the end of the line. According to Antony and Taner (2003), the quality control approach which is based on inspection is rather reactive due to defective items are produced before they are found. This will contribute to higher scrapping and rework costs and possibilities are also there that bad items are accepted and good items are rejected. Checking on whether the products pass or fail the inspection is not informative enough in terms of continuous improvement (CI) of process or product quality. Besides, Booker (2003) also cited that quality is best assured by reducing the deviation from the target. Instead of inspecting the products, quality should be designed into the product. Six Sigma philosophy contributes to the realization that variations have significant impacts on a process (Nakhai and Neves, 2009). Management should focus on improving a process instead of calculating the defect rates for a process (Mehrjerdi, 2011).

Introduced by Bill Smith from Motorola in mid 1980s, Six Sigma plays an important role in improving products quality. Basically, Six Sigma quality level is associated with 3.4 defects per million opportunities (DPMO). It is a quality initiative that strives to reduce variations in a process and lower the production cost at the same time (Deshmukh and Chavan, 2012). Six Sigma focuses on the number of opportunities which causes defects within a process instead of the number of defects produced in a process (Aboelmaged, 2010). By eliminating the causes of quality issues, Six Sigma prevents defects even before the defective units are produced. Since the introduction by Motorola in 1980s, companies such as IBM, Allied Signal and General Electric also adopted Six Sigma to expand employees' skills, improve work processes and deliver high-level strategic results (Antony et al., 2012). In addition, other than the manufacturing industry, Six Sigma is also well established in service organisations such as hospitals, banks, financial services, the utilities services and airline industry (Antony et al., 2007). Referring to Aboelmaged (2010), Six Sigma is now applied in many industries and a lot of organisations worldwide have adopted Six Sigma methodology and tools to fit into their own operations.

There are five important improvement steps for Six Sigma projects. They are define, measure, analyze, improve and control (DMAIC) cycle. The explanations for DMAIC can be found in Section 1.1. As for Six Sigma working teams, it is all about teamwork (Gutierrez et al., 2009). There are roles such as "Champion", "Master Black Belt", "Black Belt" and "Green Belts" in a Six Sigma team. "Champion" usually sponsor the projects. "Master Black Belt" monitors, reviews and guides "Black Belt" across all projects. "Black Belt" is project leader while "Green Belts" are project members. Antony and Fergusson (2004) highlighted that Six Sigma has both technical and management components. The main focus of technical component is on creating data which gives explanations on process variations, using statistical techniques and tools for problem solving and improving processes by reducing variation. As of management component, the main focus is on choosing and assigning the right people for Six Sigma projects, selecting suitable process metrics, providing sufficient resources for Six Sigma training and giving clear guidance and direction with regard to project selection.

Braunscheidel et al. (2011) mentioned that institutional theory can be used to explain why more and more organizations nowadays adopted Six Sigma. Institutional theory proposes that other than customers and resources, organizations need to cope with the stress to conform with shared notions of proper forms and behaviours. By violating the shared beliefs, it will cause organizations to lose its ability to secure social support and resources. Furthermore, organizations also adopted Six Sigma for better organizational performance. Among the benefits brought by Six Sigma are listed in following table.

Table 2.1

Benefits of adopting Six Sigma projects

Benefits	**Author and Year of Publication**
Increase profitability for organizations	Antony et al. (2012); Deshmukh and Chavan (2012); Iwaarden et al. (2008); Kumar et al. (2007).
Savings on operational costs	Deshmukh and Chavan (2012); Kumar et al. (2007); Lee and Choi (2006).
Aids management team in decision making	Antony et al. (2012); Gijo et al. (2011).
Improve process performance	Antony et al. (2012); Banuelas and Antony (2004); Kumar et al. (2007); Lee and Choi (2006).
Improve productivity	Antony et al. (2012); Dedhia (2005).
Improve product quality	Banuelas and Antony (2004); Goh (2002); Kumar et al. (2008).
Reduce defect rates	Gijo et al. (2011); Kumar et al. (2008).
Improve customer satisfaction	Dedhia (2005); Krueger et al. (2014); Kumar et al. (2007); Lee and Choi (2006).